RV003 - Squat Shouldered Jar

An Exploration of Precision
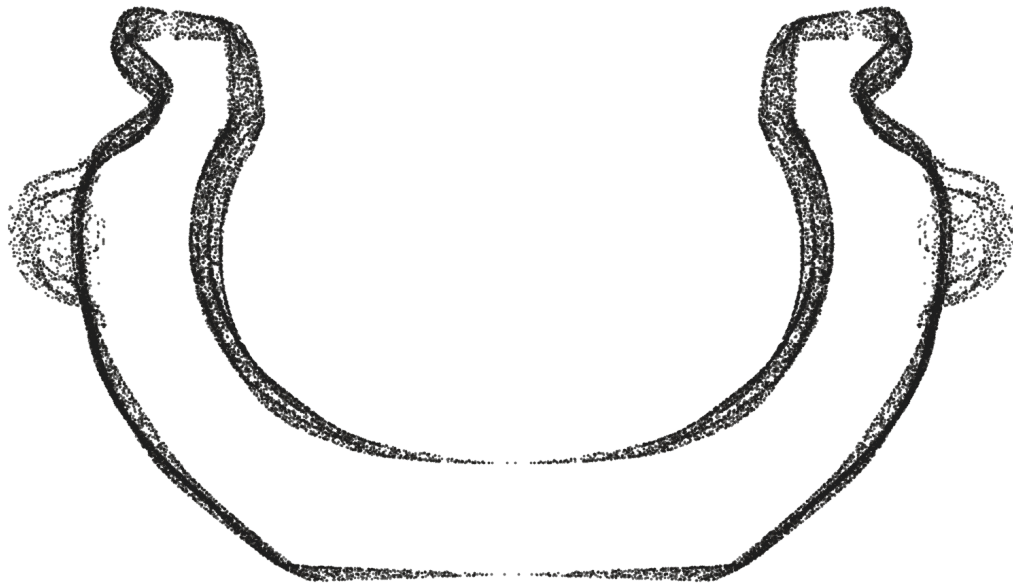
Scientists Against Myths

Contents

Artifact Information

Artifact Data

Collection	In private collection
Provenance[1]	NA
Provenience[2]	NA
Attribution	NA

Art dealer information

Ref.	
Description	Modern replica - Produced by Olga Vdovina and Yulia Gukasova in collaboration with Scientists Against Myths
URL	https://www.youtube.com/watch?v=dC3Z_DBnCp8

Maijers vessel classification[3]

Short classification	Squat Shouldered Jar
Long classification	The vessel is created in a closed form classified as a squat jar with a shouldered shape, it has a footed base and a raised blunt rim.

Physical properties

Precision score[4]	1.52	
Height (approximate)	83 mm	3.27 in
Width (approximate)	126 mm	4.96 in
Material	Marble breccia	
Mohs Hardness[5]	5 - 7 (Breccia)	
Weight		

Scan information

Source	Scanned by Scientists Against Myths
Source file name	SAM_Vaza_2_Model_00.obj
Scan method	Unknown
Scanner	Unknown
Rated scan accuracy	Not specified
Scan date	Unknown
Scanned by	Unknown
Mesh decimation	Unknown
Number of vertices	261 712
Mesh density[6]	342 µm | 13.47 thou
Max vertex distance	978 µm | 38.496 thou
Min vertex distance	0 µm | 0.000 thou
Vertices per cm2	456 (approximated)
Vertices per in2	2941 (approximated)

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[1] The verifiable chain of custody of an artifact

[2] The location or site where an artifact was recovered

[3] Vessel artifact classification developed by W. Arnold Maijer and described in his publication Masters of Stone, ISBN 978-90-829212-0-5

[4] The precision score metric is described in Precision Score Of The Artifact, p. 46

[5] The Mohs scale is an ordinal scale, from 1 to 10, describing the materials resistance to abrasion (the ability of harder material to scratch softer material)

[6] Median distance between vertices

Alignment In The Cartesian Coordinate System

For precise and valid measurements of the vessel's geometry to be possible, the points of the scanned dataset must first and foremost be placed optimally in a Cartesian coordinate system. Several alignment methods and algorithms have been tested on a number of different vessels to determine the best way to achieve optimal alignment.

Any misalignment of the artifact will increase the error of the precision measurements, due to the distortion/wobble effect caused by the misaligned object. To visualize this distortion, we can consider a representation of the three-dimensional point cloud data, folded to a two-dimensional plane. This folded representation is obtained by rotating all scanned points around an assumed center axis to y = 0, x > 0, thus resulting in a two-dimensional profile representation of all scanned vertices in the object.

Figure 1 illustrates this effect on a ideal ellipsoid. In the first image, the ellipsoid is perfectly aligned, resulting in a narrow and precise two-dimensional folded profile. As misalignments are introduced, the two-dimensional profile increases in width, visually showing the distortion, causing the error in the precision measurements to increase. While easy to understand visually, this distortion can also be objectively quantified, and as such used to compare the fitness of different assumed center axes against each other, and further to create an automated and solid process for optimal Cartesian alignment of the scan data.

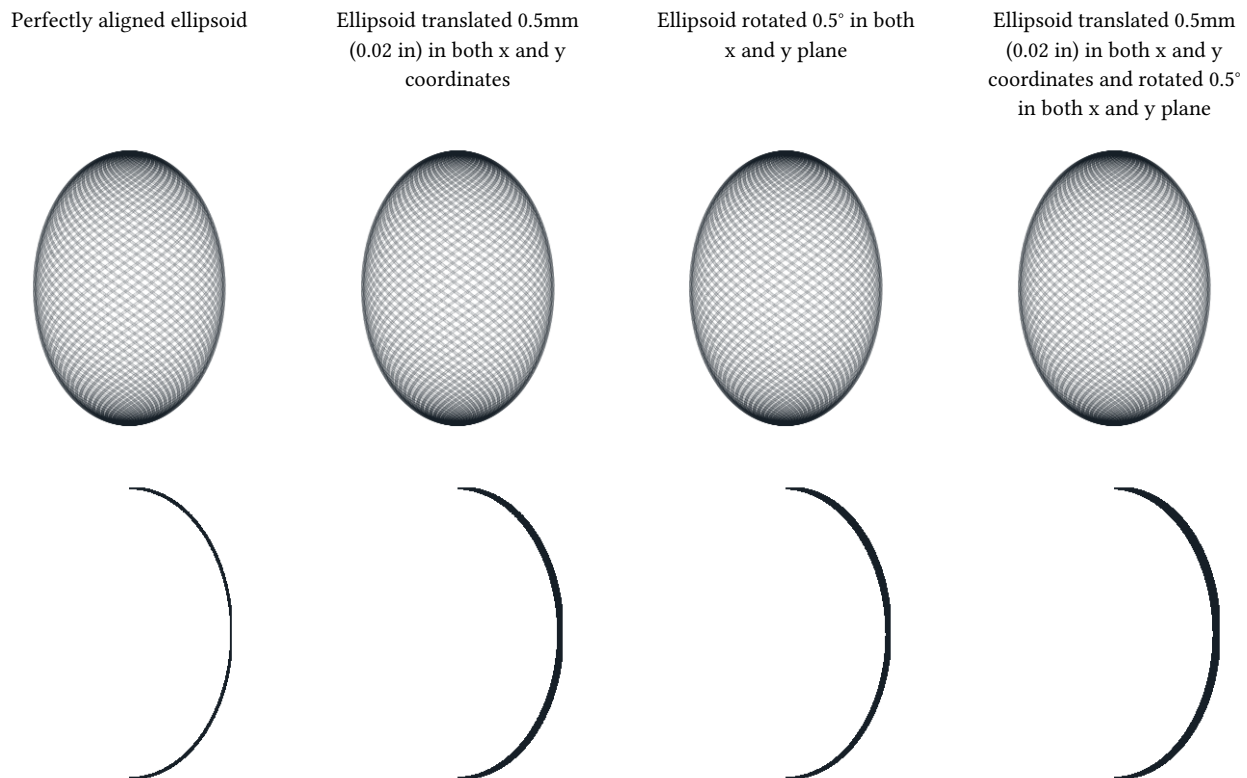| Perfectly aligned ellipsoid | Ellipsoid translated 0.5mm (0.02 in) in both x and y coordinates | Ellipsoid rotated 0.5° in both x and y plane | Ellipsoid translated 0.5mm (0.02 in) in both x and y coordinates and rotated 0.5° in both x and y plane |

Figure 1: Distortion caused by a misalignment of the artifact

In contemporary metrology analysis of modern production objects, it is common to align the object in a Cartesian coordinate system by fitting a flat surface of the object to a reference plane in the coordinate system, cylindrical features to an ideal cylinder etc., or by using specific markers placed on the object in the design process. This methodology, however, is inadequate for the ancient objects in question. Most scanned artifacts, do not have a valid flat surface which could be aligned to a plane in the Cartesian coordinate system; most surfaces seem to be curved. Some artifacts do have a flat base, however this is often a worn area of the artifact and practical tests have shown that alignment to such surfaces will not produce optimal alignment of the scan data.

As conventional methods of alignment do not always yield good results with these types of artifacts, a more adequate method of alignment has been developed to enable precise measurements and statistical analysis of the scan data.

To find the optimal position of the vessel in the coordinate system, a range of rotation and translation tests are carried out to find the best fit of the central axis.

Based on the assumption that the analyzed object was created using a rotational process, and thus have symmetry around a central axis, the alignment of the artifact is carried out in a two-step process. An overview of this process is given below.

The artifact is placed in a Cartesian coordinate system, in an initially unaligned state. The first step in the alignment process estimates the central rotational axis of the vessel, by analyzing the coaxiality of thin cross-section slices of the vessel. The slices will be as thin as possible based on the mesh density of the scan, while still ensuring enough data points in each slice to be statistically valid.

For each slice, circular regression[7] (estimate of best fit circle) is used to estimate the center point of this slice. Combined over the total Z-axis range of the vessel, these center points provide us with an indicator of the incline and position of the vessel's central axis.

The next step will optimize the center axis alignment by progressively minimizing the deviation (perpendicular to the surface curvature) of the two-dimensional profile, see Figure 1. By ascertaining and comparing the resulting fit of many thousands of different potential rotations, the best fit alignment of the scan data can be estimated, and an optimal center axis (in relation to the data points) can be reconstructed. The actual three-dimensional point-cloud is then aligned to this axis, by rotating and translating the scanned data points to match the Z-axis of the Cartesian coordinate system.

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[7]Circle regression algorithm used: Kenichi Kanatani, Prasanna Rangarajan, "Hyper least squares fitting of circles and ellipses" Computational Statistics & Data Analysis, Vol. 55, pages 2197-2208, (2011)

Precision

To explore the manufacturing precision of the artifact in depth, the following analysis have been carried out:

- Circularity around the axis of symmetry is examined in detail at selected cross-sections.
- Overall circularity around the axis of symmetry is measured for the full height of the vessel (areas of the vessel with extensive damage are not taken into account for this metric).
- Concentricity of the vessel between selected cross-sections are examined in detail to determine if the existence of an axis of rotation in the manufacture of the object can be established.
- The coaxiality of the vessel is analyzed to explore the precision of the central axis of the object.
- The surface variability is analyzed and visualized on through a heatmap.

Circularity

Circularity is the measurement of how round the surface of an object is, optionally in reference to a datum axis. The *circularity tolerance* is the radial distance of two circles, each with their centers in the datum axis, and each of them conforming, respectively, to the minimum and maximum deviations of the data-set to a true circle, see Figure 2.
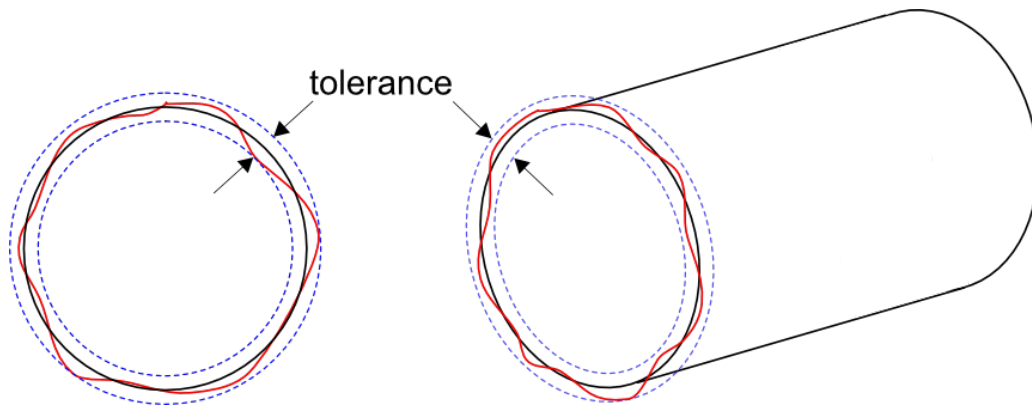

Figure 2: Circularity tolerance.

Circularity is examined at different cross-sections of the vessel, using the established Z-axis as the datum axis (axis of symmetry). The distance between the scanned points in the local datum plane is measured to determine the range between the two concentric circles encompassing the measured points, see Figure 3.

Referencing all of the individual circularity measurements to the global (reconstructed) axis of symmetry of the object, allows us to ascertain not only circularity of local features of the object, but how well circularity was *maintained* over the entire manufacturing process. This is an important distinction, which may be able to provide valuable insights into requirements of the construction methods. For reference, and seeing that the variance in local circularity also holds interest, measurements of circularity of the vessel without reference to the axis of symmetry can additionally be found in the Concentricity, p. 25.

Figure 3: Circularity measurements.

If the circularity is determined from slices of the vessel exclusively in the *Z-plane* (actually measuring the cylindricity of a very thin slices of the vessel, in an attempt to approximate circularity), this would - in some areas - introduce significant distortion (increasing measurement errors) in the samples, due to the curvature of the vessel's surface.

Each sample slice of the vessel is therefore obtained perpendicular to the surface curvature, see Figure 5 to Figure 13. The measurements are taken conservatively without filtration of potential outliers.

To explore the potential distortion caused by obtaining samples in the Z-plane only, please refer to Appendix A, where measurements in the Z-plane and measurements perpendicular to surface curvature are compared side by side.

Detailed circularity measurements of selected points

Circularity measurements across a range of selected slices of the vessel (see Table 1) have been analyzed in-depth, and detailed plots of each measurement is provided. Furthermore, full circularity measurements are shown for each available scanned surface including a detailed plot to visualize the circularity of all areas of the vessel.
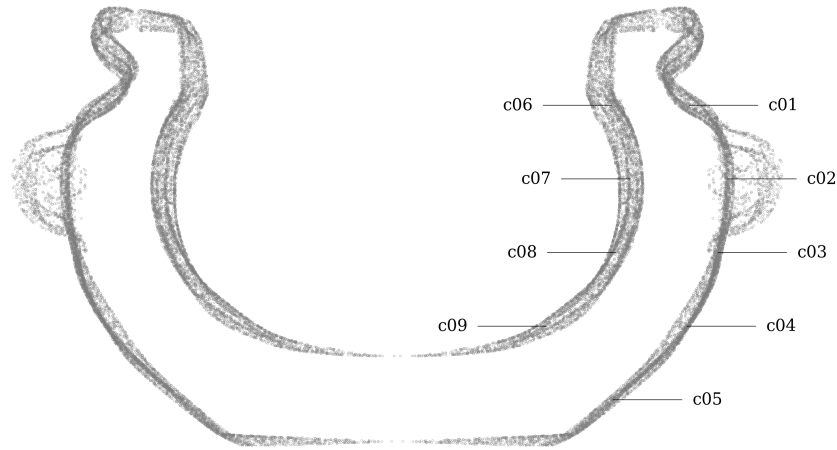
Figure 4: Circularity measurement sample location on RV003.

Metric

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø110.931±2.034	3.339	1.182	1.146	1.140	170	0.200	63.743	55.465
c02	exterior	Ø125.541±1.591	2.699	0.658	0.414	0.657	154	0.200	49.799	62.770
c03	exterior	Ø121.887±0.885	1.695	0.445	0.324	0.446	307	0.200	35.856	60.944
c04	exterior	Ø110.108±2.271	2.756	0.800	0.254	0.752	228	0.200	21.912	55.054
c05	exterior	Ø82.106±0.964	1.817	0.508	0.360	0.509	146	0.200	7.969	41.053
c06	interior	Ø81.543±3.727	5.621	1.540	1.006	1.526	290	0.200	63.743	40.771
c07	interior	Ø88.479±2.474	4.734	1.529	1.368	1.518	270	0.200	49.799	44.239
c08	interior	Ø83.181±1.960	3.323	1.105	1.018	1.099	309	0.200	35.856	41.591
c09	interior	Ø56.769±1.668	2.841	0.859	0.742	0.863	111	0.200	21.912	28.385

Imperial

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø4.3674±0.0801	0.1315	0.0465	0.0451	0.0449	170	0.0079	2.5096	2.1837
c02	exterior	Ø4.9426±0.0626	0.1062	0.0259	0.0163	0.0259	154	0.0079	1.9606	2.4713
c03	exterior	Ø4.7987±0.0348	0.0667	0.0175	0.0128	0.0176	307	0.0079	1.4116	2.3994
c04	exterior	Ø4.3350±0.0894	0.1085	0.0315	0.0100	0.0296	228	0.0079	0.8627	2.1675
c05	exterior	Ø3.2325±0.0379	0.0715	0.0200	0.0142	0.0201	146	0.0079	0.3137	1.6163
c06	interior	Ø3.2103±0.1467	0.2213	0.0606	0.0396	0.0601	290	0.0079	2.5096	1.6052
c07	interior	Ø3.4834±0.0974	0.1864	0.0602	0.0539	0.0598	270	0.0079	1.9606	1.7417
c08	interior	Ø3.2749±0.0771	0.1308	0.0435	0.0401	0.0433	309	0.0079	1.4116	1.6374
c09	interior	Ø2.2350±0.0657	0.1119	0.0338	0.0292	0.0340	111	0.0079	0.8627	1.1175

Table 1: Detailed circularity measurements at selected samples of RV003.

Figure 5 to Figure 13 shows a detailed plots of each circularity measurement.

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[8] Sample diameter Ø± maximum measured deviation from measured radius

[9] Root mean square deviation (RMSD) also called Root mean square error (RMSE)

[10] Median absolute deviation

[11] Median sample radius from z-axis

Graphical overview of circularity measurement c01
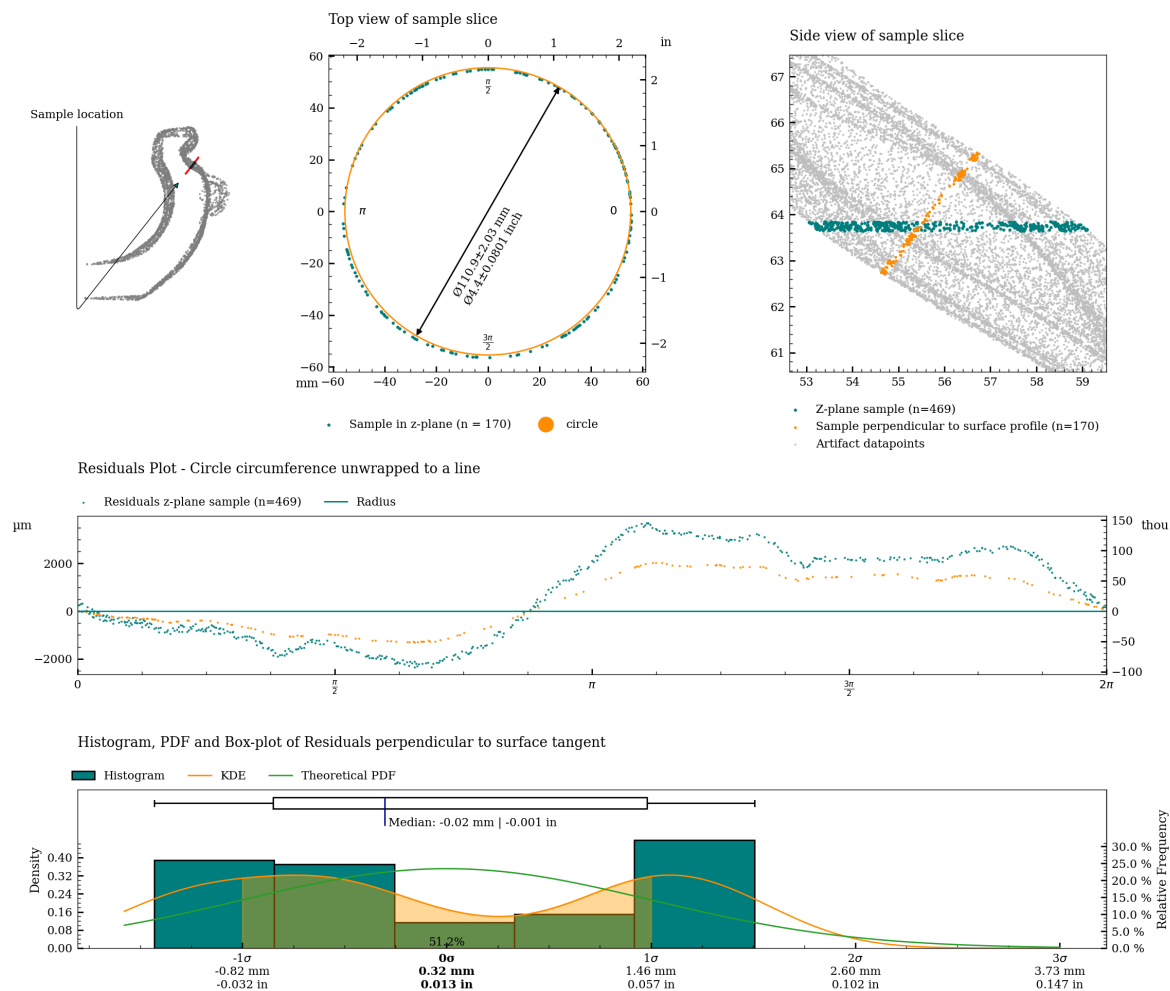
Figure 5: Charts with statistics for the measurement of c01.

Graphical overview of circularity measurement c02

Top view of sample slice

Side view of sample slice

Sample location

Ø125.3±1.59 mm
Ø4.9±0.0626 inch

- Sample in z-plane (n = 154)
- circle

- Z-plane sample (n=153)
- Sample perpendicular to surface profile (n=154)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Residuals z-plane sample (n=153) —— Radius

Histogram, PDF and Box-plot of Residuals perpendicular to surface tangent

- Histogram —— KDE —— Theoretical PDF

Median: -0.01 mm | -0.001 in

64.3%

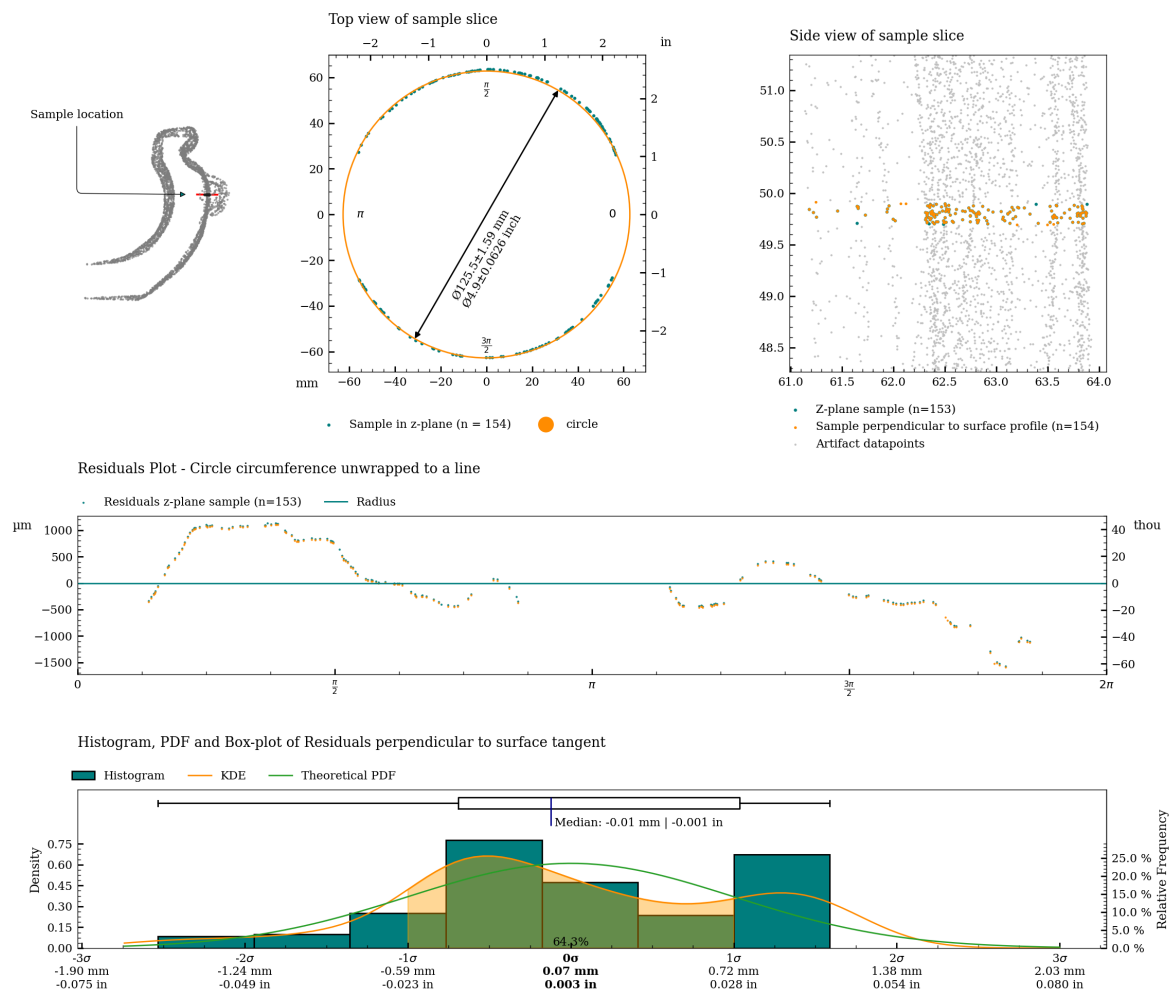|	-3σ	-2σ	-1σ	0σ	1σ	2σ	3σ
mm	-1.90 mm	-1.24 mm	-0.59 mm	**0.07 mm**	0.72 mm	1.38 mm	2.03 mm
in	-0.075 in	-0.049 in	-0.023 in	**0.003 in**	0.028 in	0.054 in	0.080 in

Figure 6: Charts with statistics for the measurement of c02.

Graphical overview of circularity measurement c03
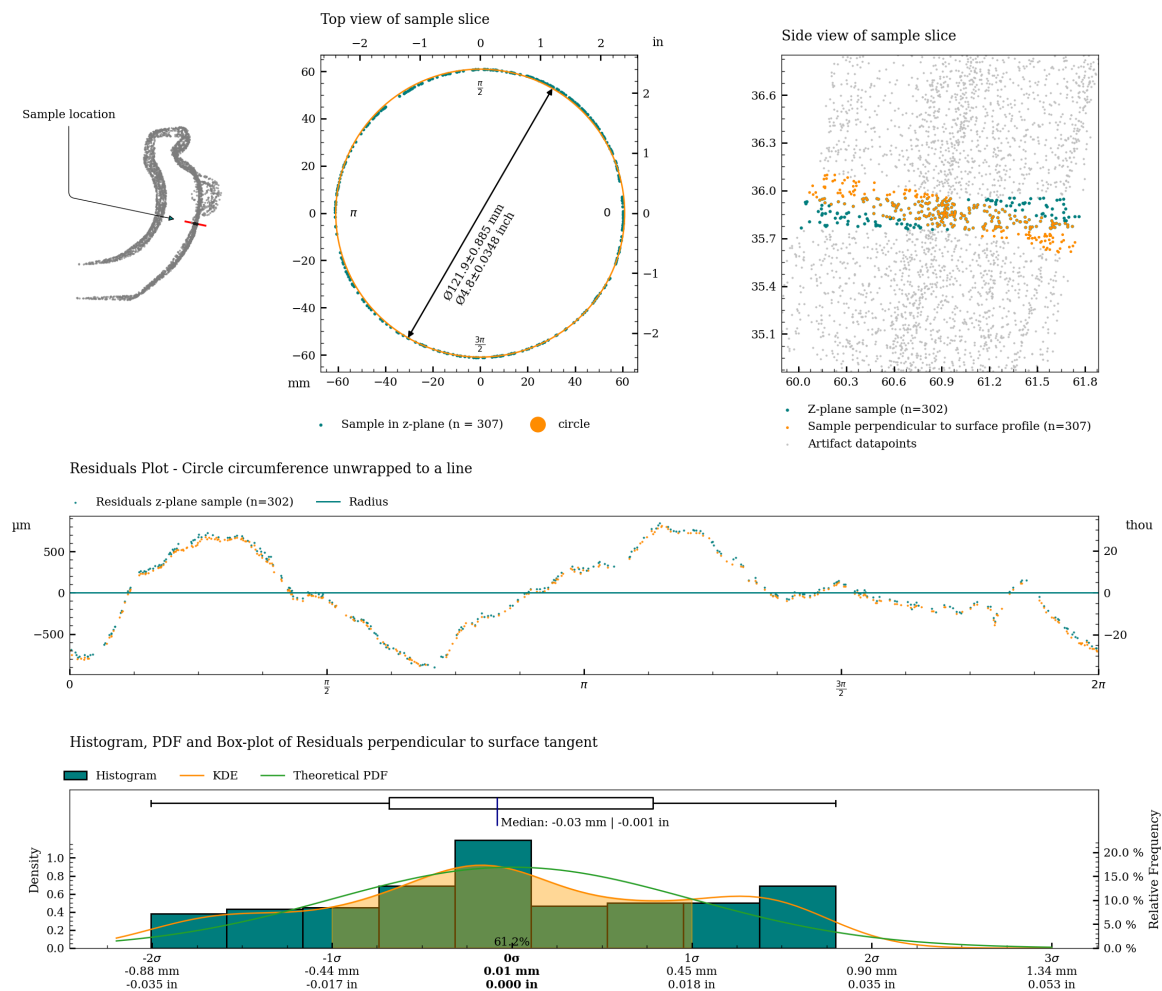

Figure 7: Charts with statistics for the measurement of c03.

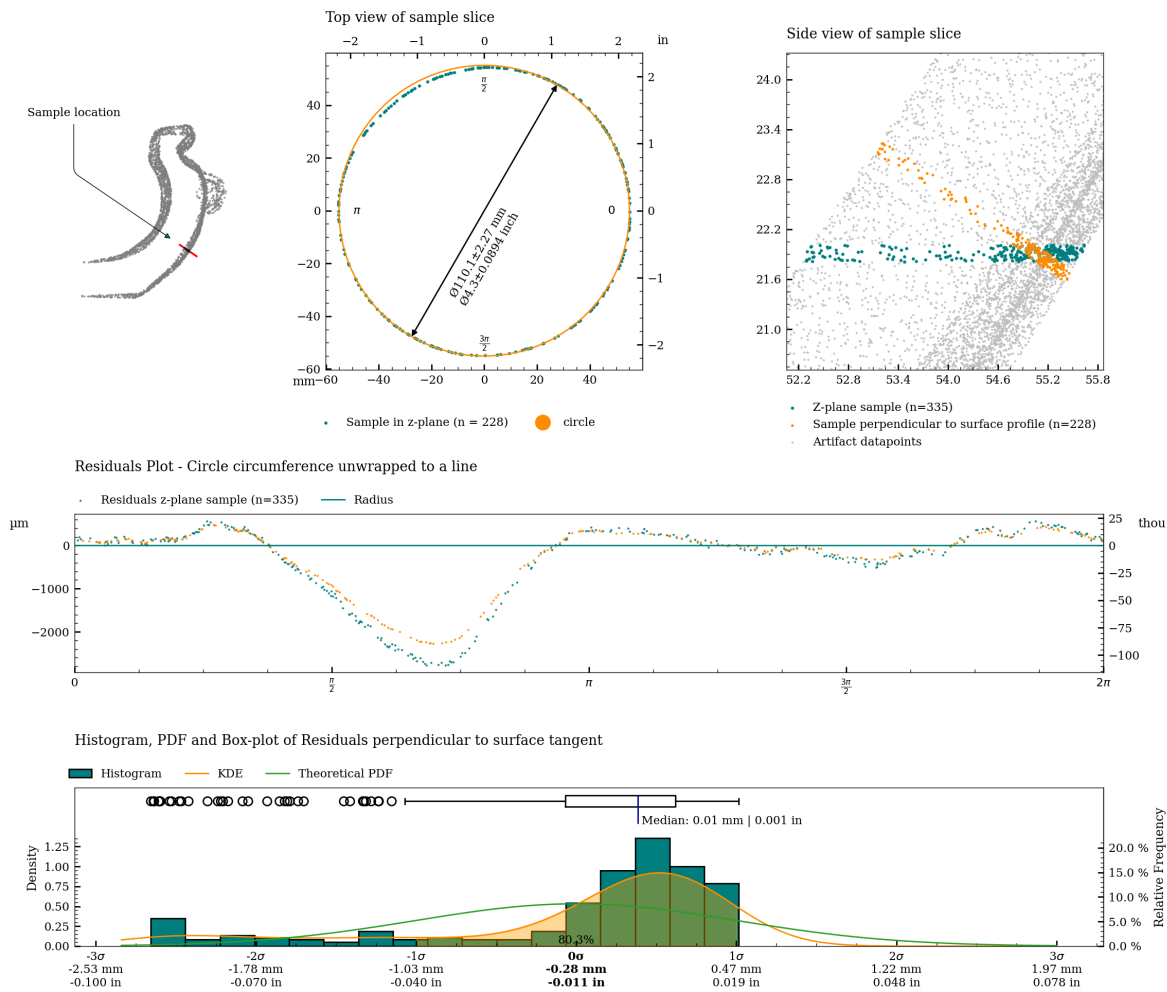# Graphical overview of circularity measurement c04

Figure 8: Charts with statistics for the measurement of c04.

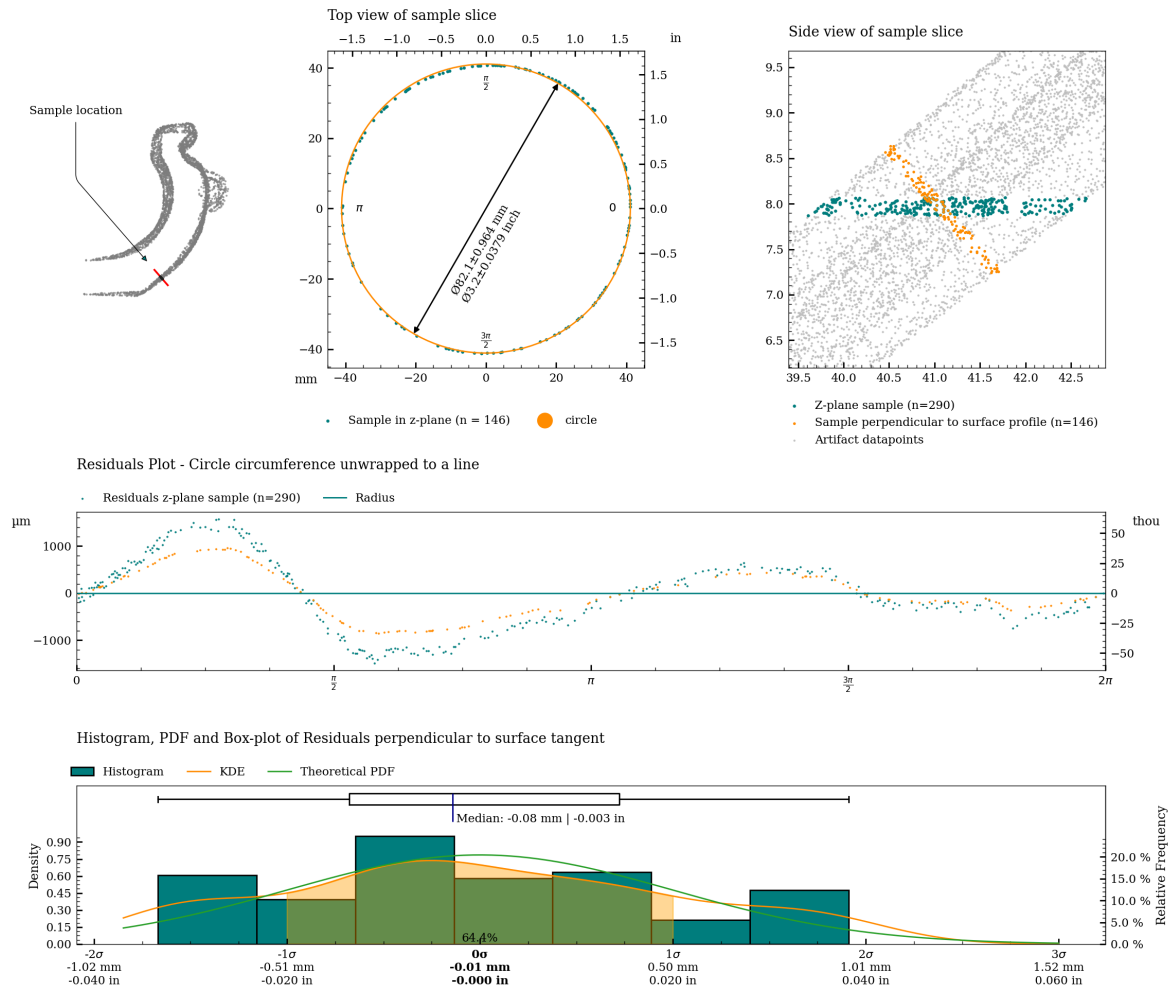# Graphical overview of circularity measurement c05

Figure 9: Charts with statistics for the measurement of c05.
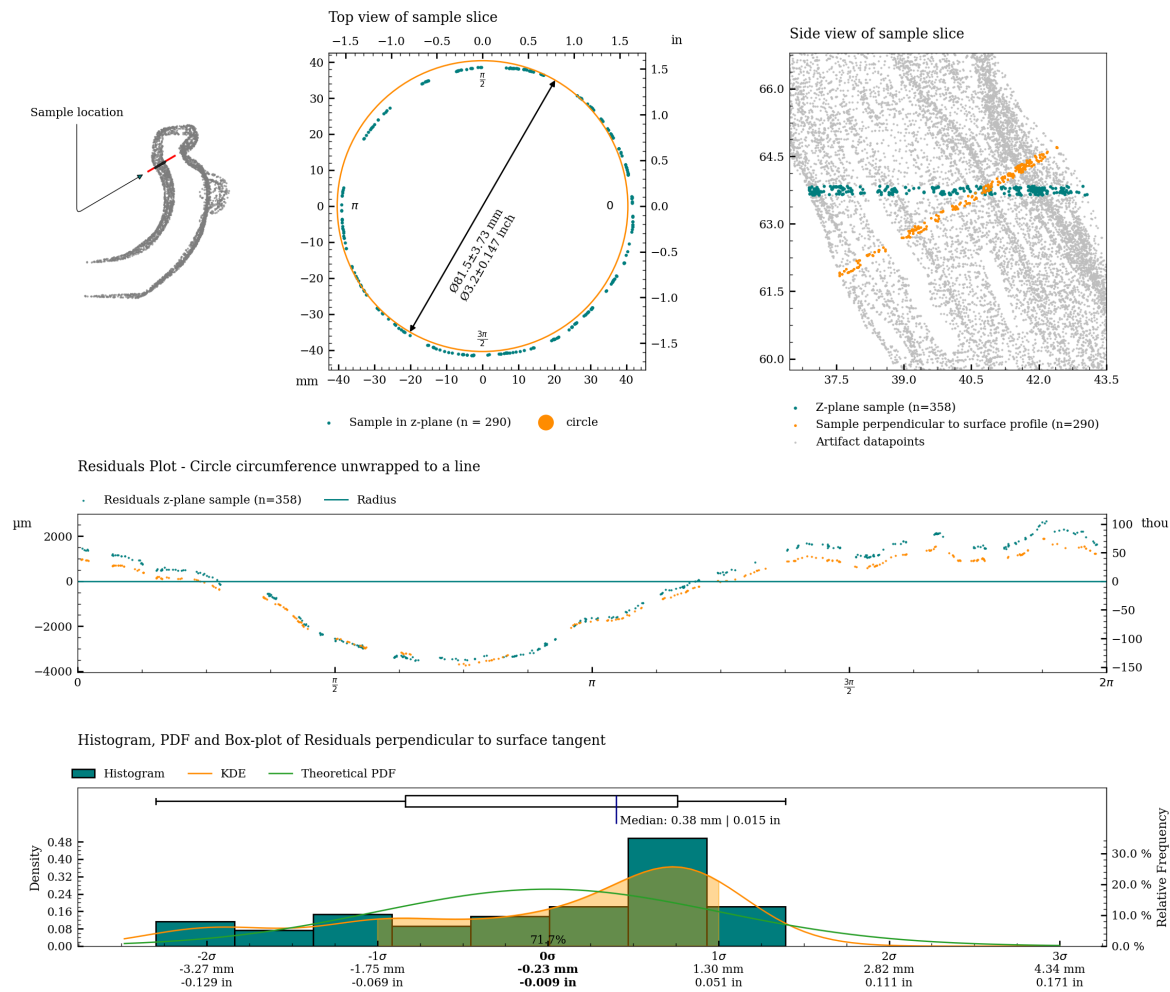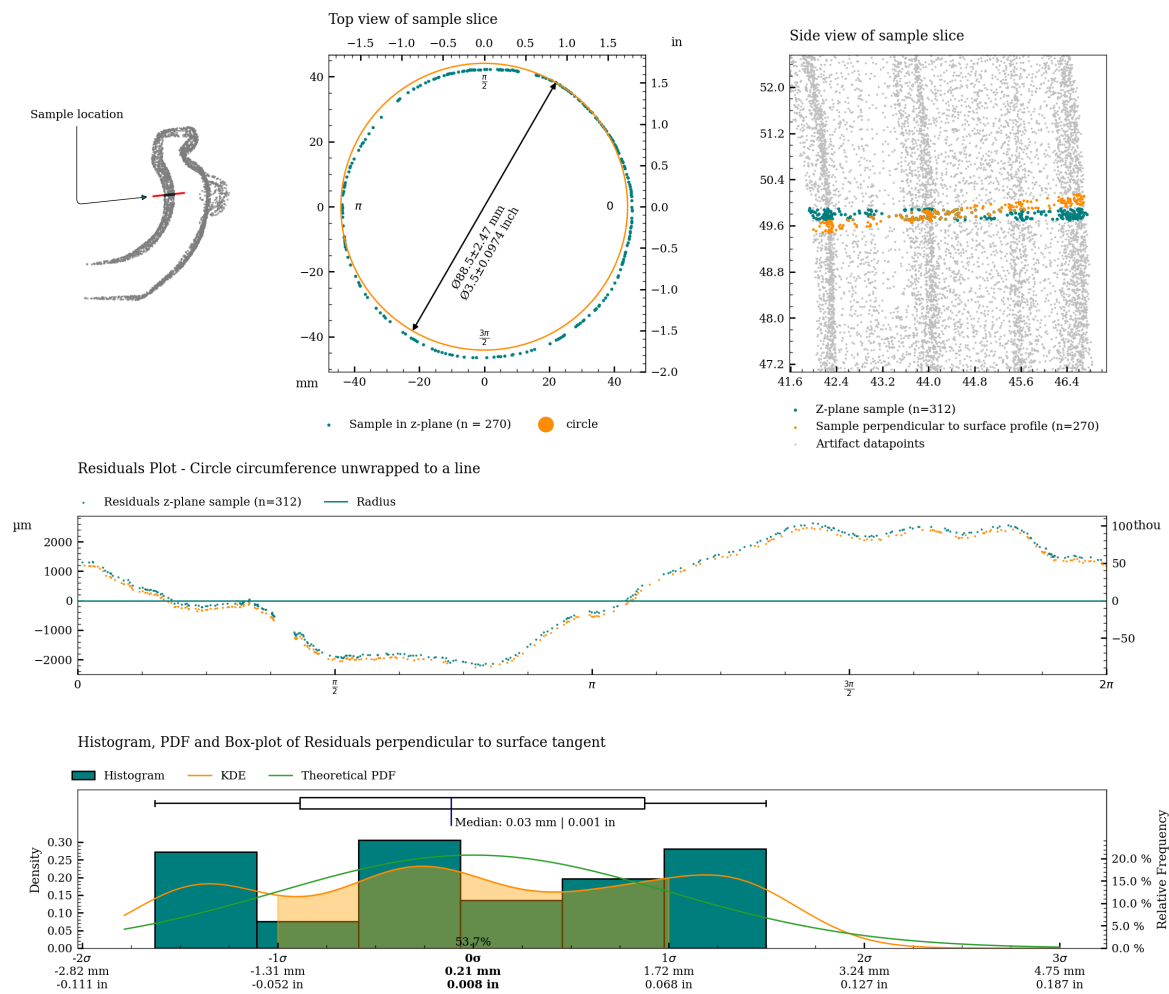
Graphical overview of circularity measurement c06

Top view of sample slice

Sample location

Side view of sample slice

- Sample in z-plane (n = 290)
- circle

- Z-plane sample (n=358)
- Sample perpendicular to surface profile (n=290)
- Artifact datapoints

Ø81.5±3.73 mm
Ø3.21±0.147 inch

Residuals Plot - Circle circumference unwrapped to a line

- Residuals z-plane sample (n=358)   — Radius

Histogram, PDF and Box-plot of Residuals perpendicular to surface tangent

- Histogram   — KDE   — Theoretical PDF

Median: 0.38 mm | 0.015 in

71.7%

| | -2σ
-3.27 mm
-0.129 in | -1σ
-1.75 mm
-0.069 in | **0σ**
-0.23 mm
-0.009 in | 1σ
1.30 mm
0.051 in | 2σ
2.82 mm
0.111 in | 3σ
4.34 mm
0.171 in |

Figure 10: Charts with statistics for the measurement of c06.

Graphical overview of circularity measurement c07

Top view of sample slice

Side view of sample slice

Sample location

Ø88.5±2.47 mm
Ø3.5±0.0974 inch

- Sample in z-plane (n = 270)
- circle

- Z-plane sample (n=312)
- Sample perpendicular to surface profile (n=270)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Residuals z-plane sample (n=312) —— Radius

Histogram, PDF and Box-plot of Residuals perpendicular to surface tangent

- Histogram —— KDE —— Theoretical PDF

Median: 0.03 mm | 0.001 in

53.7%

	-2σ	-1σ	0σ	1σ	2σ	3σ
	-2.82 mm	-1.31 mm	**0.21 mm**	1.72 mm	3.24 mm	4.75 mm
	-0.111 in	-0.052 in	**0.008 in**	0.068 in	0.127 in	0.187 in

Figure 11: Charts with statistics for the measurement of c07.

Graphical overview of circularity measurement c08

Figure 12: Charts with statistics for the measurement of c08.

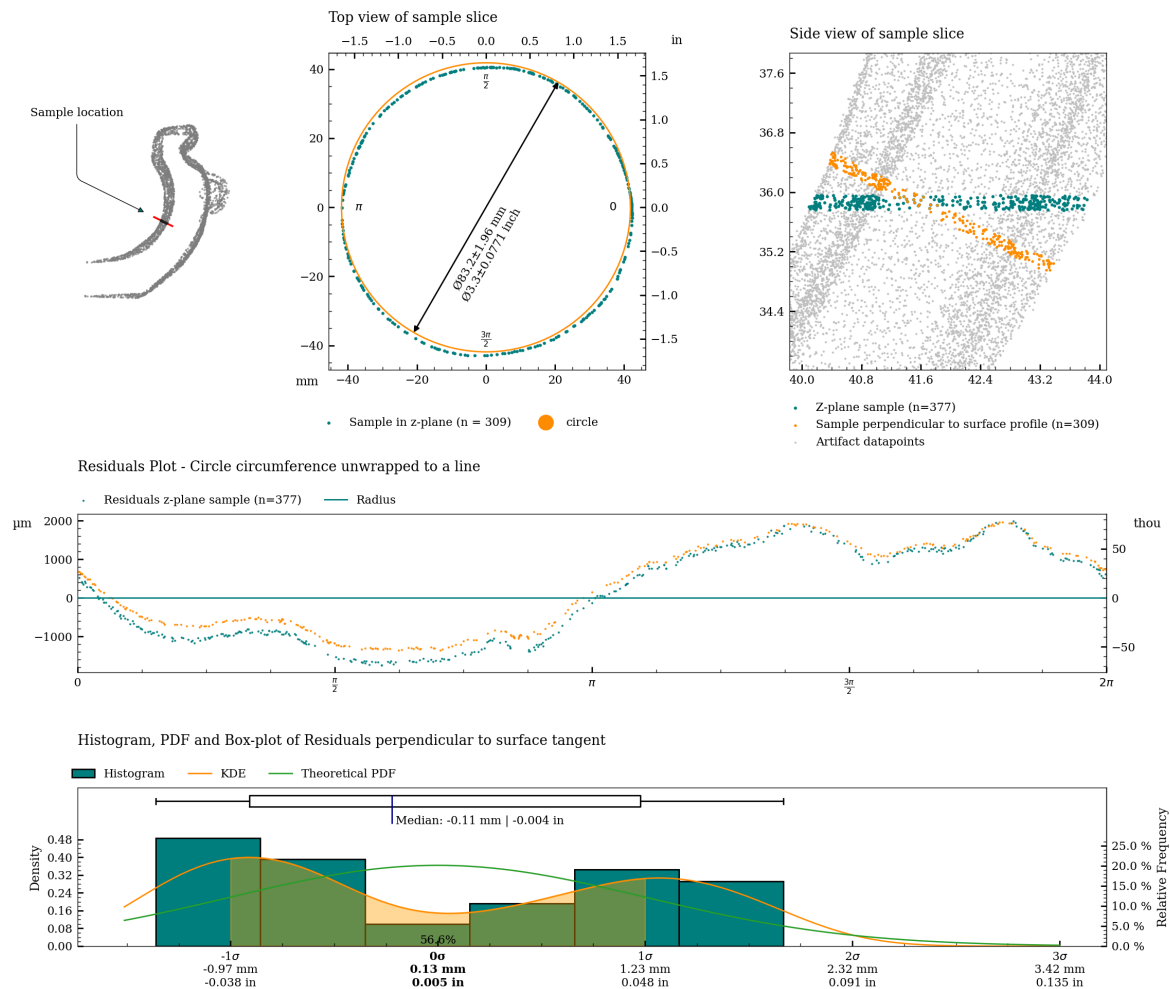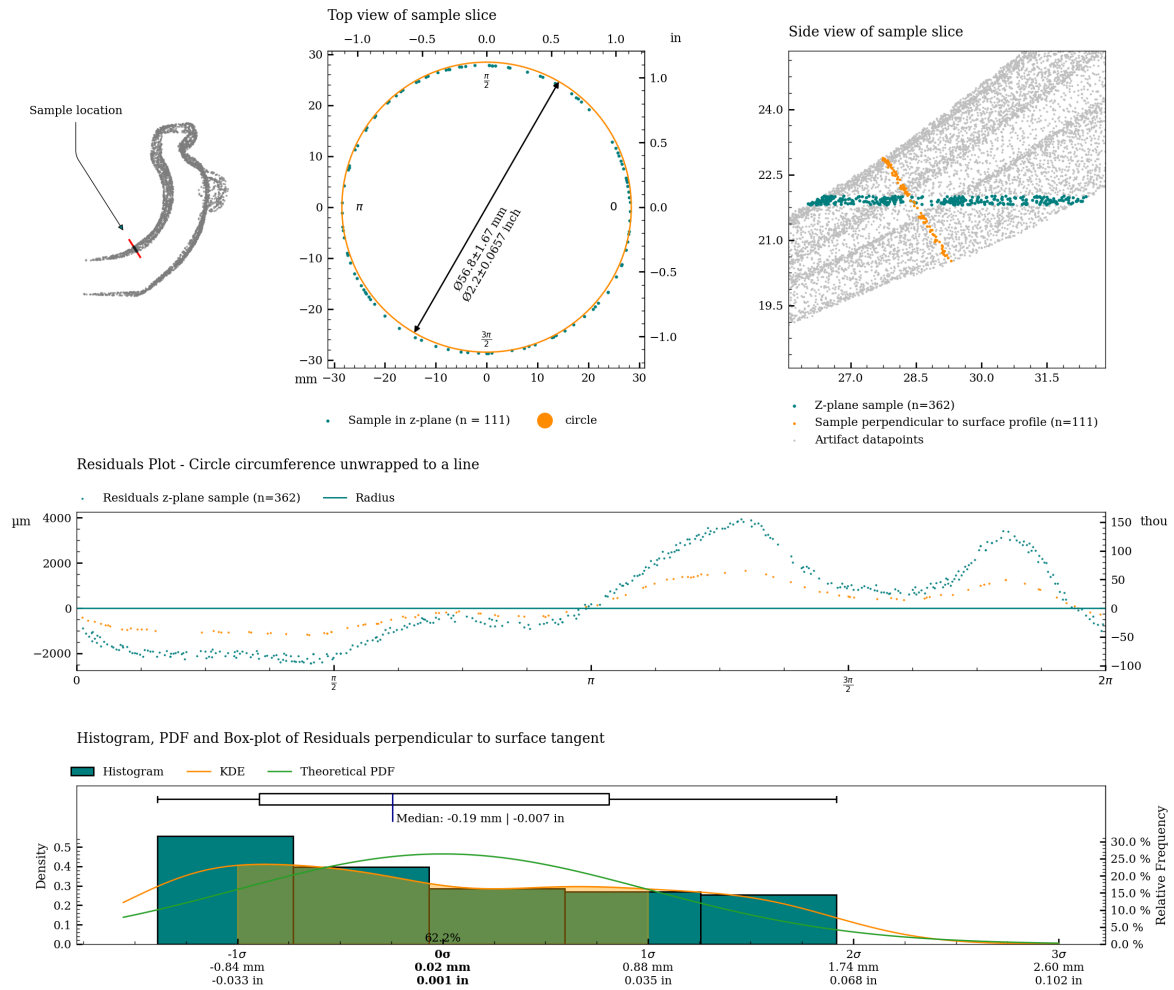# Graphical overview of circularity measurement c09

Top view of sample slice

Sample location

∅56.8±1.67 mm
∅2.2±0.0657 inch

- Sample in z-plane (n = 111)
- circle

Side view of sample slice

- Z-plane sample (n=362)
- Sample perpendicular to surface profile (n=111)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Residuals z-plane sample (n=362) — Radius

Histogram, PDF and Box-plot of Residuals perpendicular to surface tangent

- Histogram — KDE — Theoretical PDF

Median: -0.19 mm | -0.007 in

62.2%

	-1σ	0σ	1σ	2σ	3σ
	-0.84 mm	**0.02 mm**	0.88 mm	1.74 mm	2.60 mm
	-0.033 in	**0.001 in**	0.035 in	0.068 in	0.102 in

Figure 13: Charts with statistics for the measurement of c09.

Table 2 shows statistical measures of the circularity of the vessel, measured along the full height (damaged parts may reduce the measurement area).

Metric

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	1.830	1.096	3.968	0.439	0.235	0.918	0.068	0.150	0.700	271	0.200
Interior	4.267	1.594	5.179	1.333	0.427	1.760	0.381	0.129	1.840	225	0.200

Imperial

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	1.830	1.096	3.968	0.439	0.235	0.918	0.068	0.150	0.700	271	0.200
Interior	4.267	1.594	5.179	1.333	0.427	1.760	0.381	0.129	1.840	225	0.200

Table 2: Perpendicular Circularity analysis of RV003.

Circularity analysis of exterior surface
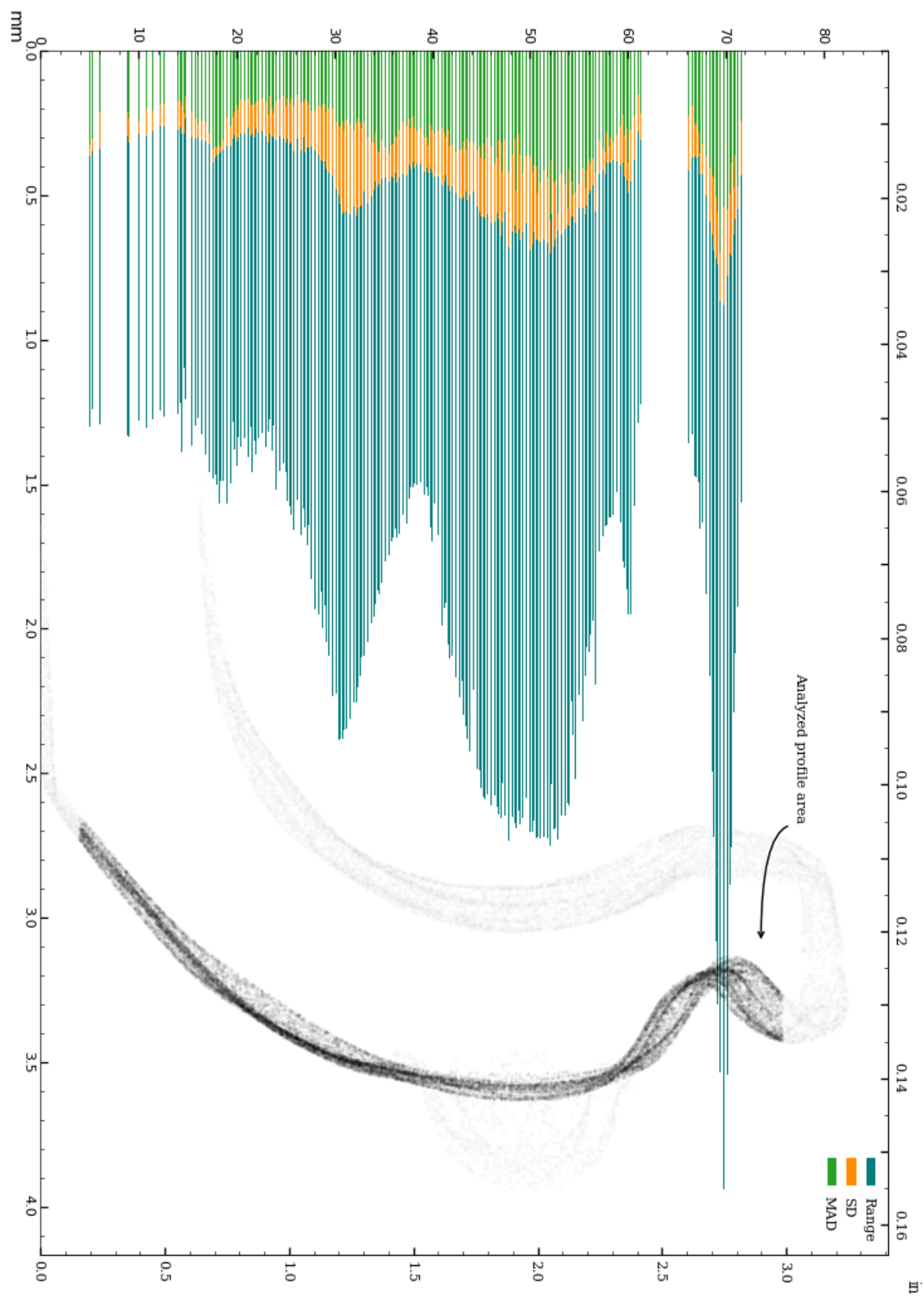
Figure 14: Circularity of exterior surface.

Circularity analysis of exterior surface, Standard Deviation and Median Absolute Deviation
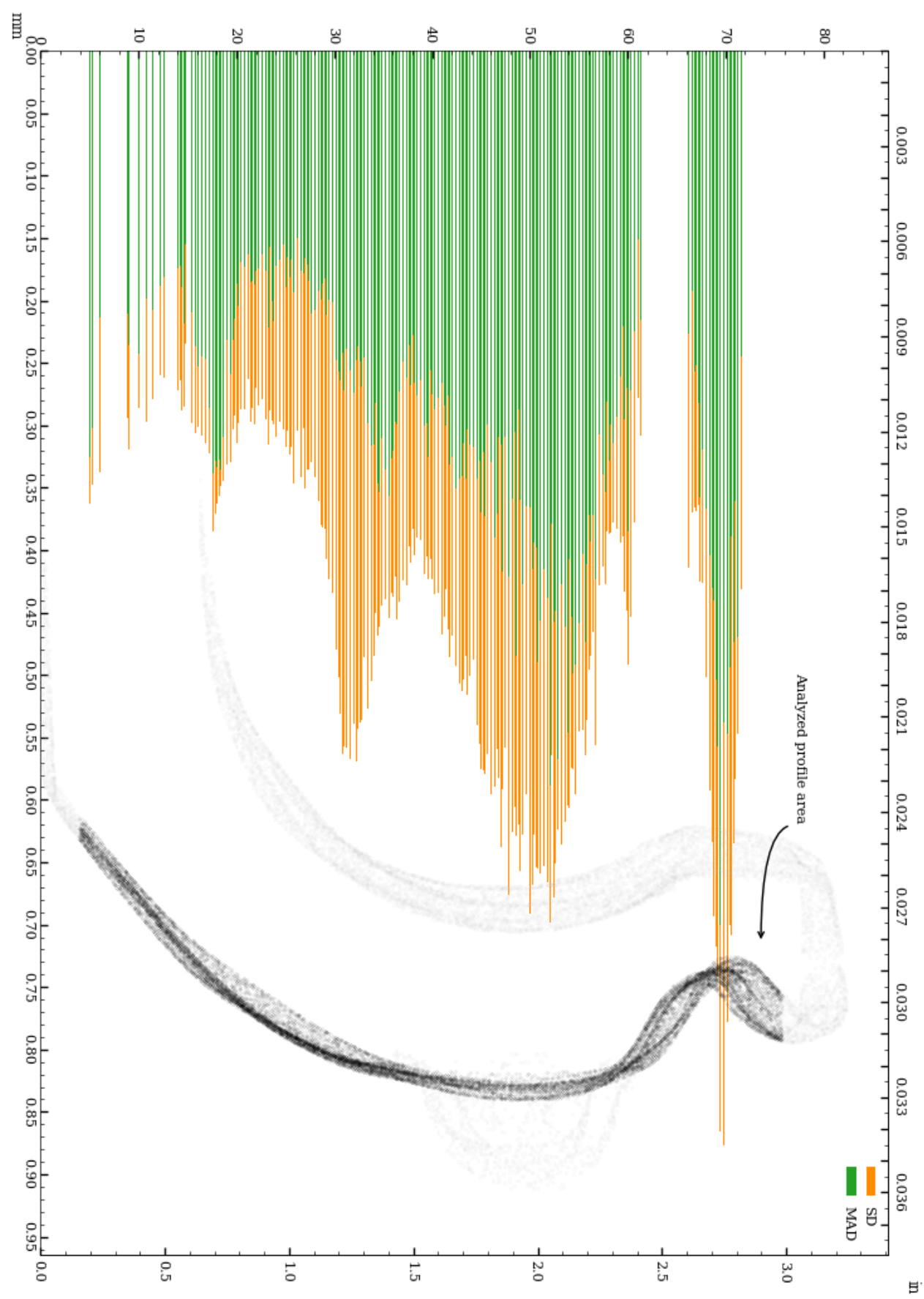

Figure 15: Vessel circularity of exterior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 271 slices of the exterior surface are shown below.

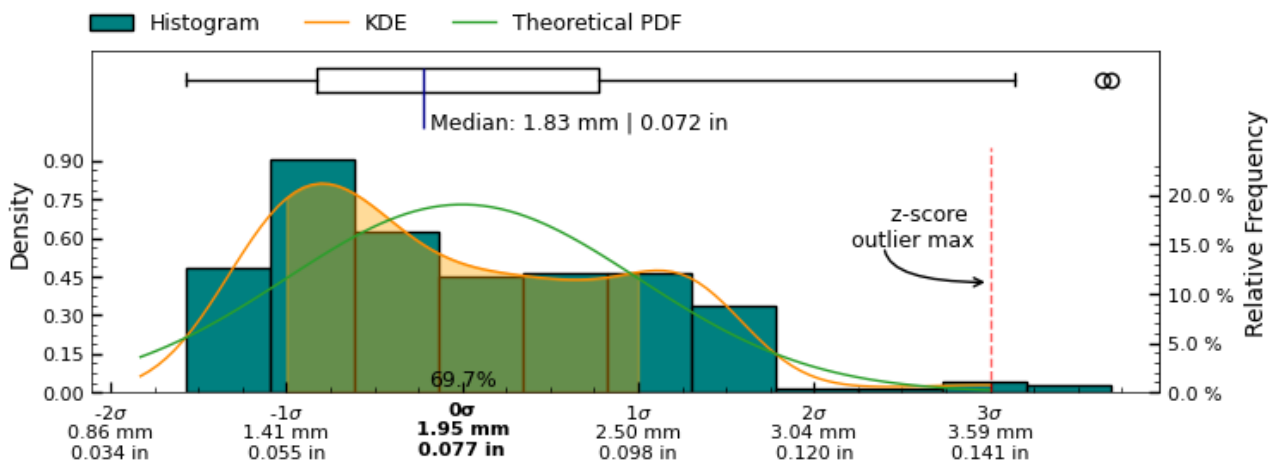**Range measurement distribution across 271 slices of exterior surface**

Figure 16: Range measurement distribution across measured slices of exterior surface

Standard deviation measurement distribution across 271 slices of exterior surface
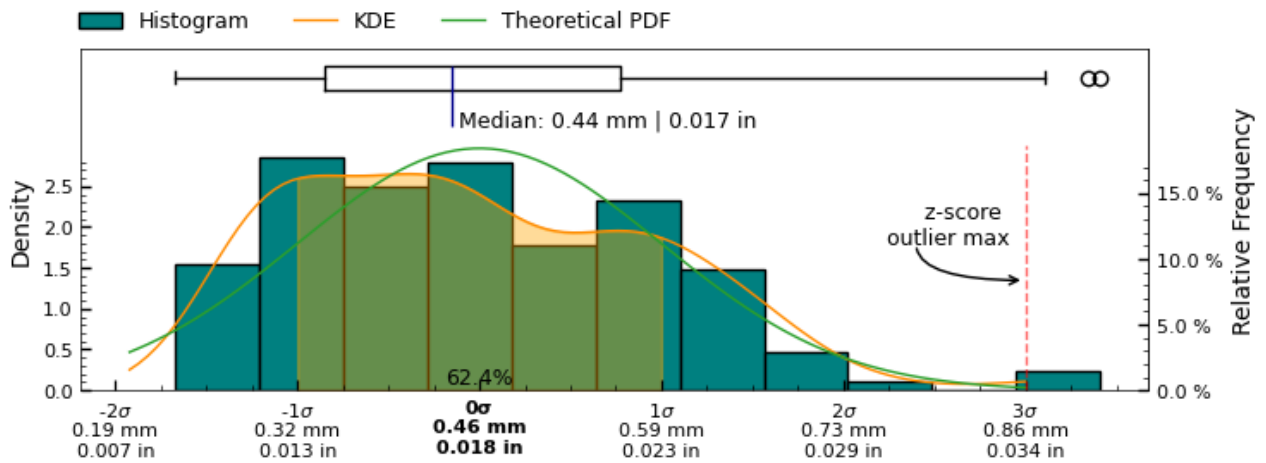
Figure 17: Standard deviation measurement distribution across measured slices of " + exterior + " surface

Median absolute deviation measurement distribution across 271 slices of exterior surface
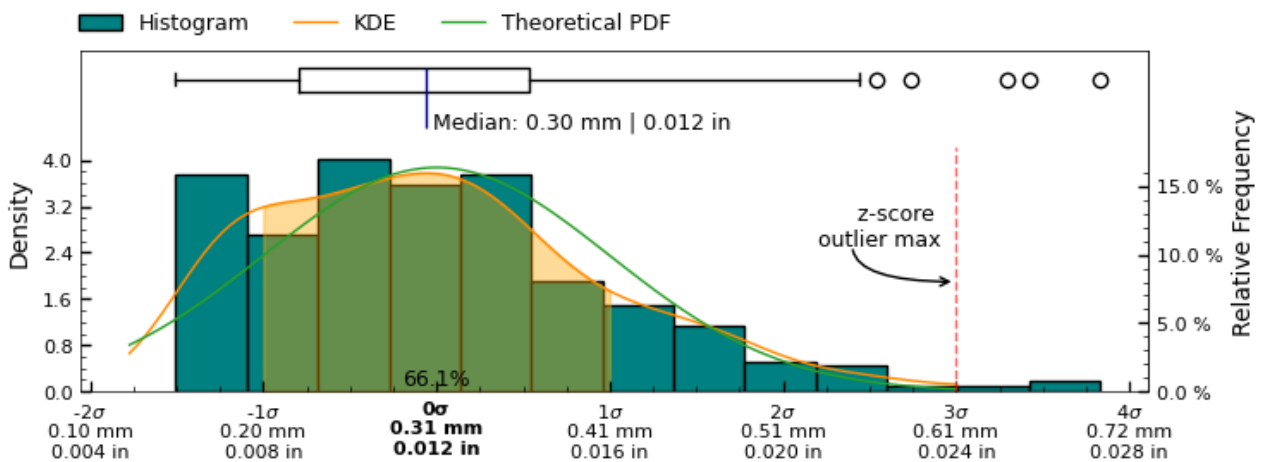
Figure 18: Median absolute deviation measurement distribution across measured slices of exterior surface

Circularity analysis of interior surface
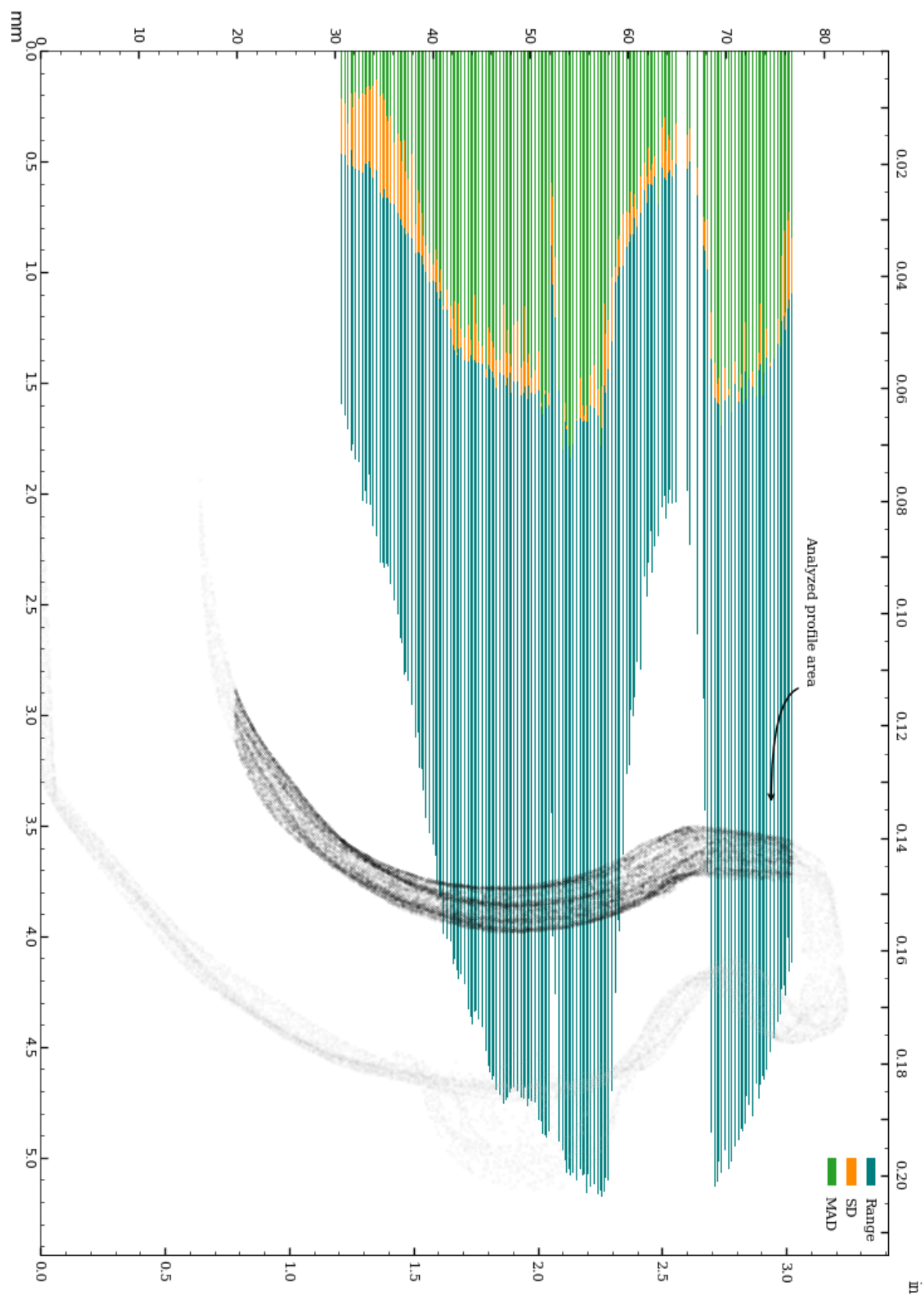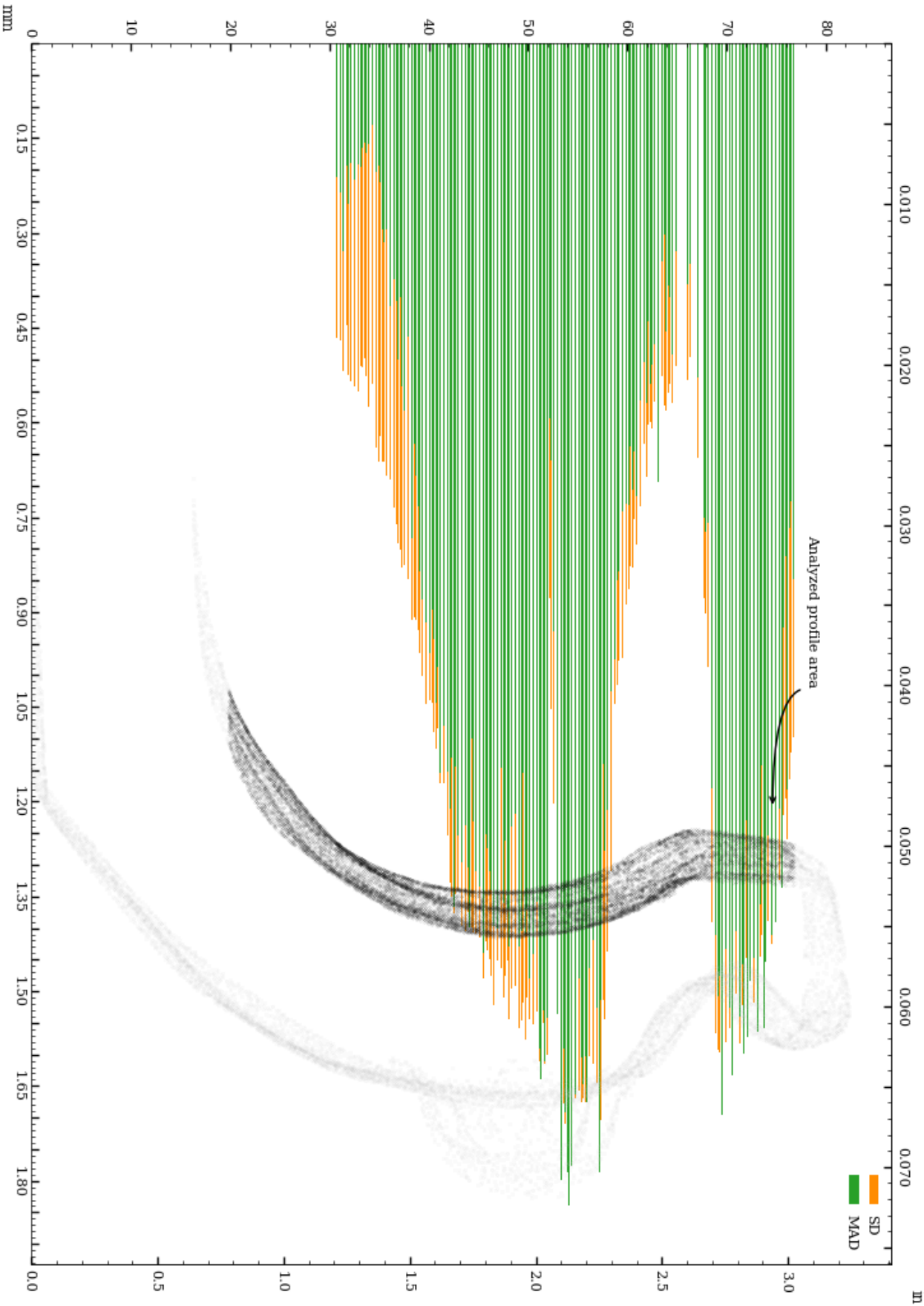

Figure 19: Circularity of interior surface.

Figure 20: Vessel circularity of interior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 225 slices of the interior surface are shown below.

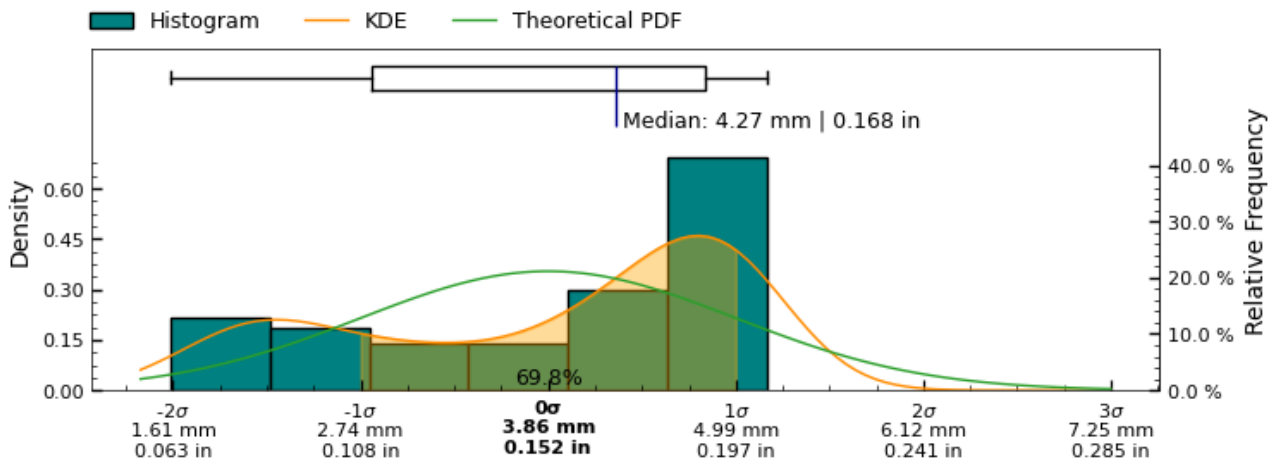**Range measurement distribution across 225 slices of interior surface**

Figure 21: Range measurement distribution across measured slices of interior surface

Standard deviation measurement distribution across 225 slices of interior surface
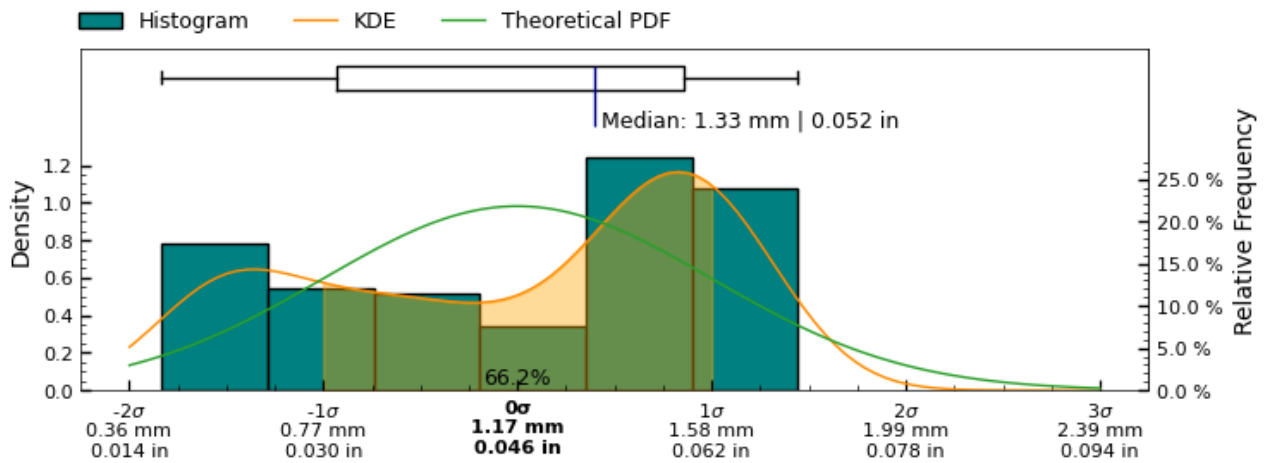

Figure 22: Standard deviation measurement distribution across measured slices of " + interior + " surface

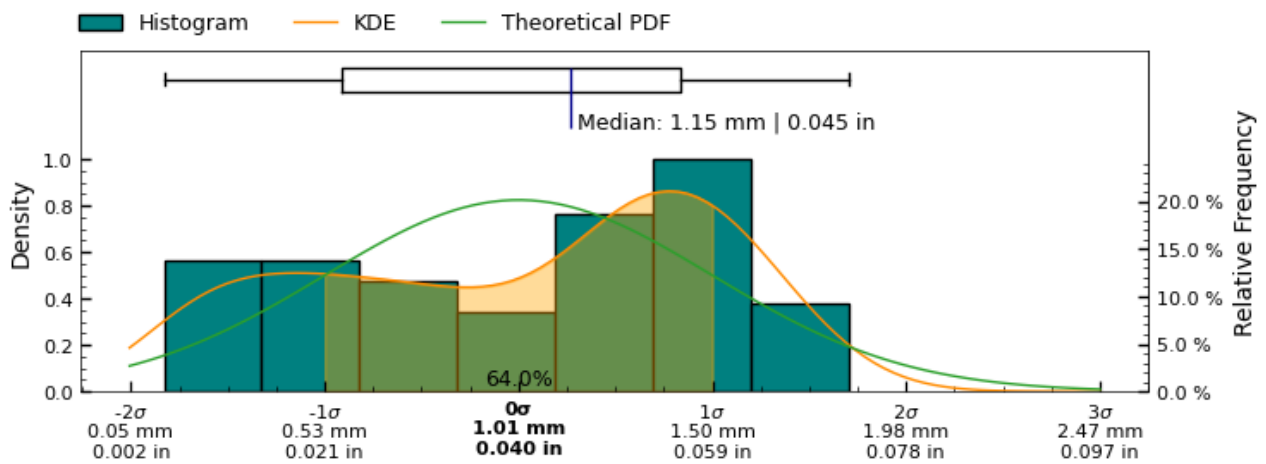**Median absolute deviation measurement distribution across 225 slices of interior surface**

Figure 23: Median absolute deviation measurement distribution across measured slices of interior surface

Concentricity

The concentricity metric describes the deviation in the center-point of the referenced features. As such, it is a measure to determine if several features of the object share the same center point/axis, and how closely. See Figure 24 for a visual representation of this metric.
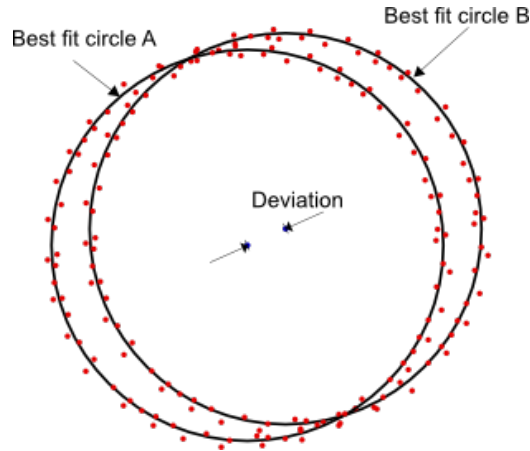

Figure 24: Concentricity measures the deviation (distance) between the center of two circles.

Determination of concentricity has been carried out by establishing the best fit circles of sample slices, using RANSAC (Random sample consensus) algorithm for outlier detection of a least squares circle regression on the scanned data-points at each cross-section, to estimate centers of each cross-section.

The concentricity between both the interior and exterior circular cross-sections is explored for cross-section measurements with the same Z-coordinates.

Additionally, the concentricity between each cross-section measurement defined in Figure 4 and the datum axis $(x, y) = (0, 0)$ has been calculated to establish the deviation of the feature center from the datum axis.
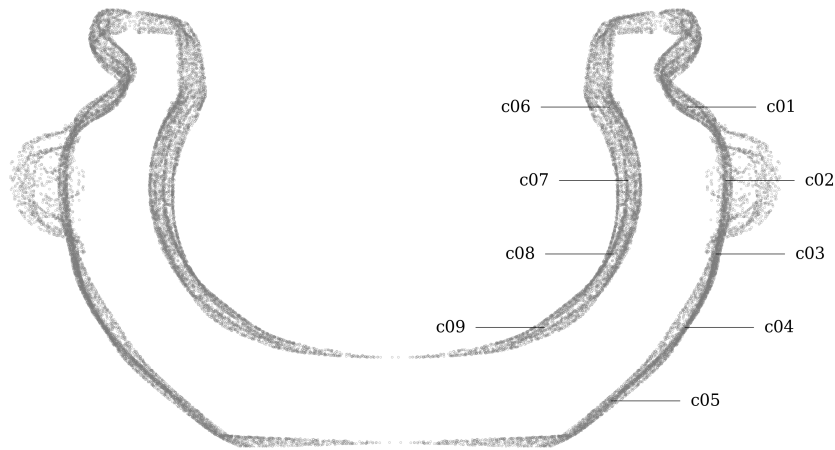
Figure 25: Concentricity measurement sample location on RV003.

Metric

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column							
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.955	170	3.339	3.339	1.140	1.140	1.122	1.122	−158, −942
c02	z-axis	0.509	154	2.699	2.699	0.657	0.657	0.403	0.403	139, 490
c03	z-axis	0.063	307	1.695	1.695	0.446	0.446	0.331	0.331	−63, −1
c04	z-axis	0.541	228	2.756	2.756	0.752	0.752	0.253	0.253	355, −408
c05	z-axis	0.212	146	1.817	1.817	0.509	0.509	0.379	0.379	208, −43
c06	z-axis	1.832	290	5.621	5.621	1.526	1.526	0.801	0.801	1256, −1333
c07	z-axis	2.161	270	4.734	4.734	1.518	1.520	1.342	1.352	874, −1977
c08	z-axis	1.379	309	3.323	3.323	1.099	1.099	0.991	0.991	260, −1354
c09	z-axis	0.604	111	2.841	2.841	0.863	0.863	0.737	0.737	−211, −566
c01	c06	1.467	170	3.339	3.339	1.140	1.140	1.122	1.122	−1414, 392
c02	c07	2.574	154	2.699	2.699	0.657	0.657	0.403	0.403	−735, 2467
c03	c08	1.391	307	1.695	1.695	0.446	0.446	0.331	0.331	−323, 1353
c04	c09	0.588	228	2.756	2.756	0.752	0.752	0.253	0.253	566, 159

Imperial

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column							
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0376	170	0.1315	0.1315	0.0449	0.0449	0.0442	0.0442	−6.2, −37.1
c02	z-axis	0.0200	154	0.1062	0.1062	0.0259	0.0259	0.0159	0.0159	5.5, 19.3
c03	z-axis	0.0025	307	0.0667	0.0667	0.0176	0.0176	0.0130	0.0130	−2.5, −0.0
c04	z-axis	0.0213	228	0.1085	0.1085	0.0296	0.0296	0.0100	0.0100	14.0, −16.1
c05	z-axis	0.0084	146	0.0715	0.0715	0.0201	0.0201	0.0149	0.0149	8.2, −1.7
c06	z-axis	0.0721	290	0.2213	0.2213	0.0601	0.0601	0.0315	0.0315	49.4, −52.5
c07	z-axis	0.0851	270	0.1864	0.1864	0.0598	0.0599	0.0528	0.0532	34.4, −77.8
c08	z-axis	0.0543	309	0.1308	0.1308	0.0433	0.0433	0.0390	0.0390	10.2, −53.3
c09	z-axis	0.0238	111	0.1119	0.1119	0.0340	0.0340	0.0290	0.0290	−8.3, −22.3
c01	c06	0.0578	170	0.1315	0.1315	0.0449	0.0449	0.0442	0.0442	−55.7, 15.4
c02	c07	0.1013	154	0.1062	0.1062	0.0259	0.0259	0.0159	0.0159	−28.9, 97.1
c03	c08	0.0548	307	0.0667	0.0667	0.0176	0.0176	0.0130	0.0130	−12.7, 53.3
c04	c09	0.0232	228	0.1085	0.1085	0.0296	0.0296	0.0100	0.0100	22.3, 6.2

Table 3: Concentricity analysis of RV003.

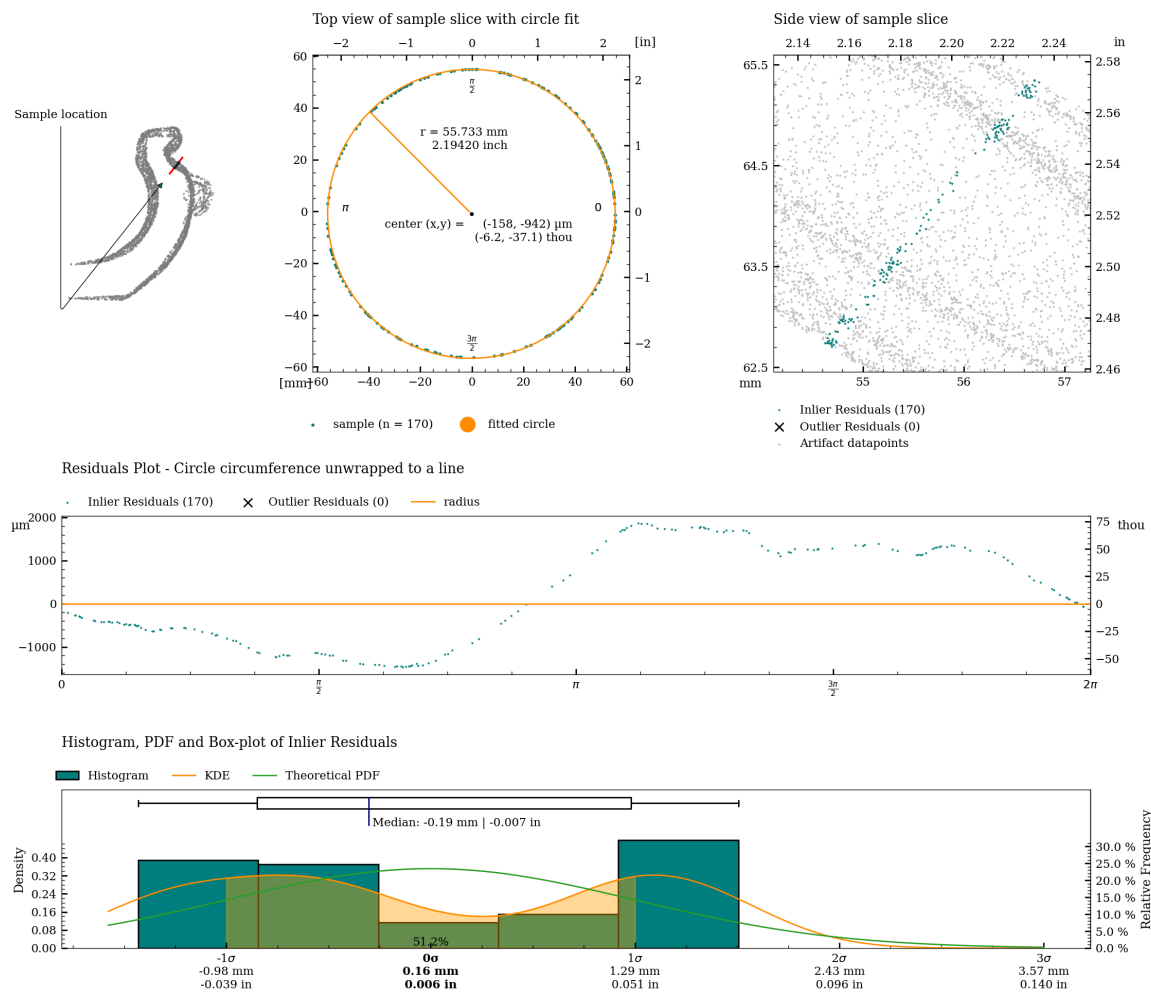# Concentricity analysis of c01

Figure 26: Detailed plot of concentricity measurement for c01.
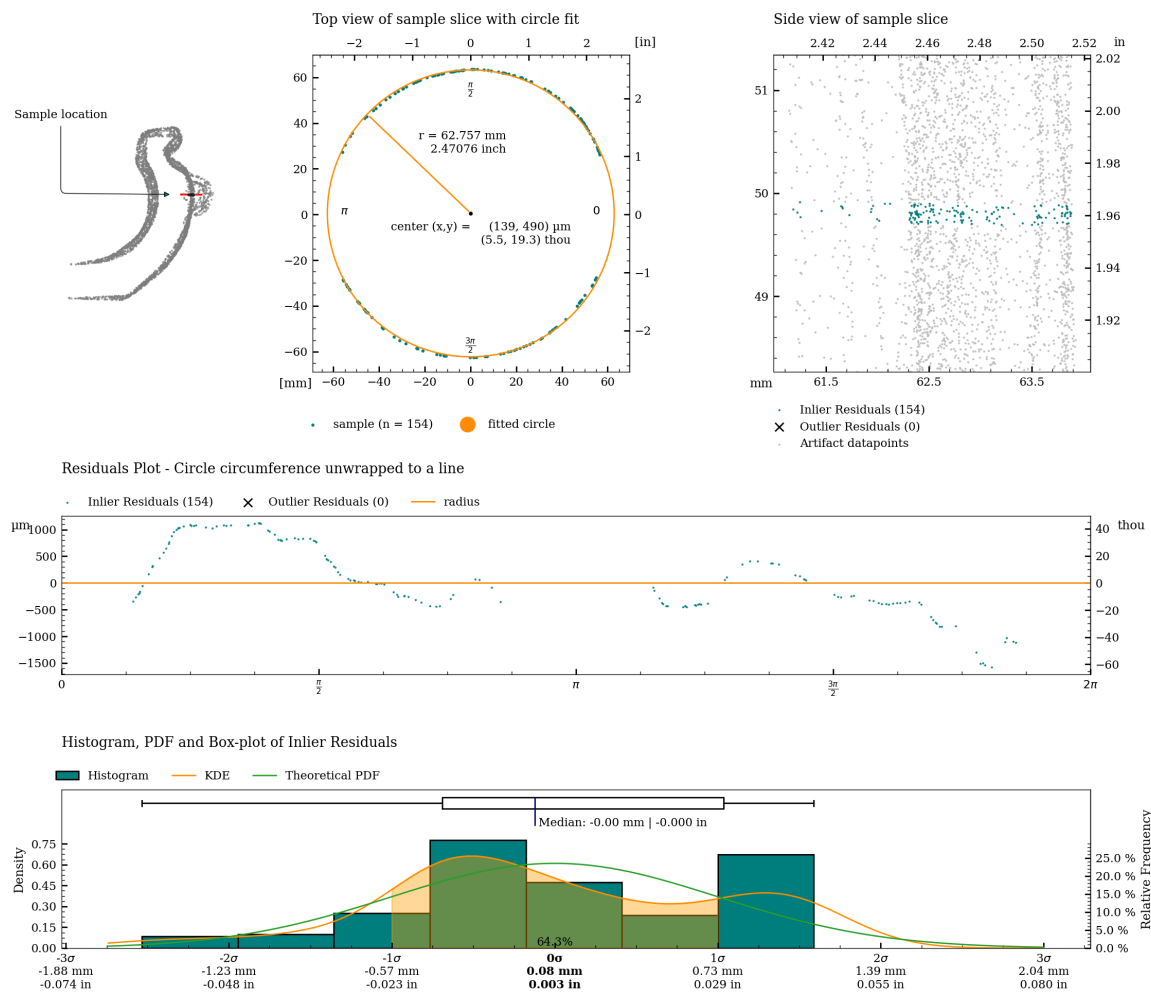
Concentricity analysis of c02

Figure 27: Detailed plot of concentricity measurement for c02.
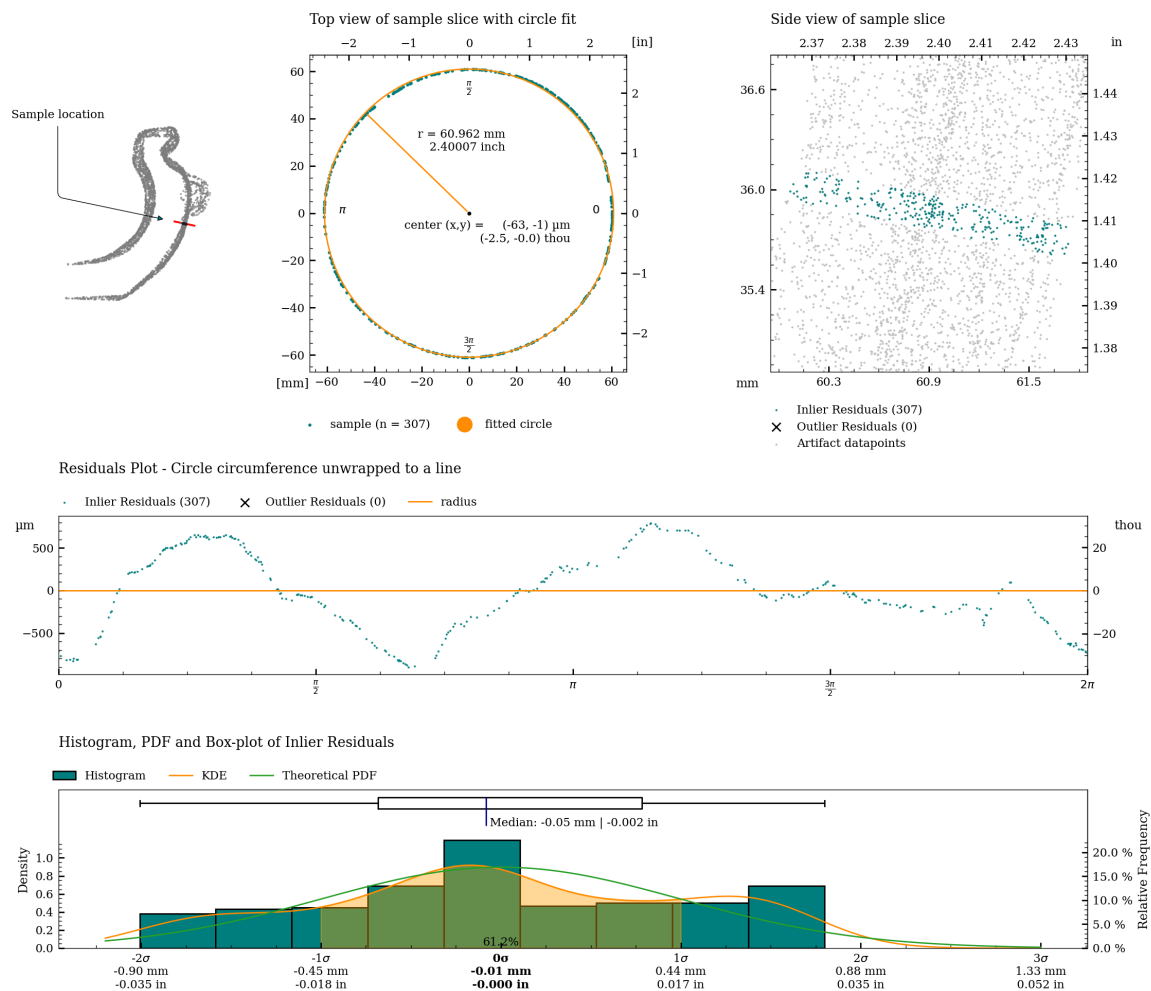
Concentricity analysis of c03

Figure 28: Detailed plot of concentricity measurement for c03.

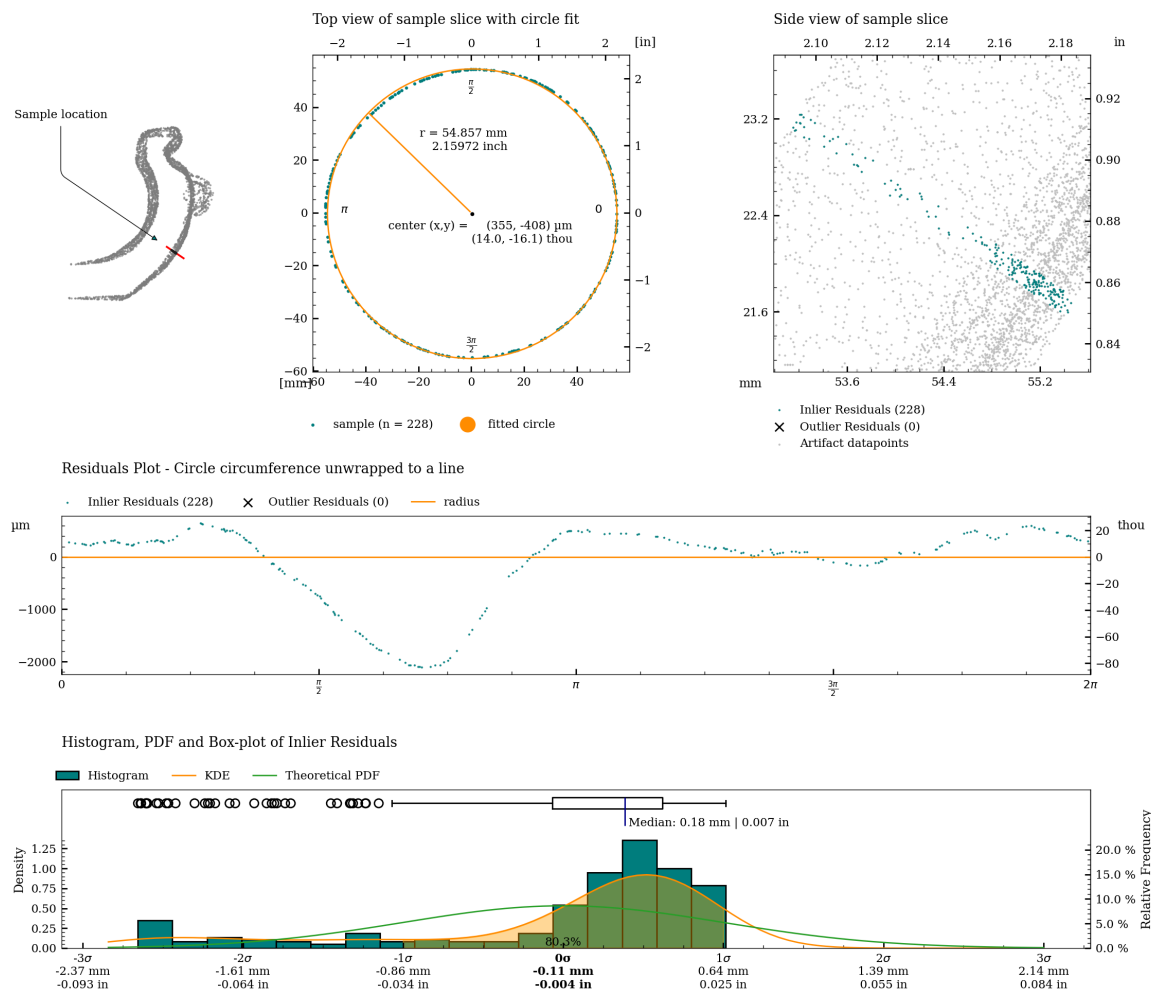# Concentricity analysis of c04

Figure 29: Detailed plot of concentricity measurement for c04.
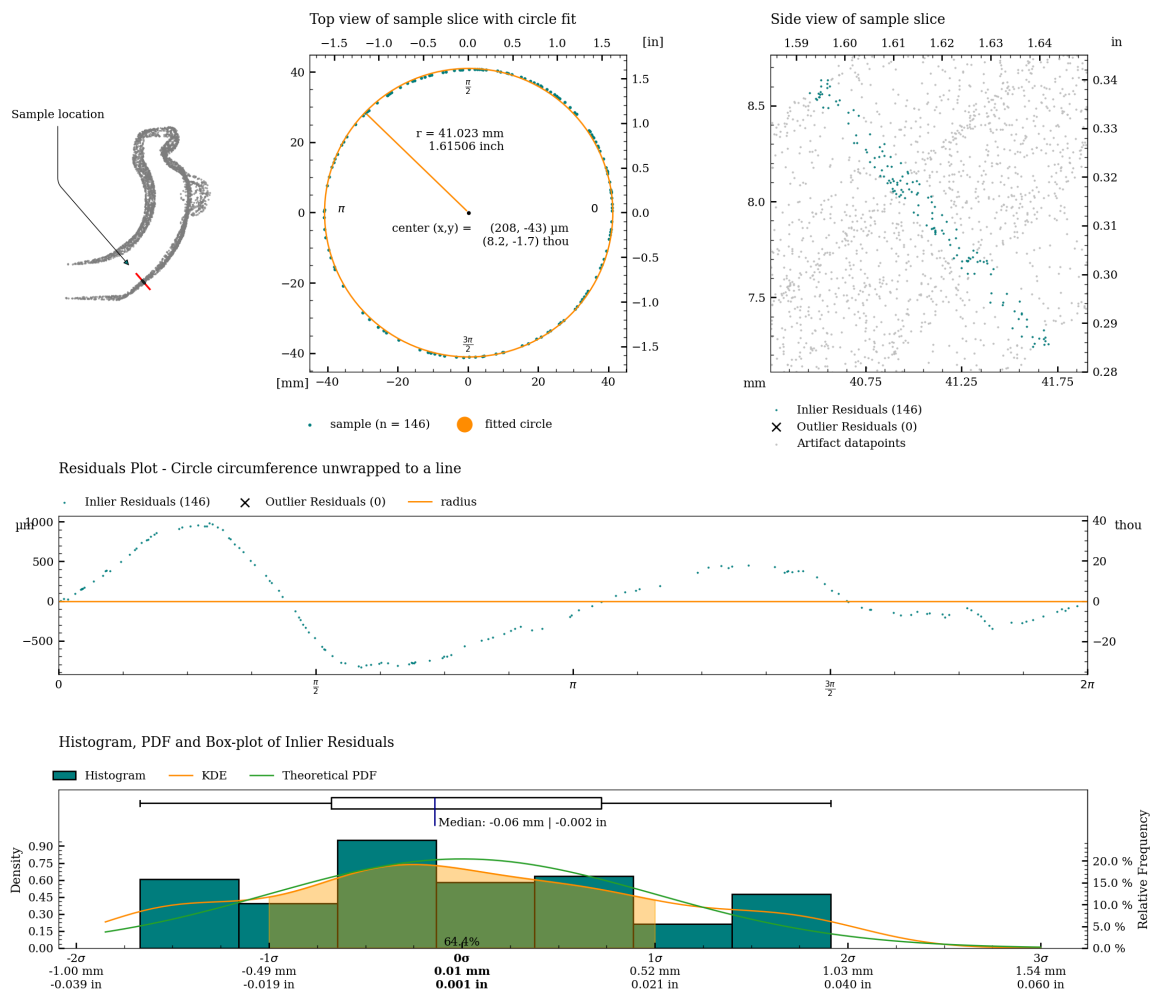
Concentricity analysis of c05

Figure 30: Detailed plot of concentricity measurement for c05.
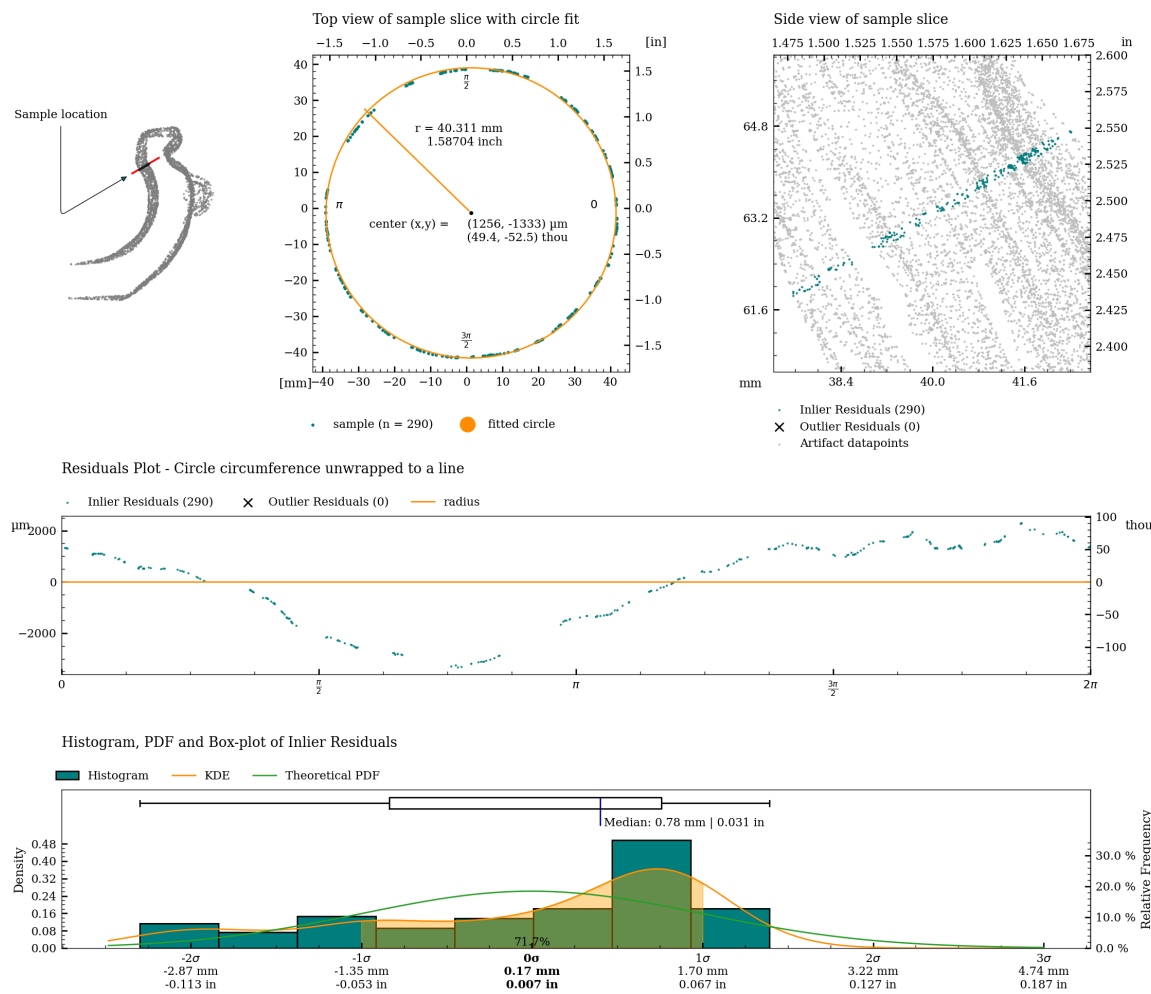
Concentricity analysis of c06

Top view of sample slice with circle fit

r = 40.311 mm
1.58704 inch

center (x,y) =   (1256, -1333) µm
(49.4, -52.5) thou

sample (n = 290)   fitted circle

Side view of sample slice

Inlier Residuals (290)
Outlier Residuals (0)
Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

Inlier Residuals (290)   Outlier Residuals (0)   radius

Histogram, PDF and Box-plot of Inlier Residuals

Histogram   KDE   Theoretical PDF

Median: 0.78 mm | 0.031 in

71.7%

	-2σ	-1σ	0σ	1σ	2σ	3σ
	-2.87 mm	-1.35 mm	**0.17 mm**	1.70 mm	3.22 mm	4.74 mm
	-0.113 in	-0.053 in	**0.007 in**	0.067 in	0.127 in	0.187 in

Figure 31: Detailed plot of concentricity measurement for c06.

Concentricity analysis of c07

Sample location

Top view of sample slice with circle fit

r = 44.620 mm
1.75671 inch

center (x,y) = (874, -1977) μm
(34.4, -77.8) thou

- sample (n = 270) ● fitted circle

Side view of sample slice

- Inlier Residuals (269)
- × Outlier Residuals (1)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Inlier Residuals (269) × Outlier Residuals (1) — radius

Histogram, PDF and Box-plot of Inlier Residuals

- Histogram — KDE — Theoretical PDF

Median: -0.33 mm | -0.013 in

54.3%

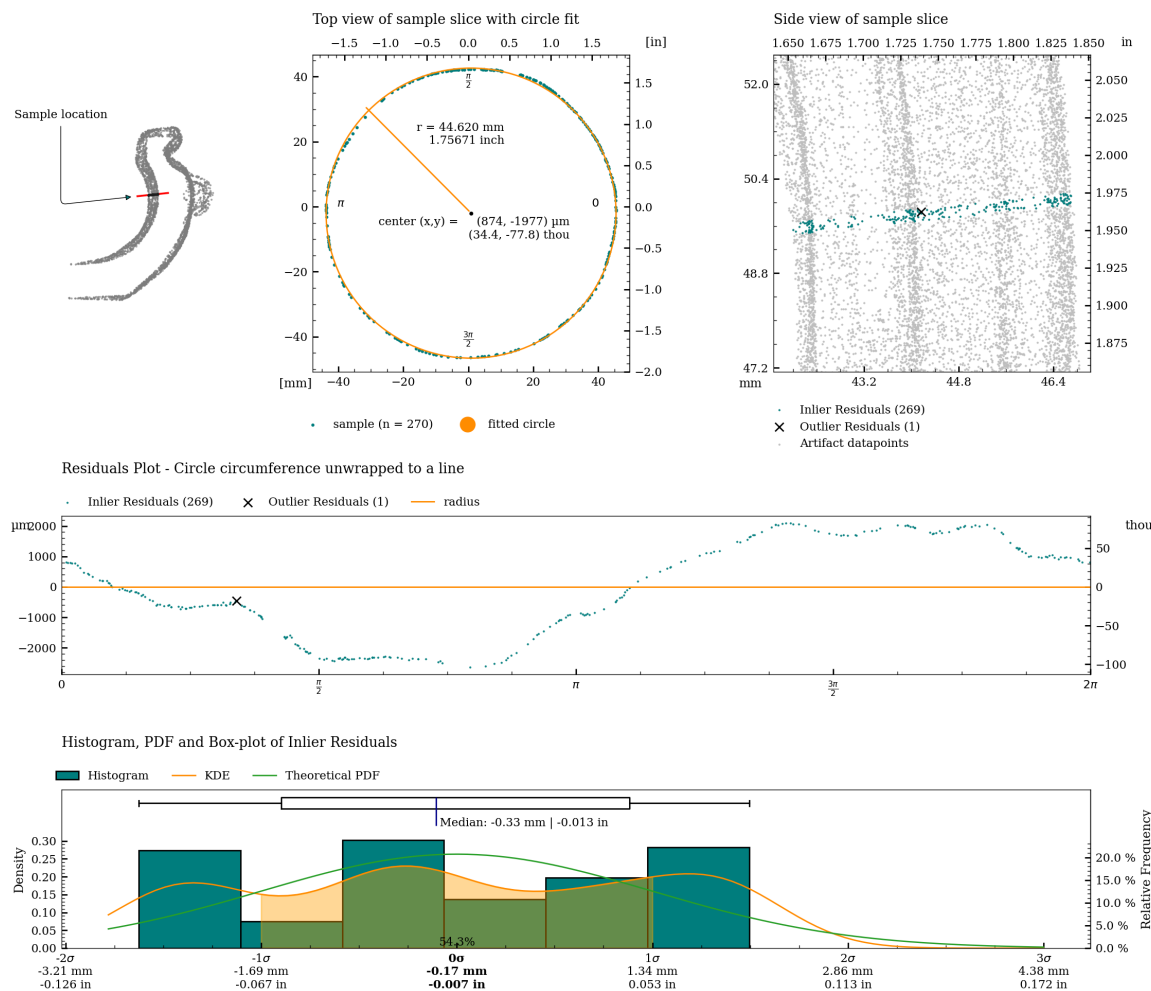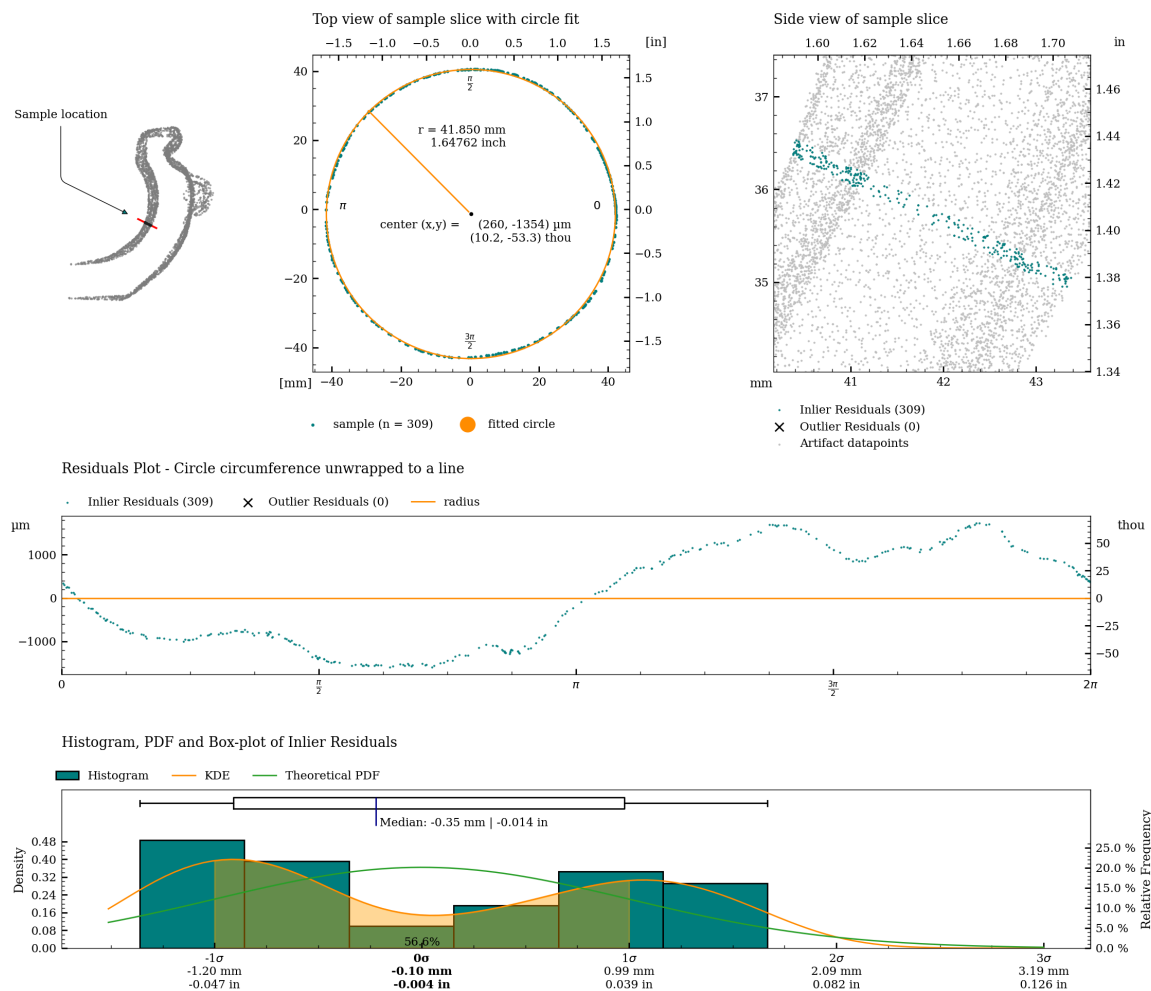|	-2σ	-1σ	0σ	1σ	2σ	3σ
	-3.21 mm	-1.69 mm	**-0.17 mm**	1.34 mm	2.86 mm	4.38 mm
	-0.126 in	-0.067 in	**-0.007 in**	0.053 in	0.113 in	0.172 in

Figure 32: Detailed plot of concentricity measurement for c07.

Concentricity analysis of c08

Figure 33: Detailed plot of concentricity measurement for c08.
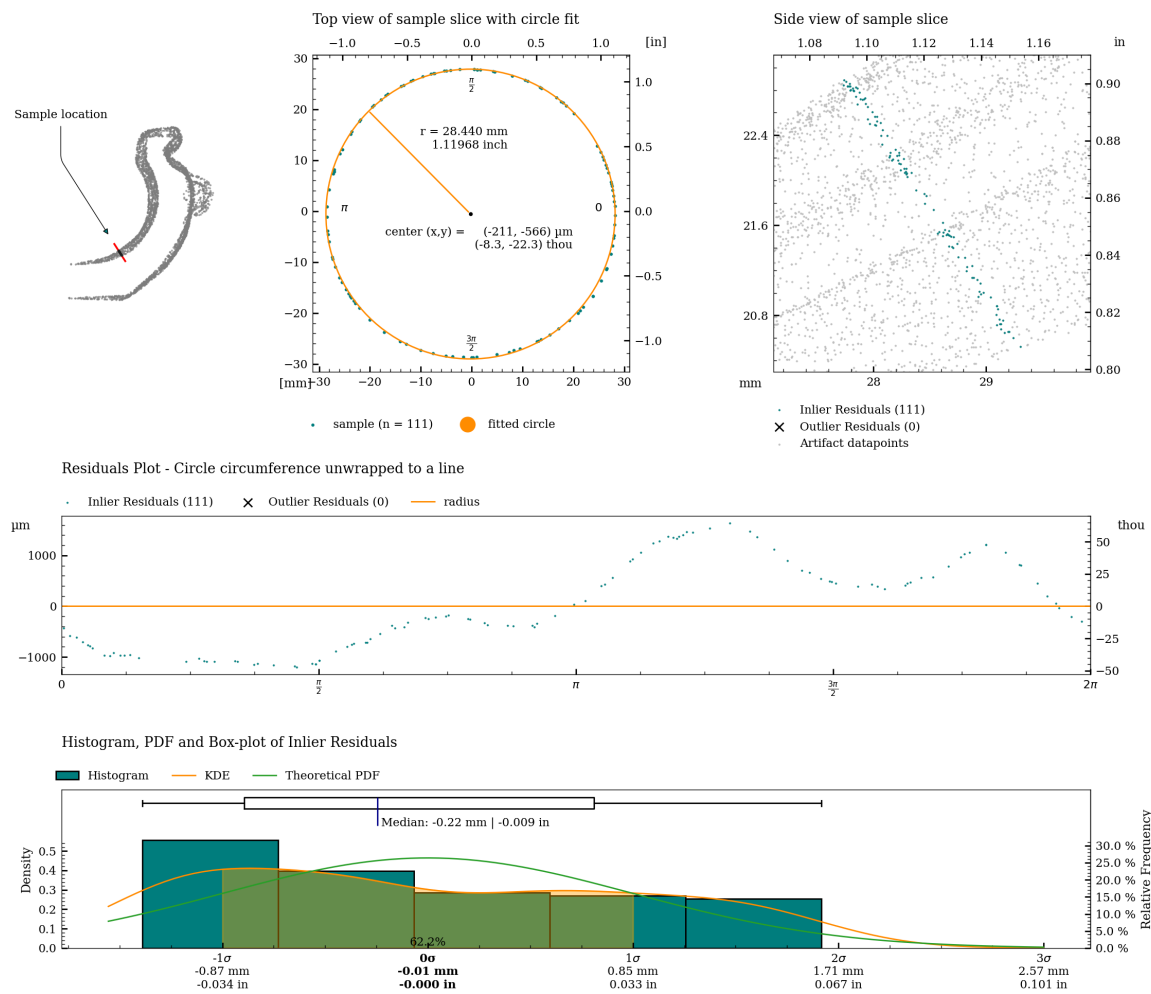
Concentricity analysis of c09

Sample location

Top view of sample slice with circle fit

r = 28.440 mm
1.11968 inch

center (x,y) = (-211, -566) μm
(-8.3, -22.3) thou

- sample (n = 111)
- fitted circle

Side view of sample slice

- Inlier Residuals (111)
- × Outlier Residuals (0)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Inlier Residuals (111)   × Outlier Residuals (0)   — radius

Histogram, PDF and Box-plot of Inlier Residuals

- Histogram   — KDE   — Theoretical PDF

Median: -0.22 mm | -0.009 in

62.2%

	-1σ	0σ	1σ	2σ	3σ
	-0.87 mm	-0.01 mm	0.85 mm	1.71 mm	2.57 mm
	-0.034 in	-0.000 in	0.033 in	0.067 in	0.101 in

Figure 34: Detailed plot of concentricity measurement for c09.

Coaxiality

Coaxiality is a measure of the deviation in the central axis of an object. Coaxiality measurements are calculated using RANSAC (Random sample consensus) algorithm for outlier detection of a least squares circle regression on cross-sections of the vessel (excluding potential handles) to estimate the best fit circle centers for each slice of the vessel.

Coaxiality is measured for:
- The exterior surface (excluding handles)
- The interior surface

	Exterior		Interior	
Analyzed Slices		271		225
Median sample size		189		210
Slice Height	200 µm	7.9 thou	200 µm	7.9 thou
Statistics with Z-axis as Reference				
Median Absolute Deviation (MAD)	228 µm	9.0 thou	1931 µm	76.0 thou
Standard Deviation (SD)	223 µm	8.8 thou	456 µm	18.0 thou
Root Mean Square Deviation (RMSD)	367 µm	14.4 thou	1851 µm	72.9 thou
Statistics with Best Fit Central Axis as Reference				
Best fit Central Axis Equation (in metric coordinate system with unit [mm])	x = 0.116 + t0.00102 y = -0.178 + t-0.00713 z = 0.000 + t-0.99997		x = -0.278 + t0.01862 y = -1.092 + t-0.00992 z = 0.000 + t0.99978	
Axis tilt		0.059°		1.077°
Median Absolute Deviation (MAD)	223 µm	8.8 thou	396 µm	15.6 thou
Standard Deviation (SD)	169 µm	6.6 thou	194 µm	7.6 thou
Root Mean Square Deviation (RMSD)	306 µm	12.1 thou	452 µm	17.8 thou

Table 4: Coaxiality analysis of vessel RV003.

Coaxiality plots, exterior surface
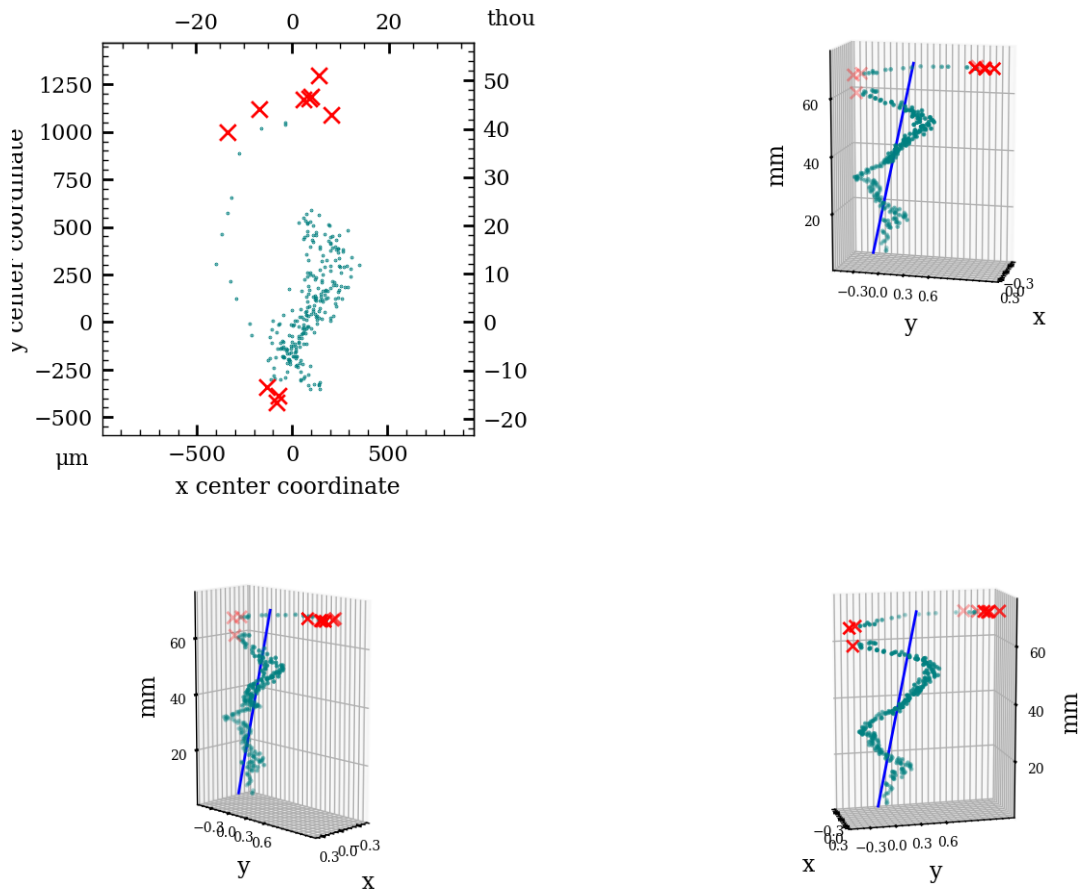
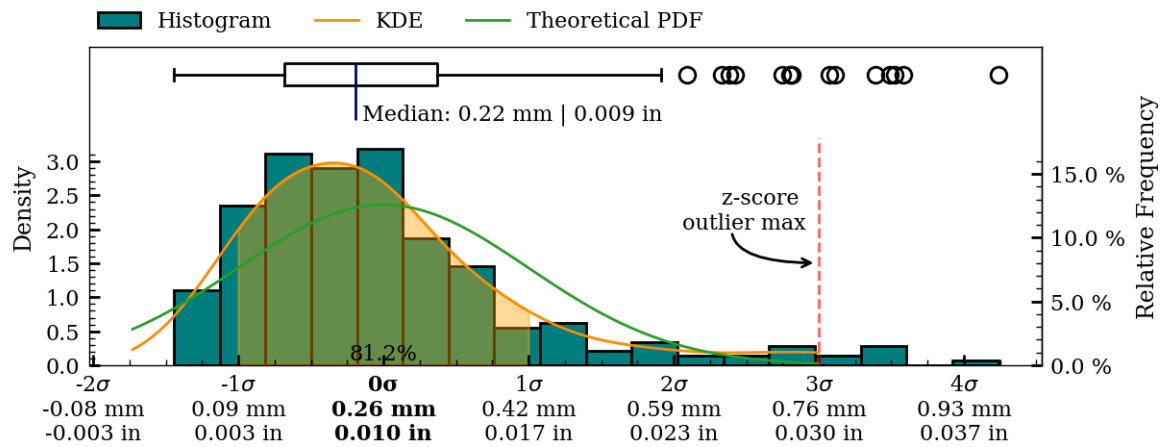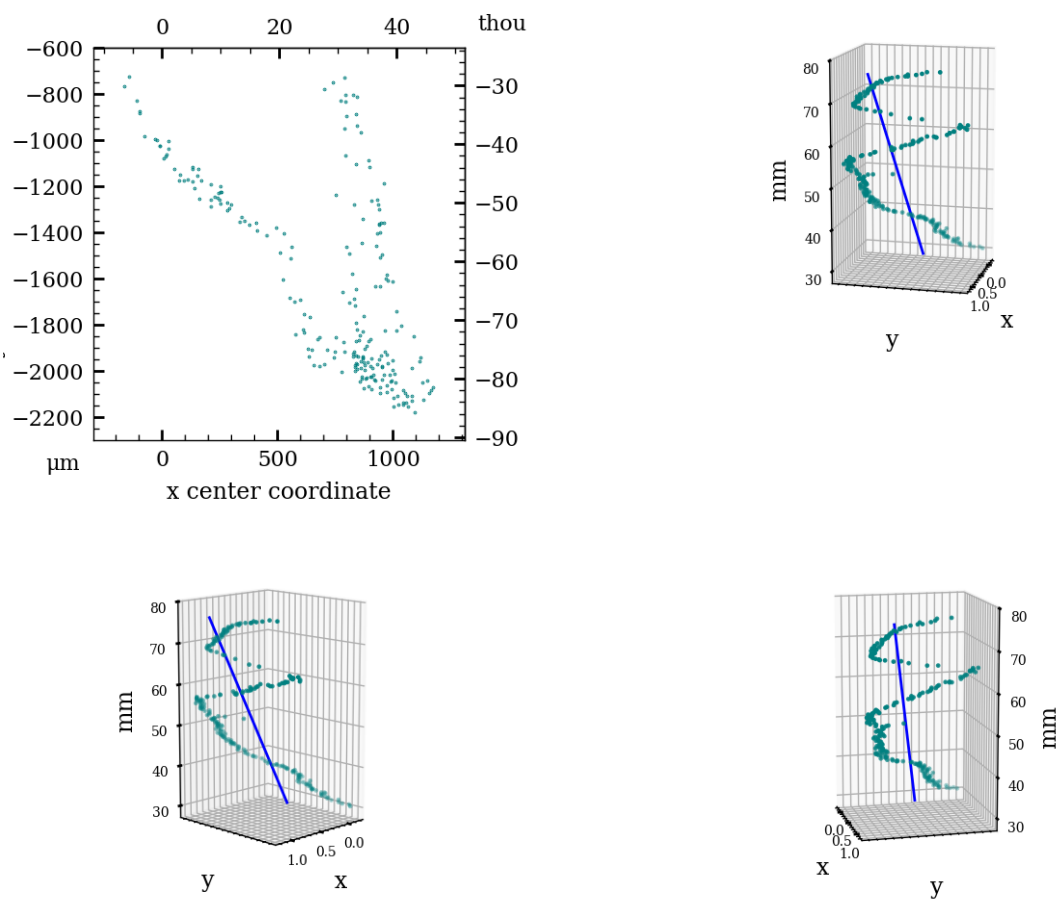## Coaxiality residuals from fitted axis, exterior surface

Figure 35: Coaxiality residual plots of exterior surface, RV003.

Coaxiality plots, interior surface

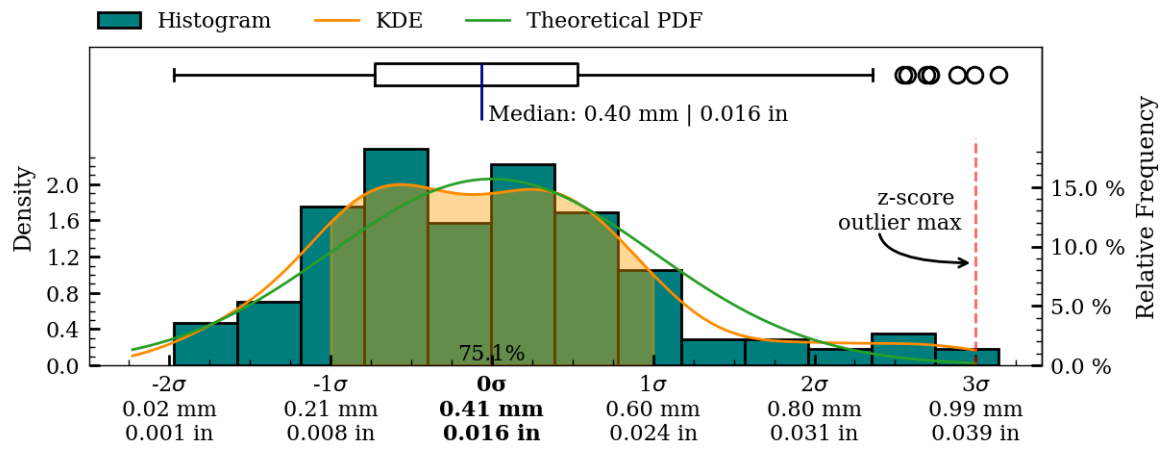## Coaxiality residuals from fitted axis, interior surface

Figure 36: Coaxiality residual plots of interior surface, RV003.

Surface Variability

To illustrate the overall surface deviations of the object, a surface variability heatmap has been created. This heatmap provides an accessible overview of the topography of the manufacturing precision and surface structure of the object.

The surface variability measurements are created by fitting a number of higher-order polynomials to the two-dimensional folded profile of the scan data. This process creates an idealized mathematical representation of actual surface curvature of object, and as such provides a continuous model representation of the actual object. It is important to note that only such a non-discretized representation is sufficient to avoid introducing inconsistently varying errors in the mapping of the final surface deviation results, that the rendered heatmaps are based on.

To produce the final surface variability map, the distance from each scanned vertex to the fitted polynomial is calculated and used as the mapping function input, for applying colours to the surface of the object.

It is important to note that this variability map does not describe deviations from the original *intended* shape of the artifact (if any), as this shape (the *intended design*, so to speak) will have been lost to time. It does however provide a very informative visualization of the texture and structure of the surface and very importantly, *does* hightlight potential manufacturing-relevant patterns in the surface texture (if present). Such patterns are, as an example, clearly evident on the interior surface of artifact PV001.
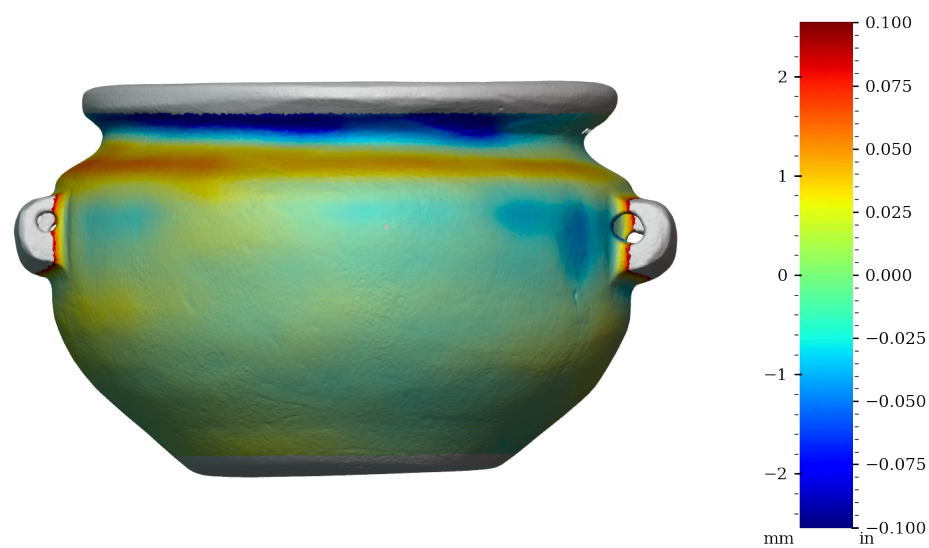
Exterior surface

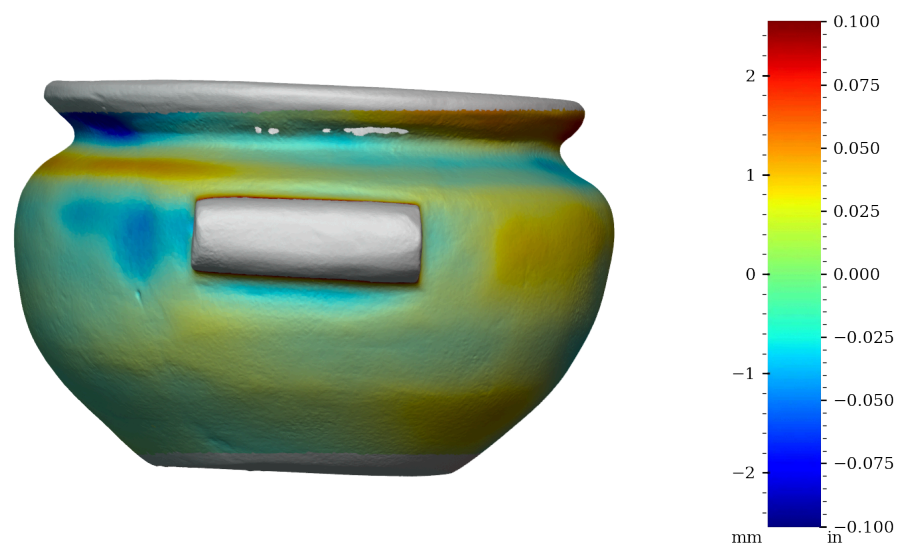Figure 37: Surface variability heatmap of RV003, front view

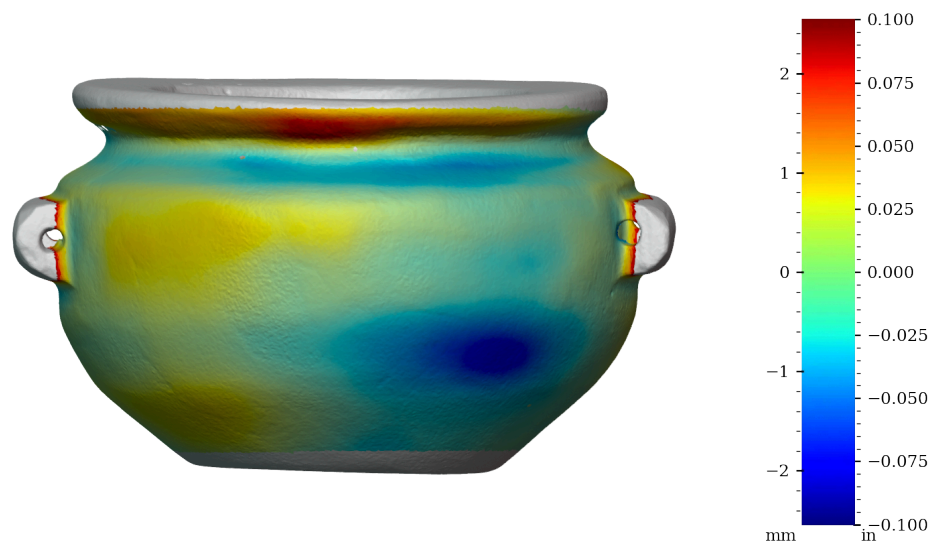Figure 38: Surface variability heatmap of RV003, rotated 90°

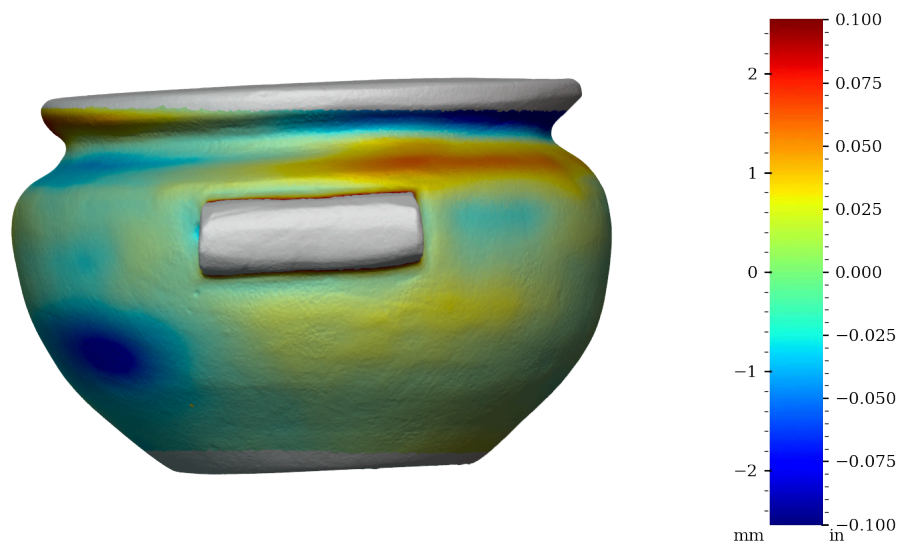Figure 39: Surface variability heatmap of RV003, rotated 180°

Figure 40: Surface variability heatmap of RV003, rotated 270°
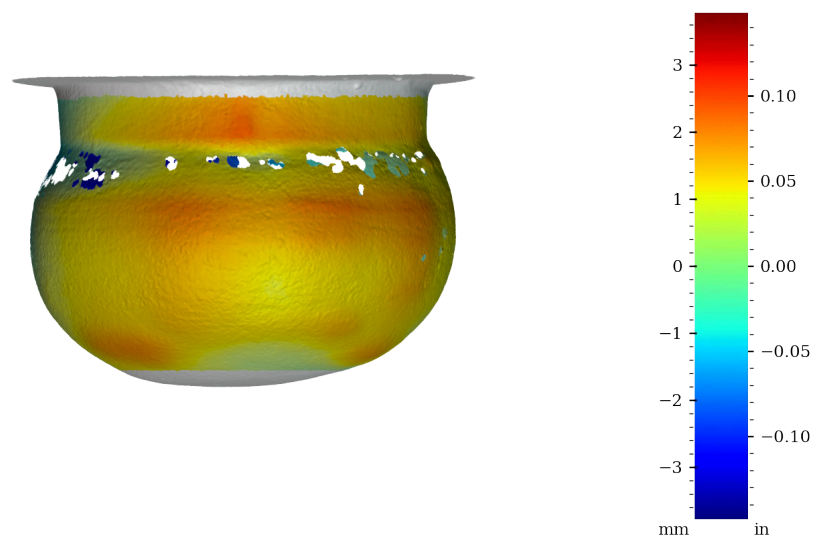
Interior surface

Figure 41: Surface variability heatmap of RV003, front view
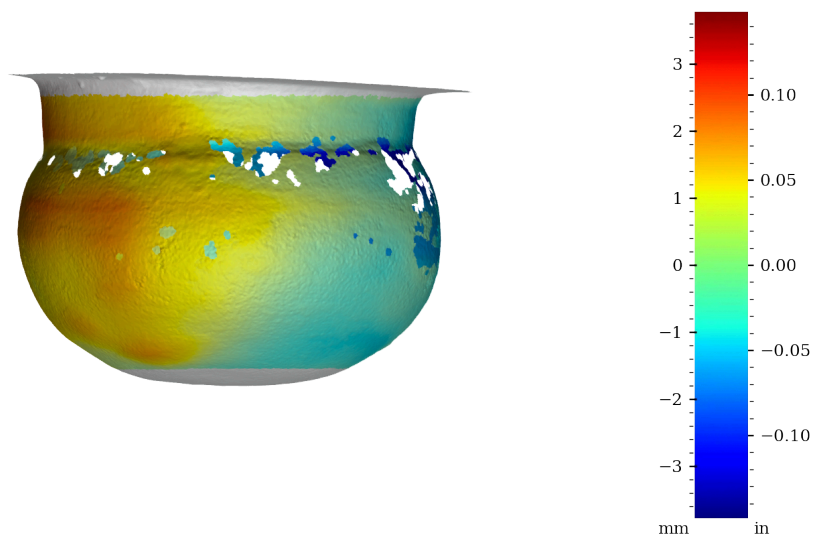

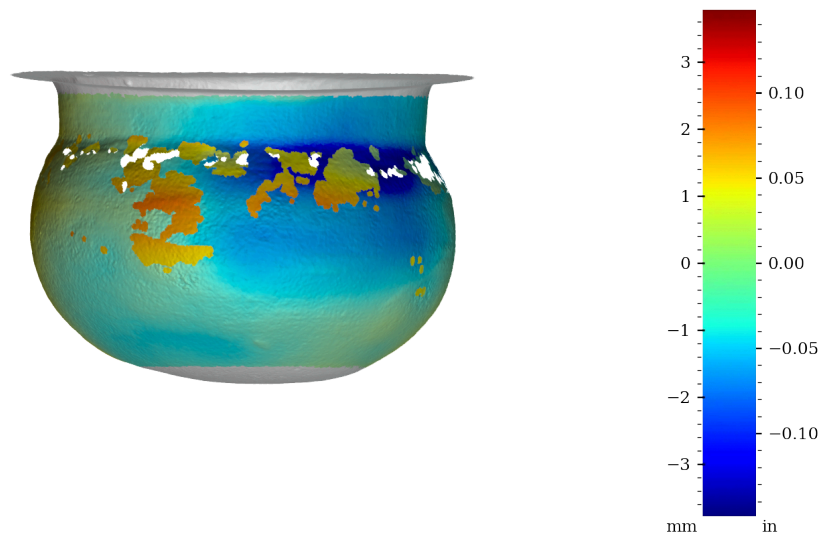Figure 42: Surface variability heatmap of RV003, rotated 90°

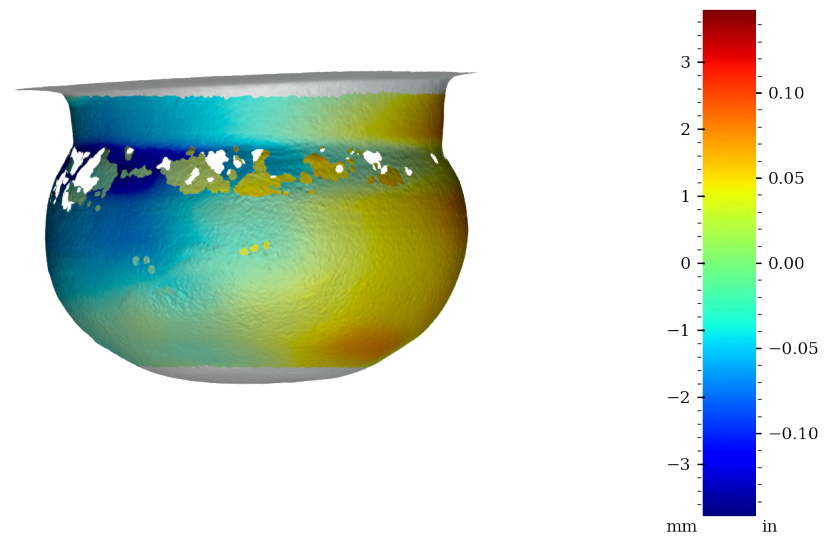Figure 43: Surface variability heatmap of RV003, rotated 180°

Figure 44: Surface variability heatmap of RV003, rotated 270°

Surface variability statistics

Area	MSD	RMSD	SD	Mean AD	Median AD	Range	Min	Max	Sample size
	mm^2	mm	mm	mm	mm	mm	mm	mm	
Exterior	0.6592	0.812	0.812	0.414	0.596	5.015	−2.469	2.545	107617
Interior	1.9506	1.397	1.395	1.107	1.186	6.586	−3.768	2.818	90346
	in^2	in	in	in	in	in	in	in	
Exterior	0.001022	0.0320	0.0320	0.0163	0.0235	0.1974	−0.0972	0.1002	107617
Interior	0.003023	0.0550	0.0549	0.0436	0.0467	0.2593	−0.1483	0.1110	90346

Table 5: Surface variability statistics, RV003

Table 5 shows the statistics of the distance from the scan vertices to the best fit object model. These statistics are briefly explained below.

Mean Squared Deviation (MSD), also known as Mean Squared Error (MSE).

$$\text{MSD} = \frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}$$

The MSD metric shows the the average squared difference between the scanned points and the fitted composite polynomial model (a value of 0 would be a perfect match). This metric emphasizes imperfections in the surface of the artifact. Outliers will negatively influence this metric, raising the value of the MSE.

Root Mean Squared Deviation (RMSD), also known as Root Mean Squared Error (RMSE).

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}}$$

Measures the dispersion of the measured surface variability y_i around a model predictor ($\hat{y}$). By obtaining the root of the MSD, the exponent will be removed from the measurement, enabling comparisons with other statistics of the same unit and making it more accessible to those familiar with the RMSD metric. This measure is used to assess the fit of a regression model to a dataset, in this case our best fit composite polynomial model. The lower the RMSD metric, the better the fit.

Standard Deviation (SD)

$$s = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n - 1}}$$

Measures the dispersion of the measured surface variability y_i around the mean ($\bar{y}$). If the residuals are normally distributed around the mean ($\bar{y} \approx 0$), the SD will be equal to the RMSD. See Figure 45 and Figure 46

Mean Absolute Deviation (MeanAD)

$$\text{MeanAD} = \frac{\sum_{i=1}^{n} |y_i - \bar{y}|}{n}$$

This metric is similar to the SD, but the difference between the residuals and the mean is *not* squared. Instead of indicating the spread of the data, we look at the average distance between each data point and the mean. The Mean Absolute Deviation is affected less by outliers than the Standard Deviation.

Median Absolute Deviation (MedianAD)

$$\text{MedianAD} = \text{median}(|y_i - \text{median}(y)|)$$

The Median Absolute Deviation is measure of the dispersion of the data around the median.

Range

$$\max(y_i) - \min(y_i)$$

Range is a measure of the total spread of the residuals

Histogram, KDE and Box-plot of measured surface variability - exterior surface
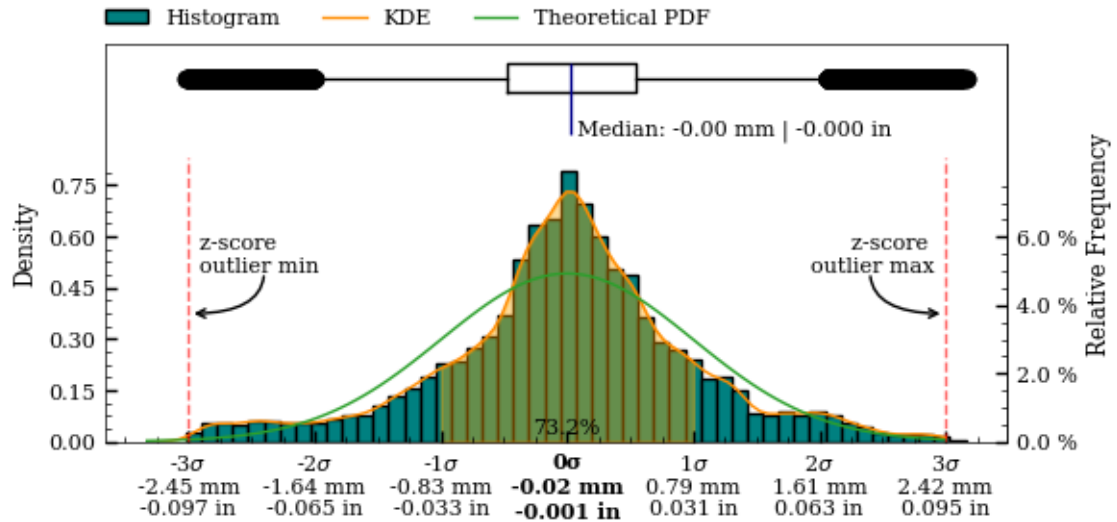

Figure 45: Exterior surface variability boxplot, kds and histogram.

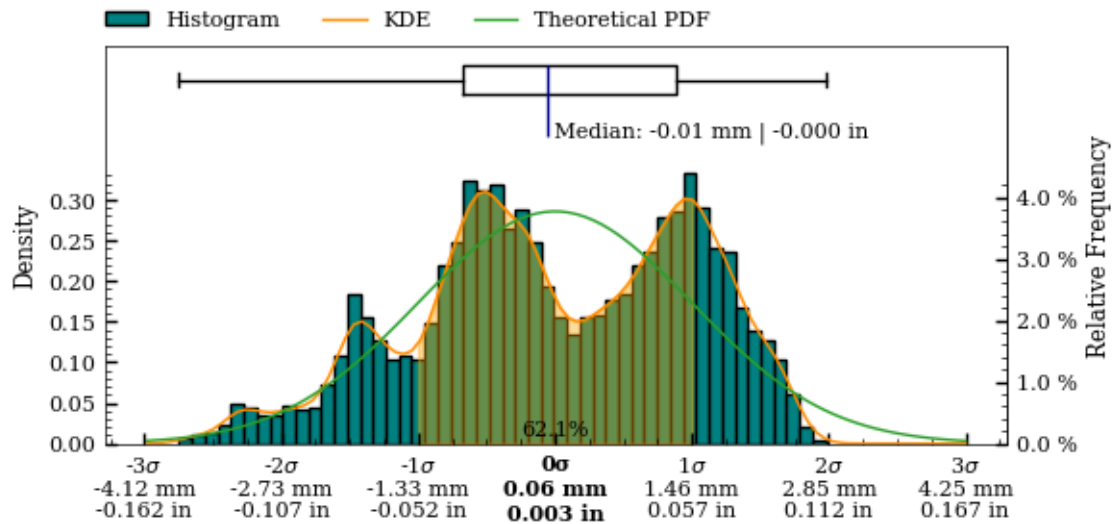**Histogram, KDE and Box-plot of measured surface variability - interior surface**

Figure 46: Interior surface variability boxplot, kds and histogram.

Precision Score Of The Artifact

To enable valid comparison of the manufacturing precision of different artifacts, a metric that robustly quantifies the overall precision of the object is required. The considerations for such a metric will be explored in this section.

Based on these considerations, a *Precision Score* metric will be defined.

For an object to be described as having been manufactured with high precision, several qualities must be present *concurrently*, and throughout the *entire* geometry of the final object. A given object may exhibit high levels of one or more *components* of precision, but be lacking in others. For example:

- An object may present high levels of coaxiality, but lack circularity.
- An object may exhibit good circularity, but show imperfections in the surface structure.
- An object may be smoothed to perfection *without* any circularity or coaxiality.
- An object may exhibit high levels of all of the above metrics in *some* areas, but not in others.

Therefore, a precision score metric **must** account for *all* aspects of the individual, underlying precision metrics (circularity, concentricity, coaxiality and surface variability) throughout the *entire* surface area of the object.

The composite high order polynomial model, used to generate the surface variability map (described in Surface Variability, p. 39) is the best continuous mathematical representation of the object available to us (lacking any original design plans, as would normally be available in metrological analysis). This idealized model encompasses all of the above component metrics.

In the creation of the model, all scan data-points are taken into account (excluding areas with extensive damage), making it the best possible idealized representation we can achieve. When this model has been accurately created, the deviation between the model and the scanned data-points can be calculated over the non-discretized polynomials, *without* the need for an "original" CAD model (and importantly, unless such a CAD model *actually* corresponded to the original design intent, it would be an insufficient comparison basis).

Within the context of defining a valid, overall precision metric, this approach satisfies the incorporation of all of the necessary metrics:

- **Circularity**: Because the reconstructed polynomial model is revolved around the Z-plane, the idealized representation is perfectly circular, and thus incorporates the circularity component.
- **Concentricity and coaxiality**: Because the Z-axis (datum axis) is the center axis of the model, it incorporates the concentricity and coaxiality components.
- **Surface variability**: Because the model is continuous and non-discretized, it can be used accurately for all points of the scan data, and incorporates the surface variability component.

The level of precision ultimately achieved in a physical object does not share a linear relationship with its manufacturing requirements. Since continuously higher levels of final precision becomes progressively harder to achieve, an overall precision metric must take this relationship into account.

A robust statistical metric that satisfies this requirement is the *Mean Squared Deviation* (MSD or MSE). Here specifically, we can utilize the mean square of the deviations between the model ($\hat{y}$) and the data-points (y_i).

Combining all of the above considerations, we can express a well-defined *Precision Score* metric, that provides an immediately accessible way to understand the overall precision of an object, while being statistically valid. Since the Mean Squared Deviation tends towards zero as the overall precision increases, the inverse of the Mean Squared Deviation is taken to obtain a precision score metric that increases as precision increases[12]:

$$\text{Precision Score} = \frac{n}{\sum_{i=1}^{n} (y_i - \hat{y})^2}$$

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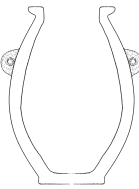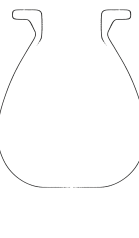
[12]The precision score unit is $\frac{1}{\text{mm}^2}$

A precision score will be calculated seperately for:

- The exterior surface
- The interior surface
- The full surface

As most scans do not include sufficient scan data for the interior surface, the exterior surface will be used for calculating the precision score in most cases. In the rare case that the scan data is more complete for the *interior* surface, this will be used instead.

Table 6 shows the precision score of this artifact (RV003), compared to the two most precise, and the two least precise vessels currently analyzed.

Artifact			Material	Precision Score	Link to Report
		PV001	Red Granite	**1905** Full: 980 Exterior: 1905 Interior: 705	Report Publication
		PV006	Dark grey granite	**621** Full: 521 Exterior: 621 Interior: 152	Report Publication
		RV003	Marble breccia	**1.52** Full: 0.80 Exterior: 1.52 Interior: 0.51	Report Publication
		MV010	Calcite (Egyptian Alabaster)	**1.12** Full: 0.64 Exterior: 1.12 Interior: 0.20	Report Publication

Analysis Roadmap

While the current iteration of this work already provides valuable results, continued future additions and improvements will enhance their utility further. This section details planned iterative updates and improvements, to both the reports themselves, and to the underlying methodology and software they are created with.

Alignment Section
- Detailed exploration of different circle regression algorithms
- If handles are present on the vessel, exploring alignment of the vessels so the handle positions match each other
- Add optimization of the perpendicular surface deviation, with the best results of the coaxial alignment
- Align by minimizing circularity results (of rotated sample slice, to compensate for sample height distortions)

Measurements of Precision
- Section detailing how measurements perpendicular to the surface curvature are obtained
- Detailed surface area analysis, exploring the residual patterns throughout subsequent sample slices of the artifact surface
- Wall thickness deviation color map
- Robust outlier identification on circularity, to better handle analysis of damaged areas of the artifacts in addition to removal of interior crystalline structure points present in CT scans
- Layout updates to the charts and tables

Visibility of Outliers and Damaged Sections
- Identification and marking of damaged parts
- Visualization of outliers on the artifact surface

Exploration of Mathematical Primitives
- Analysis of selected curvatures and flat surfaces on the vessel in both the horizontal and vertical planes
  - ‣ Circles
  - ‣ Parabolas
  - ‣ Ellipsoids
  - ‣ Hyperbolas
  - ‣ Cones
- Implementation of robust regressions models suitable for this domain, based on RANSAC.

Metrics on Primary Features
- Measurements of features in the horizontal plane
- Measurements of features in the vertical plane
- Measurements of angles
- Measurements of volume

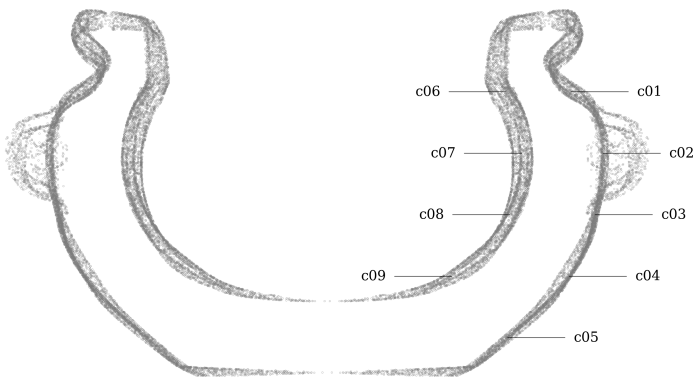**Exploration of Potential Design Ratios**
- π, φ, e, 1, 2, 3, 4 etc.

Raw Dataset Attachments
- Including all measurement and sample coordinates as CSV-files embedded in the report
- Including an STL file of the aligned object alongside the report, for easier external replication and validation of the research results

Appendix A - Comparison Of Circularity Measurements (Z-plane vs. surface-perpendicular)

Comparison of circularity samples

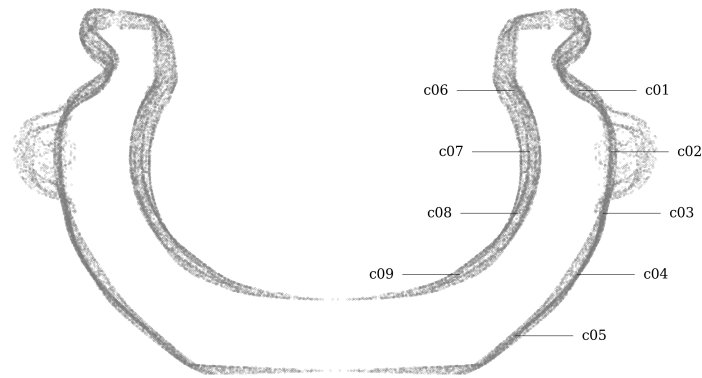## Samples perpendicular to the surface curvature

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø110.931±2.034	3.339	1.182	1.146	1.140	170	0.200	63.743	55.465
c02	exterior	Ø125.541±1.591	2.699	0.658	0.414	0.657	154	0.200	49.799	62.770
c03	exterior	Ø121.887±0.885	1.695	0.445	0.324	0.446	307	0.200	35.856	60.944
c04	exterior	Ø110.108±2.271	2.756	0.800	0.254	0.752	228	0.200	21.912	55.054
c05	exterior	Ø82.106±0.964	1.817	0.508	0.360	0.509	146	0.200	7.969	41.053
c06	interior	Ø81.543±3.727	5.621	1.540	1.006	1.526	290	0.200	63.743	40.771
c07	interior	Ø88.479±2.474	4.734	1.529	1.368	1.518	270	0.200	49.799	44.239
c08	interior	Ø83.181±1.960	3.323	1.105	1.018	1.099	309	0.200	35.856	41.591
c09	interior	Ø56.769±1.668	2.841	0.859	0.742	0.863	111	0.200	21.912	28.385

Table 7: Detailed circularity measurements at selected samples in z-plane, vessel RV003.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø110.816±3.691	6.042	1.891	1.671	1.830	469	0.200	63.743	55.408
c02	exterior	Ø125.495±1.568	2.703	0.656	0.393	0.651	153	0.200	49.799	62.748
c03	exterior	Ø121.836±0.902	1.745	0.440	0.322	0.440	302	0.200	35.856	60.918
c04	exterior	Ø110.137±2.774	3.340	0.974	0.296	0.905	335	0.200	21.912	55.069
c05	exterior	Ø82.189±1.570	3.055	0.749	0.502	0.750	290	0.200	7.969	41.094
c06	interior	Ø80.805±3.511	6.179	2.000	1.635	1.959	358	0.200	63.743	40.402
c07	interior	Ø88.233±2.630	4.823	1.594	1.526	1.577	312	0.200	49.799	44.116
c08	interior	Ø83.686±1.987	3.733	1.182	1.116	1.183	377	0.200	35.856	41.843
c09	interior	Ø56.920±3.942	6.378	1.912	1.690	1.900	362	0.200	21.912	28.460

Table 8: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel RV003.

Samples perpendicular to the surface curvature

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø4.3674±0.0801	0.1315	0.0465	0.0451	0.0449	170	0.0079	2.5096	2.1837
c02	exterior	Ø4.9426±0.0626	0.1062	0.0259	0.0163	0.0259	154	0.0079	1.9606	2.4713
c03	exterior	Ø4.7987±0.0348	0.0667	0.0175	0.0128	0.0176	307	0.0079	1.4116	2.3994
c04	exterior	Ø4.3350±0.0894	0.1085	0.0315	0.0100	0.0296	228	0.0079	0.8627	2.1675
c05	exterior	Ø3.2325±0.0379	0.0715	0.0200	0.0142	0.0201	146	0.0079	0.3137	1.6163
c06	interior	Ø3.2103±0.1467	0.2213	0.0606	0.0396	0.0601	290	0.0079	2.5096	1.6052
c07	interior	Ø3.4834±0.0974	0.1864	0.0602	0.0539	0.0598	270	0.0079	1.9606	1.7417
c08	interior	Ø3.2749±0.0771	0.1308	0.0435	0.0401	0.0433	309	0.0079	1.4116	1.6374
c09	interior	Ø2.2350±0.0657	0.1119	0.0338	0.0292	0.0340	111	0.0079	0.8627	1.1175

Table 9: Detailed circularity measurements at selected samples in z-plane, vessel RV003.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø4.3628±0.1453	0.2379	0.0744	0.0658	0.0721	469	0.0079	2.5096	2.1814
c02	exterior	Ø4.9408±0.0617	0.1064	0.0258	0.0155	0.0256	153	0.0079	1.9606	2.4704
c03	exterior	Ø4.7967±0.0355	0.0687	0.0173	0.0127	0.0173	302	0.0079	1.4116	2.3983
c04	exterior	Ø4.3361±0.1092	0.1315	0.0384	0.0116	0.0356	335	0.0079	0.8627	2.1681
c05	exterior	Ø3.2358±0.0618	0.1203	0.0295	0.0198	0.0295	290	0.0079	0.3137	1.6179
c06	interior	Ø3.1813±0.1382	0.2433	0.0787	0.0644	0.0771	358	0.0079	2.5096	1.5906
c07	interior	Ø3.4737±0.1035	0.1899	0.0628	0.0601	0.0621	312	0.0079	1.9606	1.7369
c08	interior	Ø3.2947±0.0782	0.1470	0.0465	0.0440	0.0466	377	0.0079	1.4116	1.6474
c09	interior	Ø2.2409±0.1552	0.2511	0.0753	0.0666	0.0748	362	0.0079	0.8627	1.1205

Table 10: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel RV003.

Comparison of circularity on the full vessel surface

Metric

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	1.830	1.096	3.968	0.439	0.235	0.918	0.068	0.150	0.700	271	0.200
Interior	4.267	1.594	5.179	1.333	0.427	1.760	0.381	0.129	1.840	225	0.200

Table 11: Detailed circularity measurements at selected samples in z-plane, vessel RV003.

Samples in the z-plane

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	2.943	1.545	6.751	0.761	0.393	2.290	0.110	0.227	2.383	359	0.200
Interior	4.862	3.444	6.464	1.540	1.124	1.965	0.192	0.763	1.925	285	0.200

Table 12: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel RV003.

Imperial

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	1.830	1.096	3.968	0.439	0.235	0.918	0.068	0.150	0.700	271	0.200
Interior	4.267	1.594	5.179	1.333	0.427	1.760	0.381	0.129	1.840	225	0.200

Table 13: Detailed circularity measurements at selected samples in z-plane, vessel RV003.

Samples in the z-plane

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	2.943	1.545	6.751	0.761	0.393	2.290	0.110	0.227	2.383	359	0.200
Interior	4.862	3.444	6.464	1.540	1.124	1.965	0.192	0.763	1.925	285	0.200

Table 14: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel RV003.

Circularity analysis of exterior samples perpendicular to surface curvature
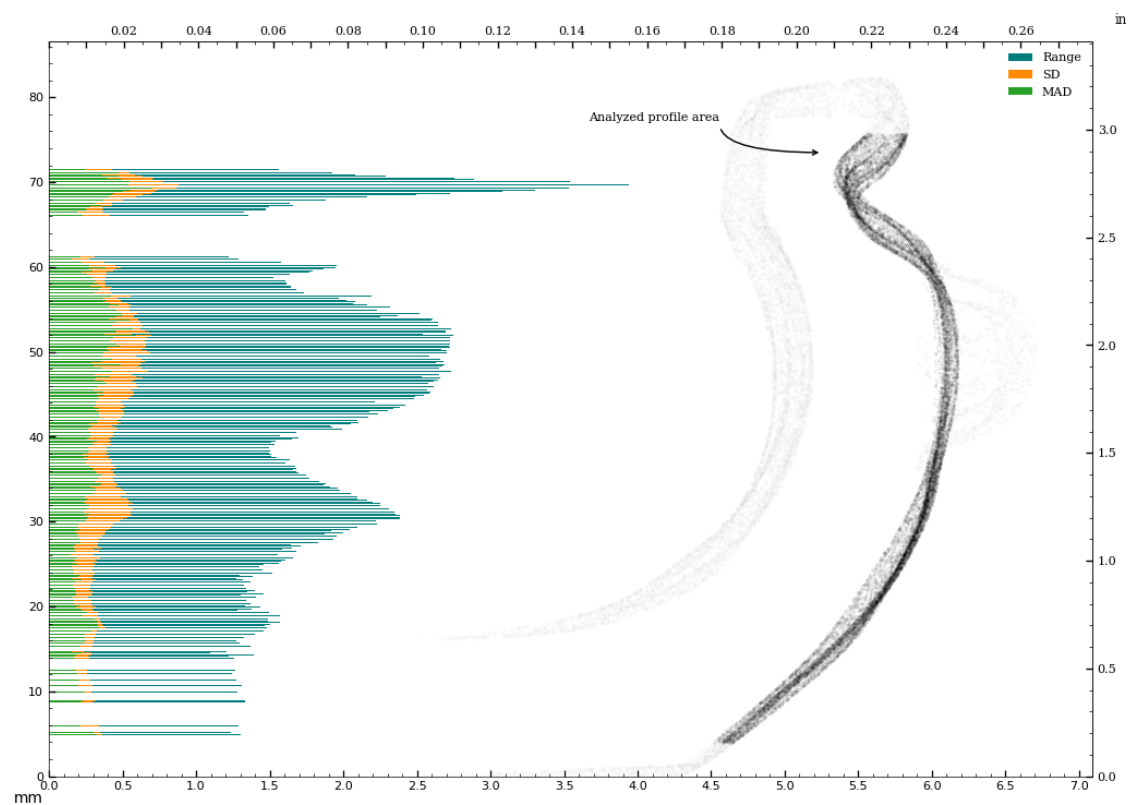

Figure 47: Circularity analysis of exterior samples perpendicular to surface curvature

Circularity analysis of exterior surface - in z-plane

Figure 48: Circularity analysis of exterior surface - in z-plane

Circularity analysis of exterior samples perpendicular to surface curvature

Figure 49: Circularity analysis of exterior samples perpendicular to surface curvature

Circularity analysis of exterior surface - in z-plane

Figure 50: Circularity analysis of exterior surface - in z-plane

Circularity analysis of interior samples perpendicular to surface curvature

Figure 51: Circularity analysis of interior samples perpendicular to surface curvature

Circularity analysis of interior surface - in z-plane
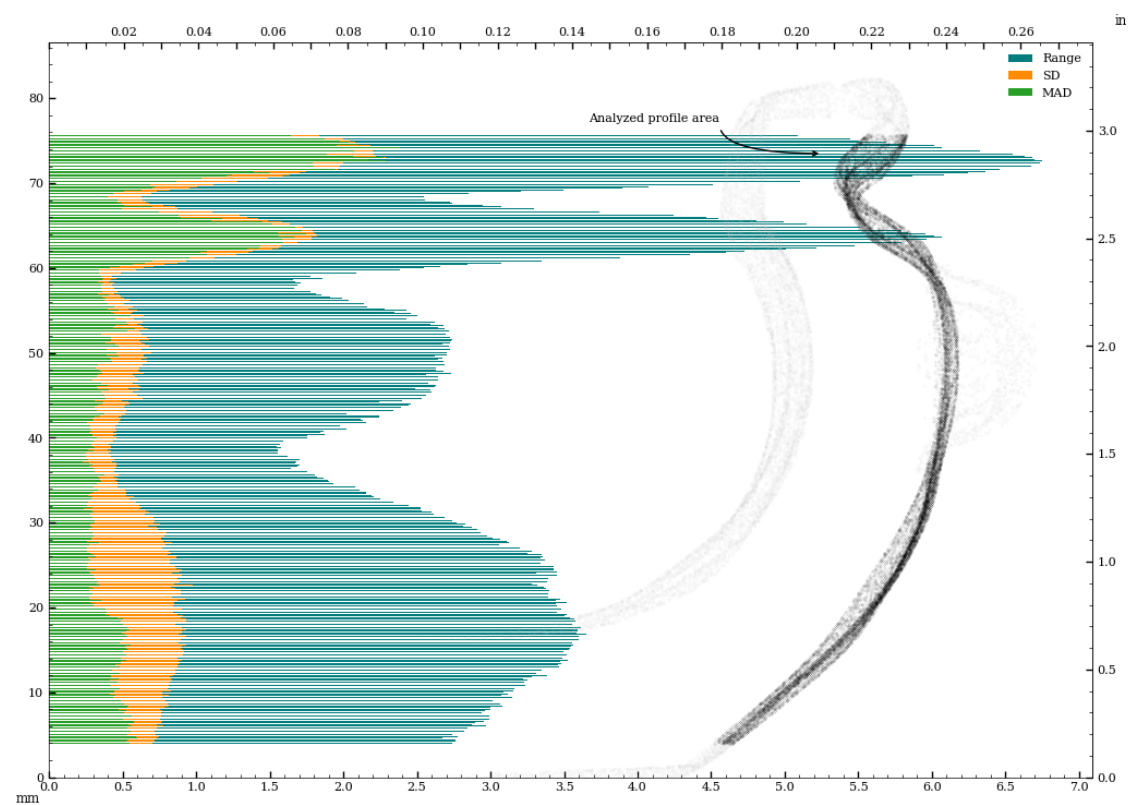

Figure 52: Circularity analysis of interior surface - in z-plane

Circularity analysis of interior samples perpendicular to surface curvature
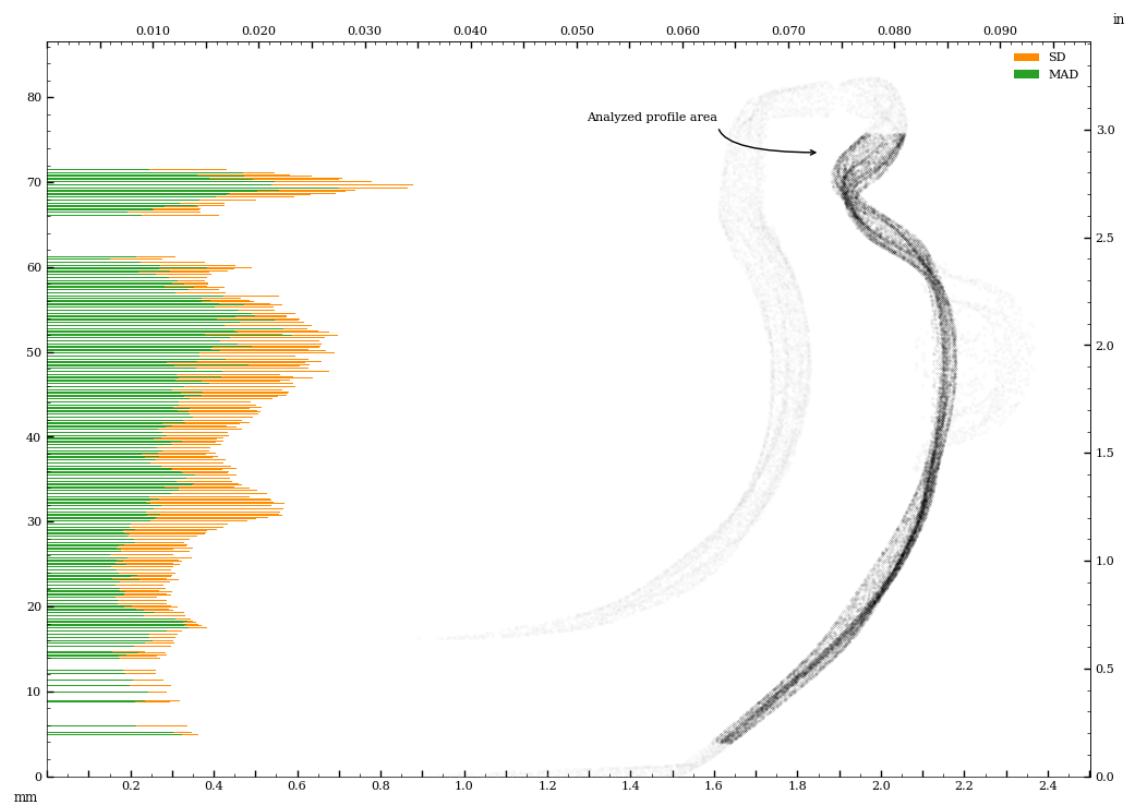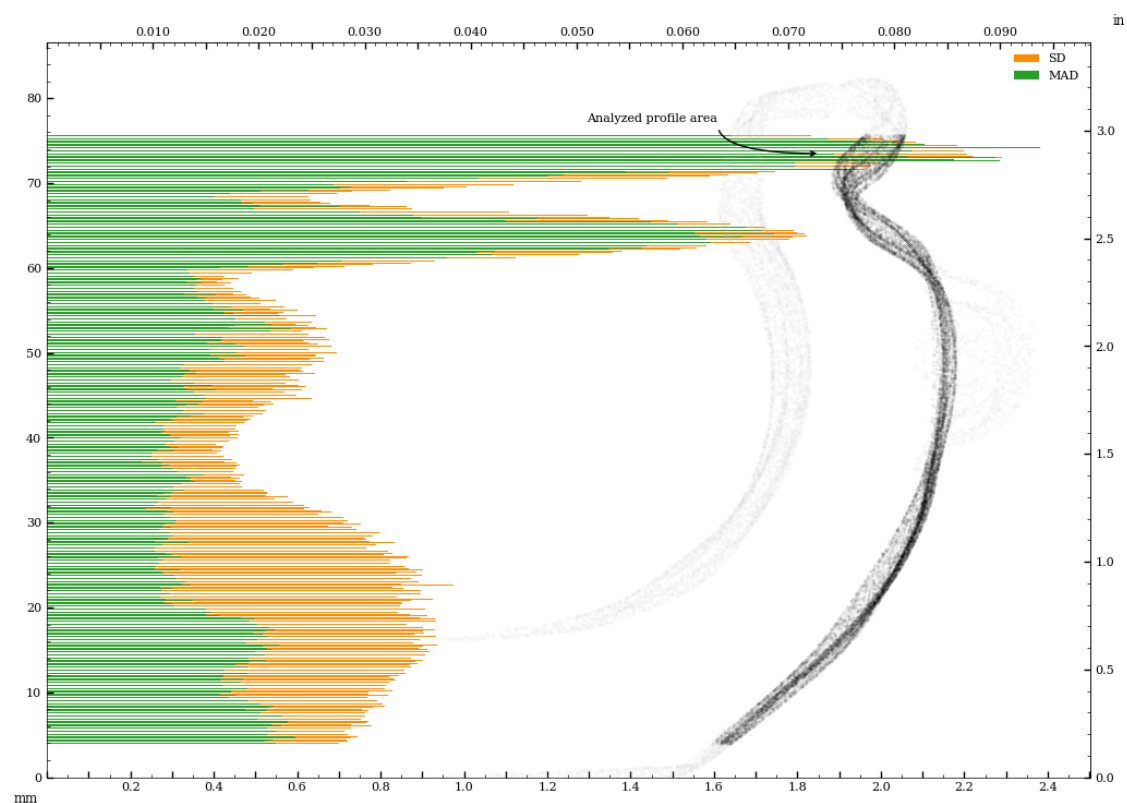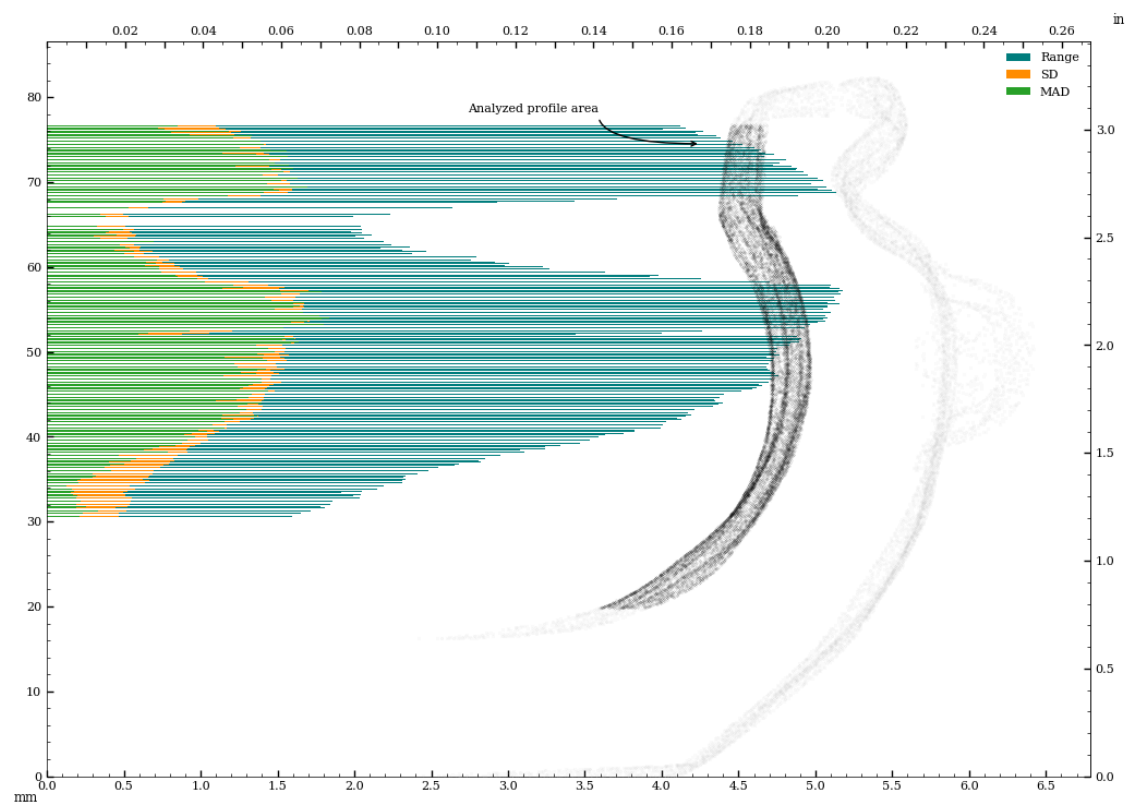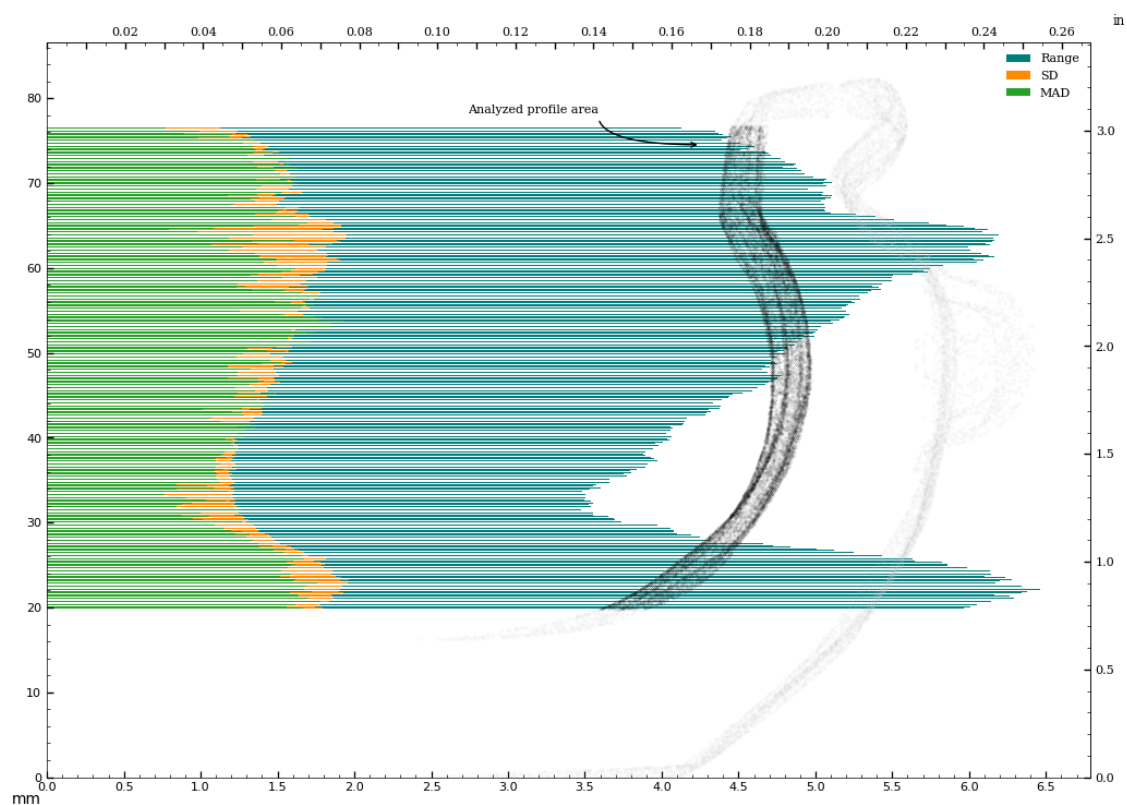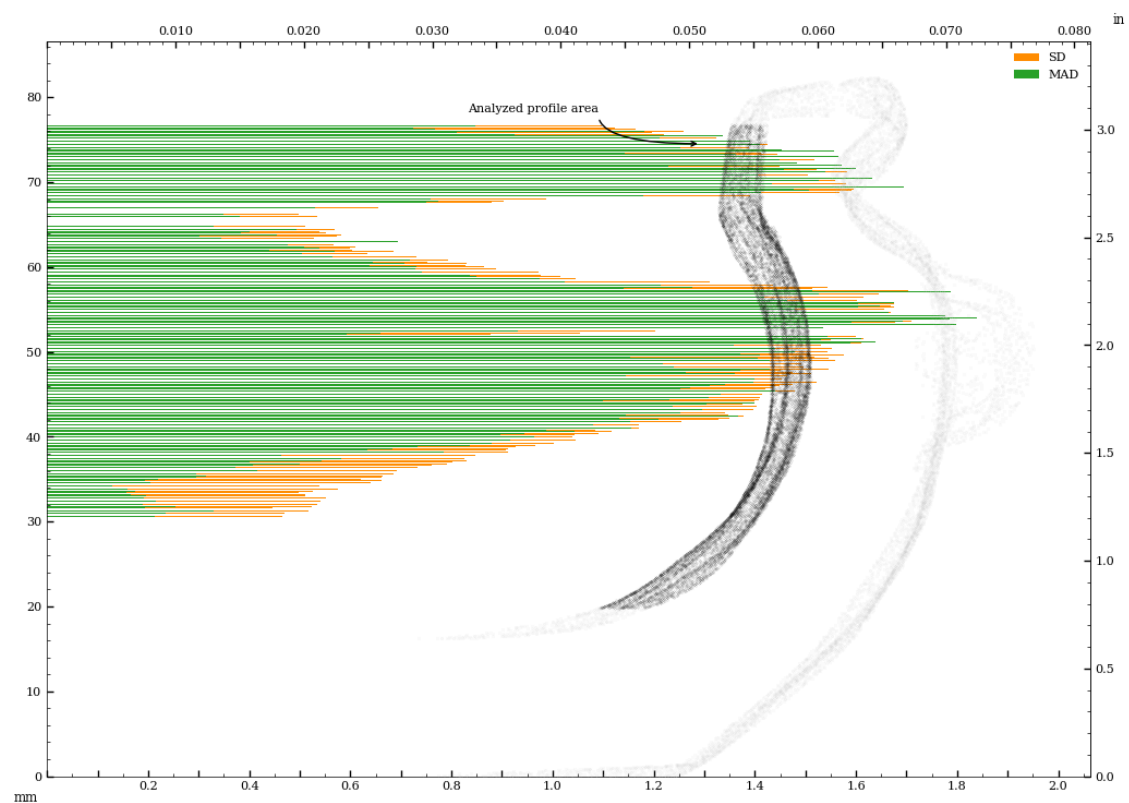

Figure 53: Circularity analysis of interior samples perpendicular to surface curvature

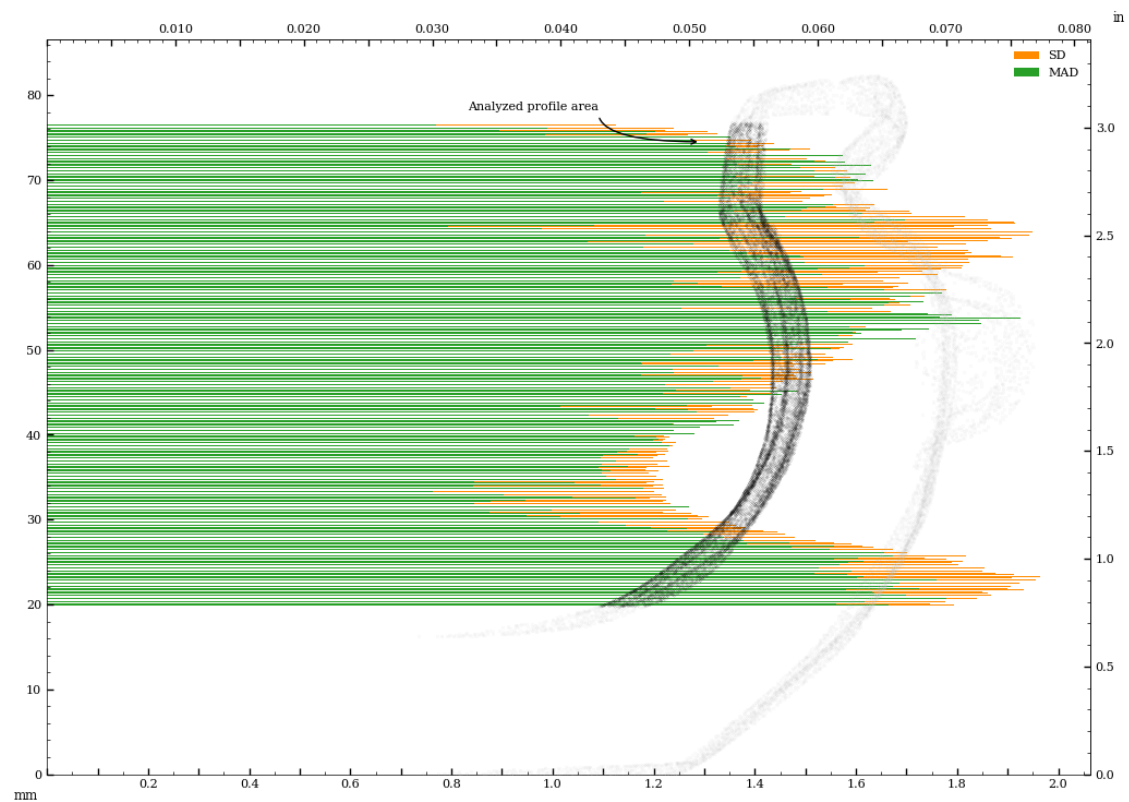## Circularity analysis of interior surface - in z-plane

Figure 54: Circularity analysis of interior surface - in z-plane

Appendix B - Comparison Of Concentricity Measurements (Z-plane vs. surface-perpendicular)

Metric

Concentricity measurements perpendicular to surface curvature

| Tag | Reference | Deviation | Sample size | Circle fit residuals analysis for sample listed in Tag column | | | | | | Center (x,y) |
| | | | | Range full | Range inliers | SD full | SD inliers | MAD full | MAD inliers | |
		mm		mm	mm	mm	mm	mm	mm	μm
c01	z-axis	0.955	170	3.339	3.339	1.140	1.140	1.122	1.122	−158, −942
c02	z-axis	0.509	154	2.699	2.699	0.657	0.657	0.403	0.403	139, 490
c03	z-axis	0.063	307	1.695	1.695	0.446	0.446	0.331	0.331	−63, −1
c04	z-axis	0.541	228	2.756	2.756	0.752	0.752	0.253	0.253	355, −408
c05	z-axis	0.212	146	1.817	1.817	0.509	0.509	0.379	0.379	208, −43
c06	z-axis	1.832	290	5.621	5.621	1.526	1.526	0.801	0.801	1256, −1333
c07	z-axis	2.161	270	4.734	4.734	1.518	1.520	1.342	1.352	874, −1977
c08	z-axis	1.379	309	3.323	3.323	1.099	1.099	0.991	0.991	260, −1354
c09	z-axis	0.604	111	2.841	2.841	0.863	0.863	0.737	0.737	−211, −566
c01	c06	1.467	170	3.339	3.339	1.140	1.140	1.122	1.122	−1414, 392
c02	c07	2.574	154	2.699	2.699	0.657	0.657	0.403	0.403	−735, 2467
c03	c08	1.391	307	1.695	1.695	0.446	0.446	0.331	0.331	−323, 1353
c04	c09	0.588	228	2.756	2.756	0.752	0.752	0.253	0.253	566, 159

Concentricity measurements in z-plane

| Tag | Reference | Deviation | Sample size | Circle fit residuals analysis for sample listed in Tag column | | | | | | Center (x,y) |
| | | | | Range full | Range inliers | SD full | SD inliers | MAD full | MAD inliers | |
		mm		mm	mm	mm	mm	mm	mm	μm
c01	z-axis	2.522	469	11.935	11.935	4.063	4.063	2.877	2.877	−450, −2482
c02	z-axis	0.502	153	3.226	3.226	1.010	1.010	0.801	0.801	164, 475
c03	z-axis	0.102	302	1.843	1.843	0.456	0.456	0.333	0.333	−79, −64
c04	z-axis	0.812	335	4.055	4.055	1.099	1.099	0.638	0.638	577, −571
c05	z-axis	0.477	290	2.969	2.969	0.733	0.733	0.520	0.520	460, −126
c06	z-axis	2.615	358	8.579	8.579	2.766	2.766	2.564	2.564	1773, −1923
c07	z-axis	2.183	312	8.911	8.911	2.964	2.964	2.028	2.028	934, −1973
c08	z-axis	1.665	377	7.530	7.530	2.538	2.538	2.532	2.532	256, −1645
c09	z-axis	2.374	362	12.583	12.583	3.964	3.964	3.408	3.408	−662, −2280
c01	c06	2.292	469	11.935	11.935	4.063	4.063	2.877	2.877	−2223, −559
c02	c07	2.566	153	3.226	3.226	1.010	1.010	0.801	0.801	−770, 2448
c03	c08	1.616	302	1.843	1.843	0.456	0.456	0.333	0.333	−335, 1581
c04	c09	2.110	335	4.055	4.055	1.099	1.099	0.638	0.638	1239, 1708

Concentricity measurements perpendicular to surface curvature

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0376	170	0.1315	0.1315	0.0449	0.0449	0.0442	0.0442	−6.2, −37.1
c02	z-axis	0.0200	154	0.1062	0.1062	0.0259	0.0259	0.0159	0.0159	5.5, 19.3
c03	z-axis	0.0025	307	0.0667	0.0667	0.0176	0.0176	0.0130	0.0130	−2.5, −0.0
c04	z-axis	0.0213	228	0.1085	0.1085	0.0296	0.0296	0.0100	0.0100	14.0, −16.1
c05	z-axis	0.0084	146	0.0715	0.0715	0.0201	0.0201	0.0149	0.0149	8.2, −1.7
c06	z-axis	0.0721	290	0.2213	0.2213	0.0601	0.0601	0.0315	0.0315	49.4, −52.5
c07	z-axis	0.0851	270	0.1864	0.1864	0.0598	0.0599	0.0528	0.0532	34.4, −77.8
c08	z-axis	0.0543	309	0.1308	0.1308	0.0433	0.0433	0.0390	0.0390	10.2, −53.3
c09	z-axis	0.0238	111	0.1119	0.1119	0.0340	0.0340	0.0290	0.0290	−8.3, −22.3
c01	c06	0.0578	170	0.1315	0.1315	0.0449	0.0449	0.0442	0.0442	−55.7, 15.4
c02	c07	0.1013	154	0.1062	0.1062	0.0259	0.0259	0.0159	0.0159	−28.9, 97.1
c03	c08	0.0548	307	0.0667	0.0667	0.0176	0.0176	0.0130	0.0130	−12.7, 53.3
c04	c09	0.0232	228	0.1085	0.1085	0.0296	0.0296	0.0100	0.0100	22.3, 6.2

Concentricity measurements in z-plane

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0993	469	0.4699	0.4699	0.1599	0.1599	0.1133	0.1133	−17.7, −97.7
c02	z-axis	0.0198	153	0.1270	0.1270	0.0397	0.0397	0.0315	0.0315	6.4, 18.7
c03	z-axis	0.0040	302	0.0725	0.0725	0.0180	0.0180	0.0131	0.0131	−3.1, −2.5
c04	z-axis	0.0320	335	0.1596	0.1596	0.0433	0.0433	0.0251	0.0251	22.7, −22.5
c05	z-axis	0.0188	290	0.1169	0.1169	0.0289	0.0289	0.0205	0.0205	18.1, −5.0
c06	z-axis	0.1030	358	0.3378	0.3378	0.1089	0.1089	0.1009	0.1009	69.8, −75.7
c07	z-axis	0.0859	312	0.3508	0.3508	0.1167	0.1167	0.0799	0.0799	36.8, −77.7
c08	z-axis	0.0655	377	0.2965	0.2965	0.0999	0.0999	0.0997	0.0997	10.1, −64.8
c09	z-axis	0.0935	362	0.4954	0.4954	0.1561	0.1561	0.1342	0.1342	−26.1, −89.7
c01	c06	0.0902	469	0.4699	0.4699	0.1599	0.1599	0.1133	0.1133	−87.5, −22.0
c02	c07	0.1010	153	0.1270	0.1270	0.0397	0.0397	0.0315	0.0315	−30.3, 96.4
c03	c08	0.0636	302	0.0725	0.0725	0.0180	0.0180	0.0131	0.0131	−13.2, 62.2
c04	c09	0.0831	335	0.1596	0.1596	0.0433	0.0433	0.0251	0.0251	48.8, 67.3