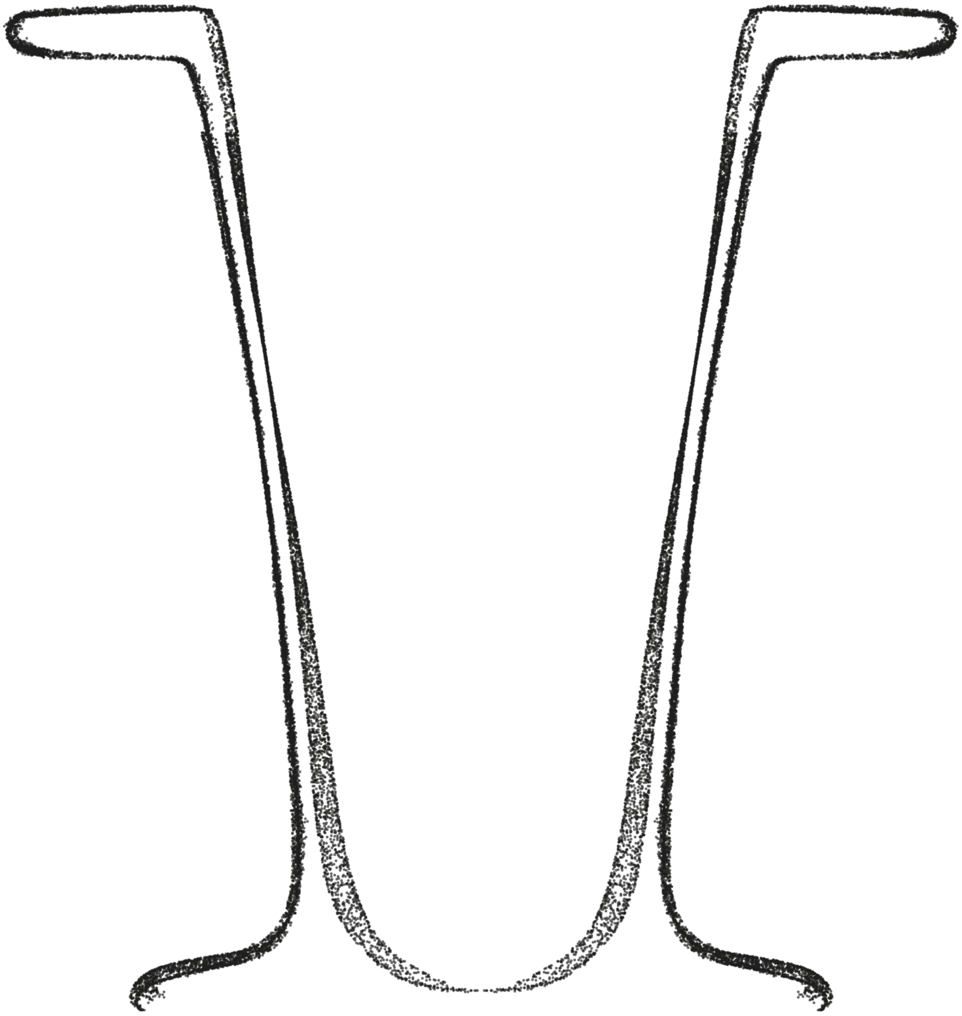
MV013 - Conical Beaker

An Exploration of Precision



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Contents

Artifact Information

Artifact Data

Collection	Petrie Museum of Egyptian Archaeology
Provenance[1]	Petrie Museum of Egyptian Archaeology (London), recovered by Flinders Petrie
Provenience[2]	Unknown
Attribution	6th Dynasty

Museum information

Ref.	LDUCE-UC41053
Description	Slender diorite cylinder jar with sharply flared base and protruding rim. Large chip on base and approximately one third of rim missing.
URL	https://collections.ucl.ac.uk/Details/collect/46314

Maijers vessel classification[3]

Short classification	Conical Beaker
Long classification	The vessel is created in an open form classified as a beaker with a conical shape, it has a footed base and the vessels Conical curves are ending in a squared flat rim.

Physical properties

Precision score[4]	20
Height (approximate)	94 mm 3.70 in
Width (approximate)	78 mm 3.07 in
Material	Diorite
Mohs Hardness[5]	5.5 - 7 (Diorite)
Weight	

Scan information

Source	Scanned by Artifact Foundation
Source file name	UC41053_base_0.09.stl
Scan method	Laser
Scanner	FreeScale Combo+
Rated scan accuracy	37 µm | 1.51 thou
Scan date	2024-10-14
Scanned by	Károly Póka

Mesh decimation	None, raw scan file used in the analysis
Number of vertices	5 029 428
Mesh density[6]	36 µm | 1.43 thou
Max vertex distance	151 µm | 5.952 thou
Min vertex distance	0 µm | 0.000 thou
Vertices per cm2	18 176 (approximated)
Vertices per in2	117 264 (approximated)

[1] The verifiable chain of custody of an artifact

[2] The location or site where an artifact was recovered

[3] Vessel artifact classification developed by W. Arnold Maijer and described in his publication Masters of Stone, ISBN 978-90-829212-0-5

[4] The precision score metric is described in Precision Score Of The Artifact, p. 48

[5] The Mohs scale is an ordinal scale, from 1 to 10, describing the materials resistance to abrasion (the ability of harder material to scratch softer material)

[6] Median distance between vertices

Alignment In The Cartesian Coordinate System

For precise and valid measurements of the vessel's geometry to be possible, the points of the scanned dataset must first and foremost be placed optimally in a Cartesian coordinate system. Several alignment methods and algorithms have been tested on a number of different vessels to determine the best way to achieve optimal alignment.

Any misalignment of the artifact will increase the error of the precision measurements, due to the distortion/ wobble effect caused by the misaligned object. To visualize this distortion, we can consider a representation of the three-dimensional point cloud data, folded to a two-dimensional plane. This folded representation is obtained by rotating all scanned points around an assumed center axis to $y = 0$, $x > 0$, thus resulting in a two-dimensional profile representation of all scanned vertices in the object.

Figure 1 illustrates this effect on a ideal ellipsoid. In the first image, the ellipsoid is perfectly aligned, resulting in a narrow and precise two-dimensional folded profile. As misalignments are introduced, the two-dimensional profile increases in width, visually showing the distortion, causing the error in the precision measurements to increase. While easy to understand visually, this distortion can also be objectively quantified, and as such used to compare the fitness of different assumed center axes against each other, and further to create an automated and solid process for optimal Cartesian alignment of the scan data.

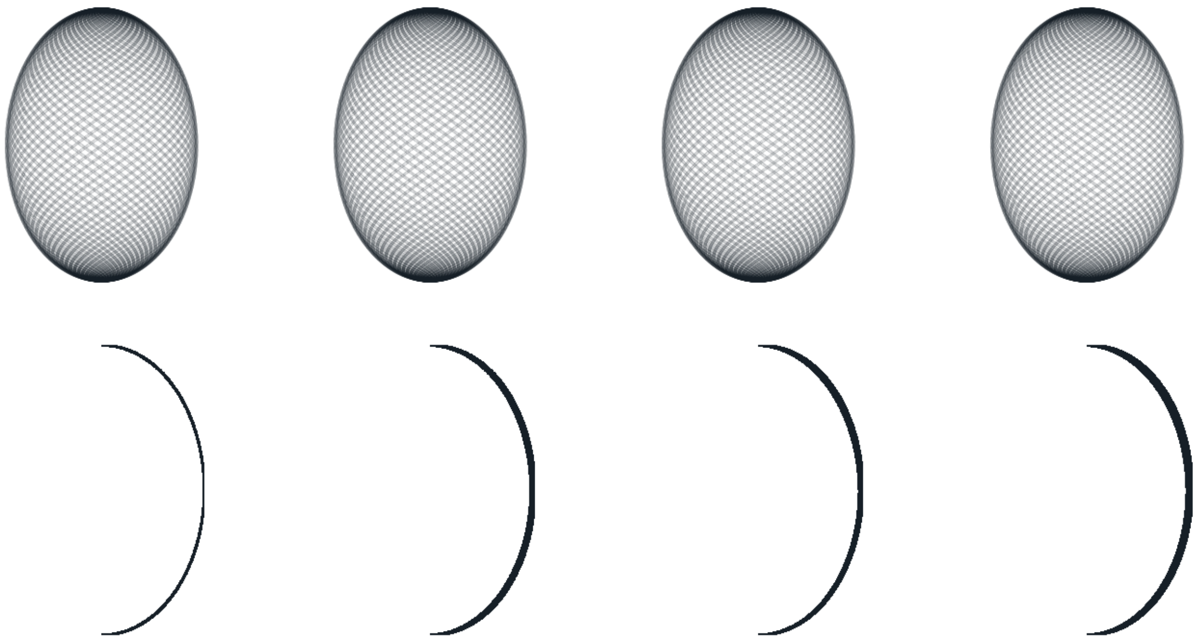


Figure 1: Distortion caused by a misalignment of the artifact

In contemporary metrology analysis of modern production objects, it is common to align the object in a Cartesian coordinate system by fitting a flat surface of the object to a reference plane in the coordinate system, cylindrical features to an ideal cylinder etc., or by using specific markers placed on the object in the design process. This methodology, however, is inadequate for the ancient objects in question. Most scanned artifacts, do not have a valid flat surface which could be aligned to a plane in the Cartesian coordinate system; most surfaces seem to be curved. Some artifacts do have a flat base, however this is often a worn area of the artifact and practical tests have shown that alignment to such surfaces will not produce optimal alignment of the scan data.

As conventional methods of alignment do not always yield good results with these types of artifacts, a more adequate method of alignment has been developed to enable precise measurements and statistical analysis of the scan data.

To find the optimal position of the vessel in the coordinate system, a range of rotation and translation tests are carried out to find the best fit of the central axis.

Based on the assumption that the analyzed object was created using a rotational process, and thus have symmetry around a central axis, the alignment of the artifact is carried out in a two-step process. An overview of this process is given below.

The artifact is placed in a Cartesian coordinate system, in an initially unaligned state. The first step in the alignment process estimates the central rotational axis of the vessel, by analyzing the coaxiality of thin cross-section slices of the vessel. The slices will be as thin as possible based on the mesh density of the scan, while still ensuring enough data points in each slice to be statistically valid.

For each slice, circular regression[7] (estimate of best fit circle) is used to estimate the center point of this slice. Combined over the total Z-axis range of the vessel, these center points provide us with an indicator of the incline and position of the vessel's central axis.

The next step will optimize the center axis alignment by progressively minimizing the deviation (perpendicular to the surface curvature) of the two-dimensional profile, see Figure 1. By ascertaining and comparing the resulting fit of many thousands of different potential rotations, the best fit alignment of the scan data can be estimated, and an optimal center axis (in relation to the data points) can be reconstructed. The actual three-dimensional point-cloud is then aligned to this axis, by rotating and translating the scanned data points to match the Z-axis of the Cartesian coordinate system.

[7]Circle regression algorithm used: Kenichi Kanatani, Prasanna Rangarajan, "Hyper least squares fitting of circles and ellipses" Computational Statistics & Data Analysis, Vol. 55, pages 2197-2208, (2011)

Precision

To explore the manufacturing precision of the artifact in depth, the following analysis have been carried out:

- Circularity around the axis of symmetry is examined in detail at selected cross-sections.
- Overall circularity around the axis of symmetry is measured for the full height of the vessel (areas of the vessel with extensive damage are not taken into account for this metric).
- Concentricity of the vessel between selected cross-sections are examined in detail to determine if the existence of an axis of rotation in the manufacture of the object can be established.
- The coaxiality of the vessel is analyzed to explore the precision of the central axis of the object.
- The surface variability is analyzed and visualized on through a heatmap.

Circularity

Circularity is the measurement of how round the surface of an object is, optionally in reference to a datum axis. The *circularity tolerance* is the radial distance of two circles, each with their centers in the datum axis, and each of them conforming, respectively, to the minimum and maximum deviations of the data-set to a true circle, see Figure 2.

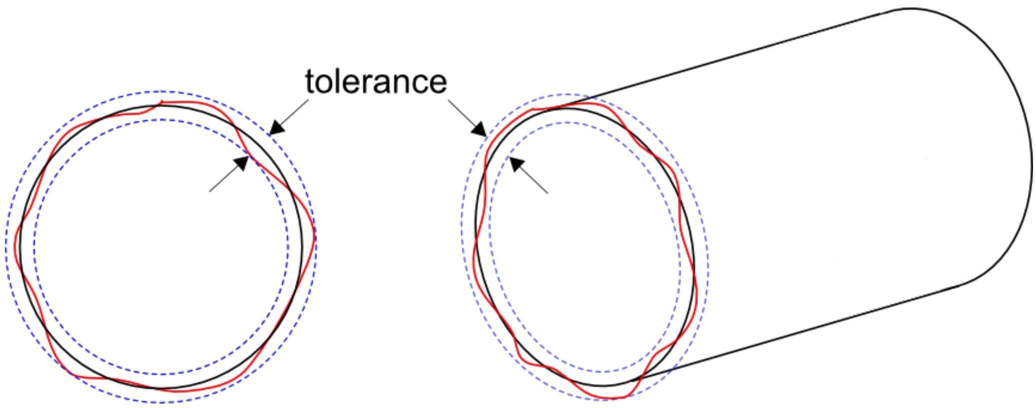


Figure 2: Circularity tolerance.

Circularity is examined at different cross-sections of the vessel, using the established Z-axis as the datum axis (axis of symmetry). The distance between the scanned points in the local datum plane is measured to determine the range between the two concentric circles encompassing the measured points, see Figure 3.

Referencing all of the individual circularity measurements to the global (reconstructed) axis of symmetry of the object, allows us to ascertain not only circularity of local features of the object, but how well circularity was *maintained* over the entire manufacturing process. This is an important distinction, which may be able to provide valuable insights into requirements of the construction methods. For reference, and seeing that the variance in local circularity also holds interest, measurements of circularity of the vessel without reference to the axis of symmetry can additionally be found in the Concentricity, p. 26.

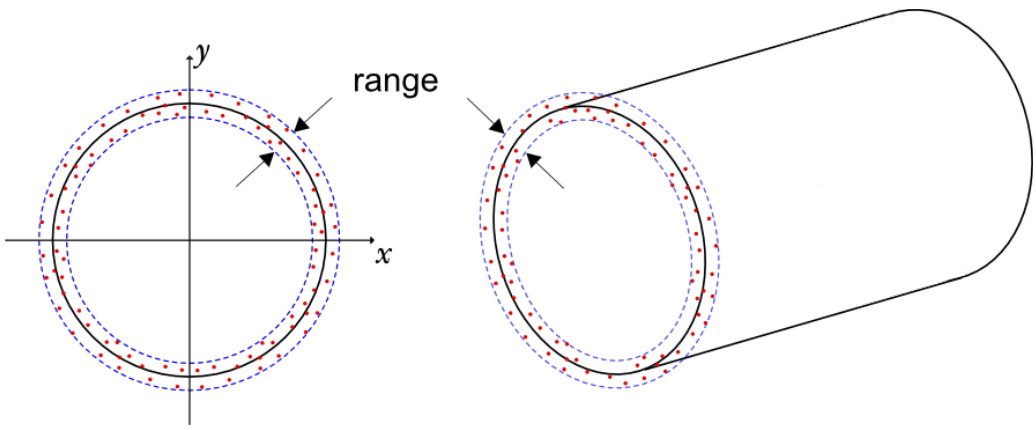


Figure 3: Circularity measurements.

If the circularity is determined from slices of the vessel exclusively in the *Z-plane* (actually measuring the cylindricity of a very thin slices of the vessel, in an attempt to approximate circularity), this would - in some areas - introduce significant distortion (increasing measurement errors) in the samples, due to the curvature of the vessel's surface.

Each sample slice of the vessel is therefore obtained perpendicular to the surface curvature, see Figure 5 to Figure 14. The measurements are taken conservatively without filtration of potential outliers.

To explore the potential distortion caused by obtaining samples in the Z-plane only, please refer to Appendix A, where measurements in the Z-plane and measurements perpendicular to surface curvature are compared side by side.

Detailed circularity measurements of selected points

Circularity measurements across a range of selected slices of the vessel (see Table 1) have been analyzed in-depth, and detailed plots of each measurement is provided. Furthermore, full circularity measurements are shown for each available scanned surface including a detailed plot to visualize the circularity of all areas of the vessel.

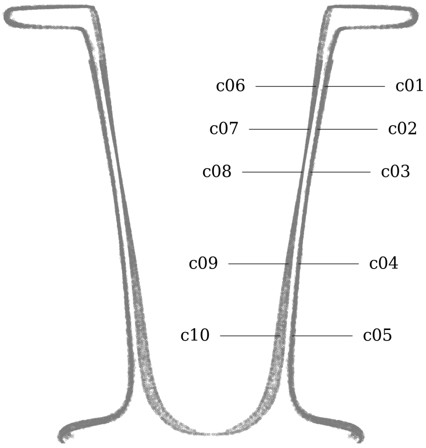


Figure 4: Circularity measurement sample location on MV013.

Metric

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø48.636±0.715	1.209	0.214	0.127	0.214	1458	0.050	76.460	24.318
c02	exterior	Ø45.516±0.525	1.015	0.176	0.122	0.176	1441	0.050	67.317	22.758
c03	exterior	Ø42.470±0.455	0.844	0.126	0.071	0.126	1287	0.050	58.174	21.235
c04	exterior	Ø37.476±0.398	0.756	0.134	0.089	0.134	1173	0.050	38.608	18.738
c05	exterior	Ø34.809±0.463	0.874	0.200	0.122	0.200	1112	0.050	23.244	17.405
c06	interior	Ø44.829±0.482	0.875	0.289	0.278	0.287	1306	0.050	76.460	22.415
c07	interior	Ø42.014±0.256	0.464	0.135	0.136	0.133	1260	0.050	67.317	21.007
c08	interior	Ø39.093±0.121	0.210	0.045	0.037	0.045	1114	0.050	58.174	19.546
c09	interior	Ø33.191±0.597	1.153	0.390	0.397	0.390	983	0.050	38.608	16.596
c10	interior	Ø29.691±1.060	2.047	0.674	0.674	0.674	818	0.050	23.244	14.846

Imperial

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø1.9148±0.0281	0.0476	0.0084	0.0050	0.0084	1458	0.0020	3.0102	0.9574
c02	exterior	Ø1.7920±0.0207	0.0400	0.0069	0.0048	0.0069	1441	0.0020	2.6503	0.8960
c03	exterior	Ø1.6721±0.0179	0.0332	0.0049	0.0028	0.0050	1287	0.0020	2.2903	0.8360
c04	exterior	Ø1.4754±0.0157	0.0298	0.0053	0.0035	0.0053	1173	0.0020	1.5200	0.7377
c05	exterior	Ø1.3704±0.0182	0.0344	0.0079	0.0048	0.0079	1112	0.0020	0.9151	0.6852
c06	interior	Ø1.7649±0.0190	0.0345	0.0114	0.0109	0.0113	1306	0.0020	3.0102	0.8825
c07	interior	Ø1.6541±0.0101	0.0183	0.0053	0.0054	0.0052	1260	0.0020	2.6503	0.8271
c08	interior	Ø1.5391±0.0048	0.0083	0.0018	0.0015	0.0018	1114	0.0020	2.2903	0.7695
c09	interior	Ø1.3067±0.0235	0.0454	0.0153	0.0156	0.0154	983	0.0020	1.5200	0.6534
c10	interior	Ø1.1689±0.0417	0.0806	0.0265	0.0265	0.0266	818	0.0020	0.9151	0.5845

Table 1: Detailed circularity measurements at selected samples of MV013.

Figure 5 to Figure 14 shows a detailed plots of each circularity measurement.

[8]Sample diameter Ø± maximum measured deviation from measured radius
[9]Root mean square deviation (RMSD) also called Root mean square error (RMSE)
[10]Median absolute deviation
[11]Median sample radius from z-axis

Graphical overview of circularity measurement c01

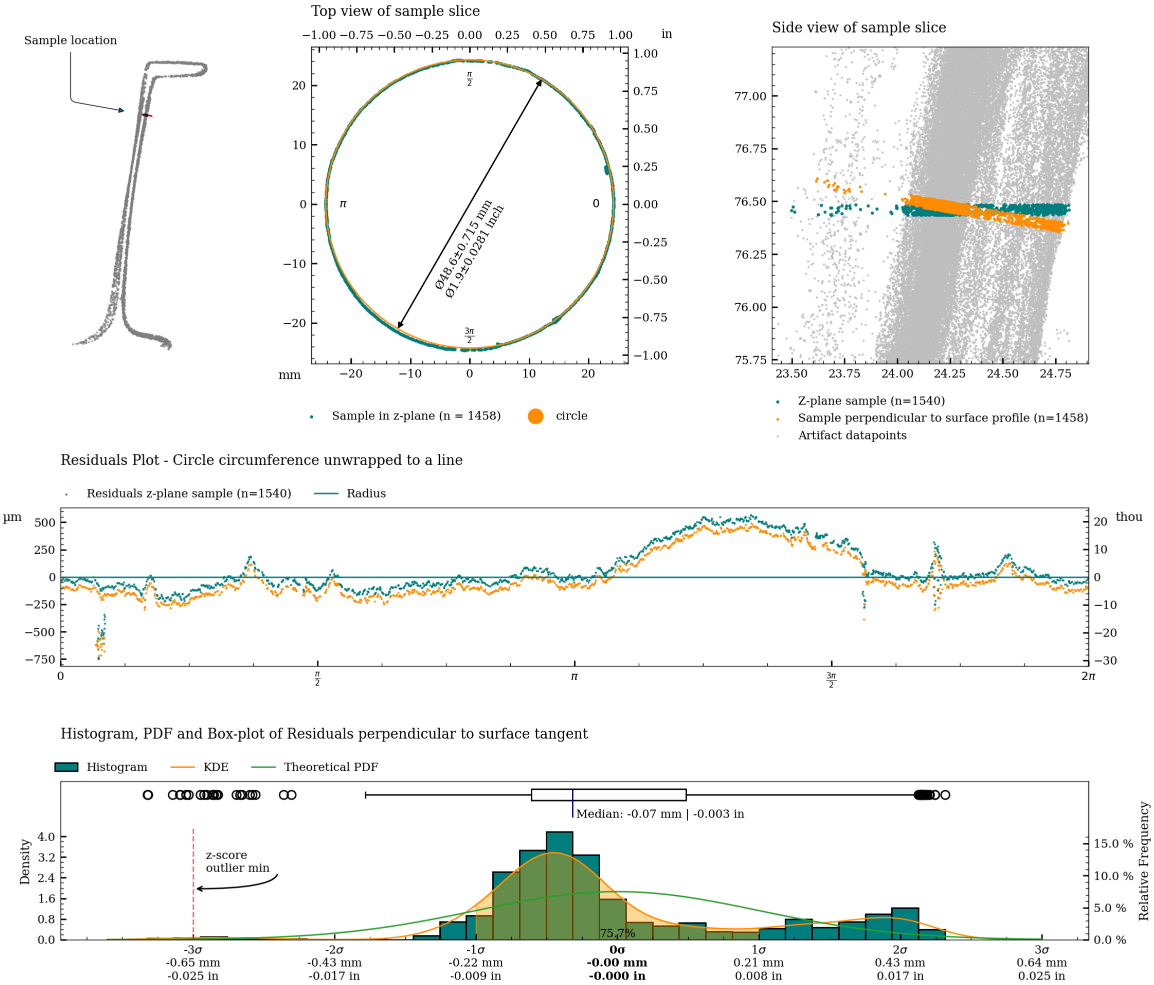


Figure 5: Charts with statistics for the measurement of c01.

Graphical overview of circularity measurement c02

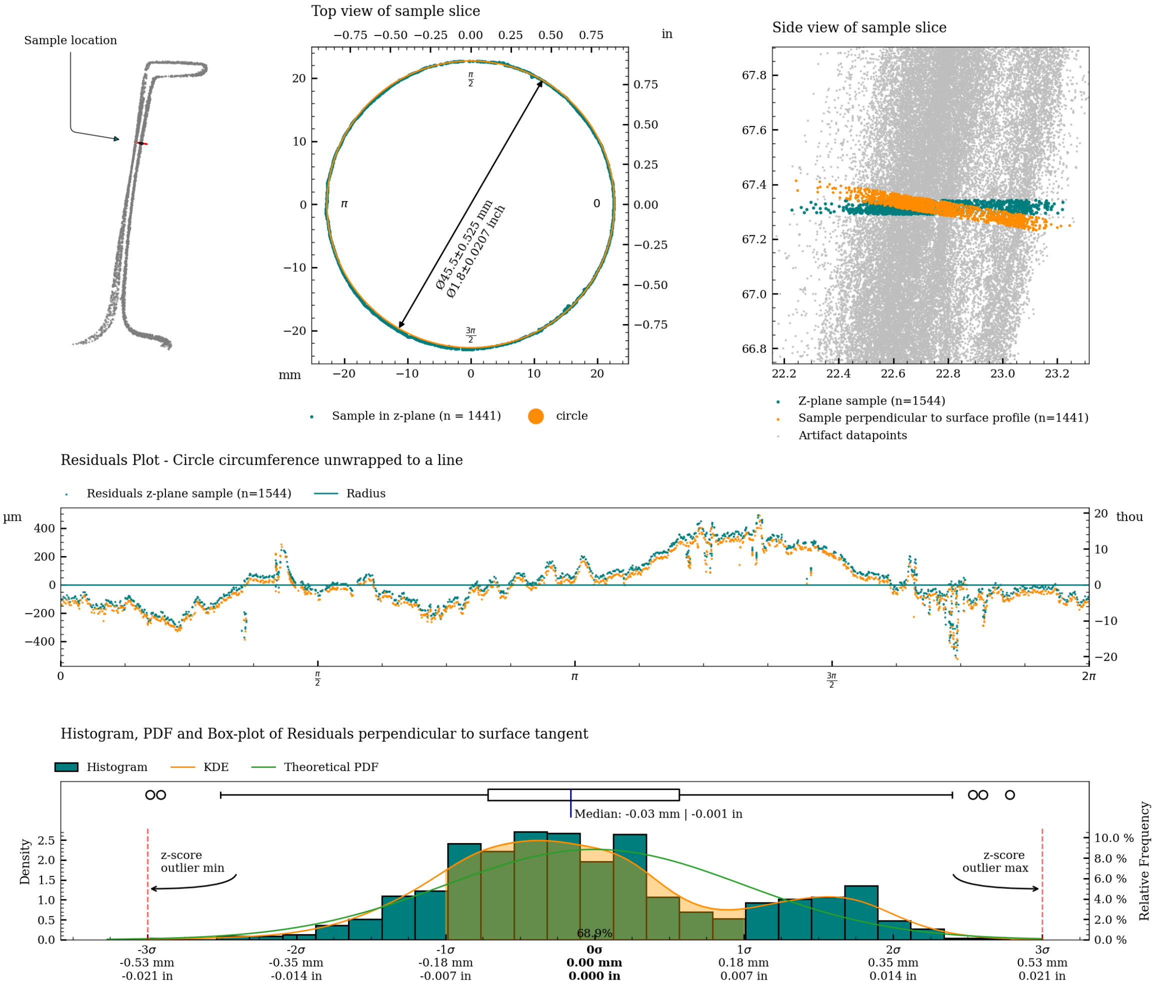


Figure 6: Charts with statistics for the measurement of c02.

Graphical overview of circularity measurement c03

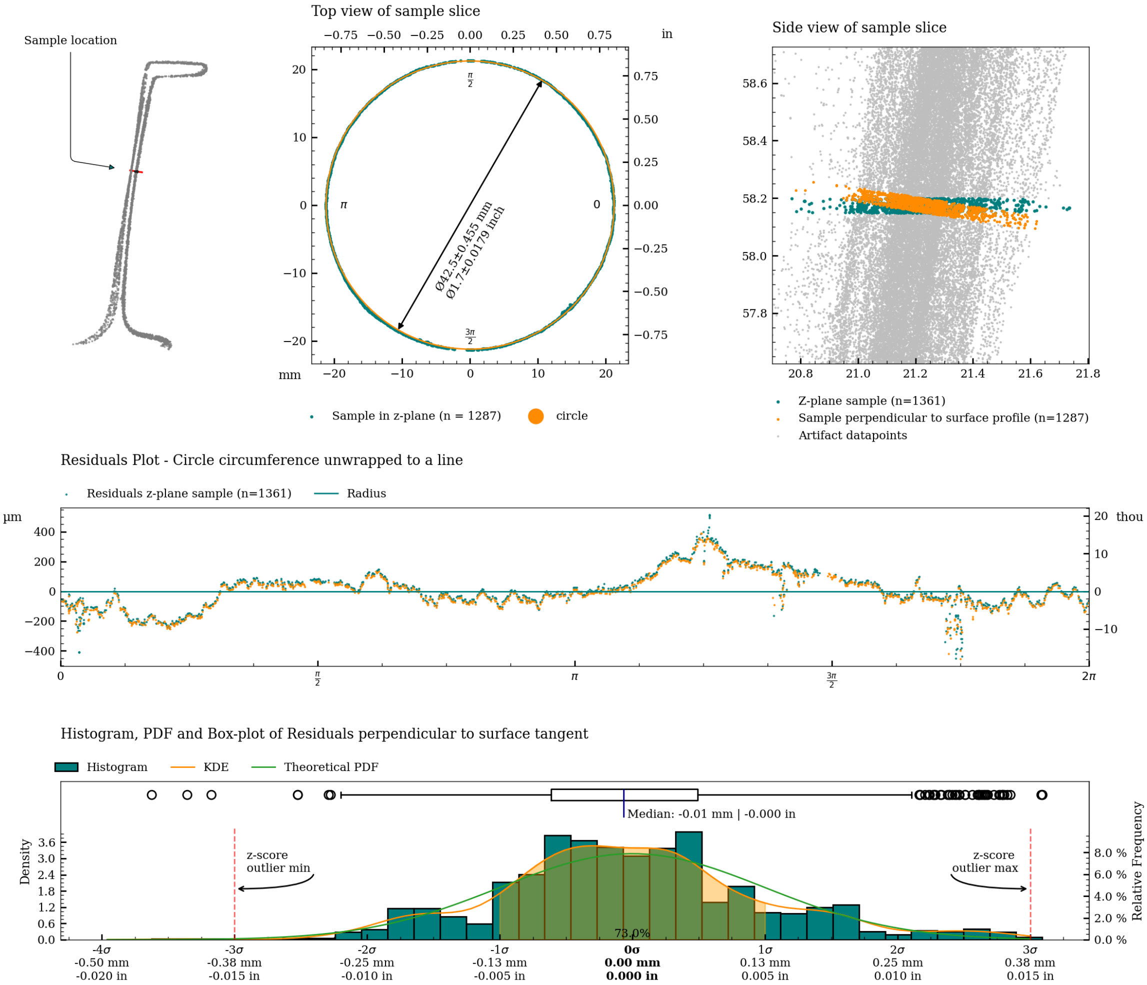


Figure 7: Charts with statistics for the measurement of c03.

Graphical overview of circularity measurement c04

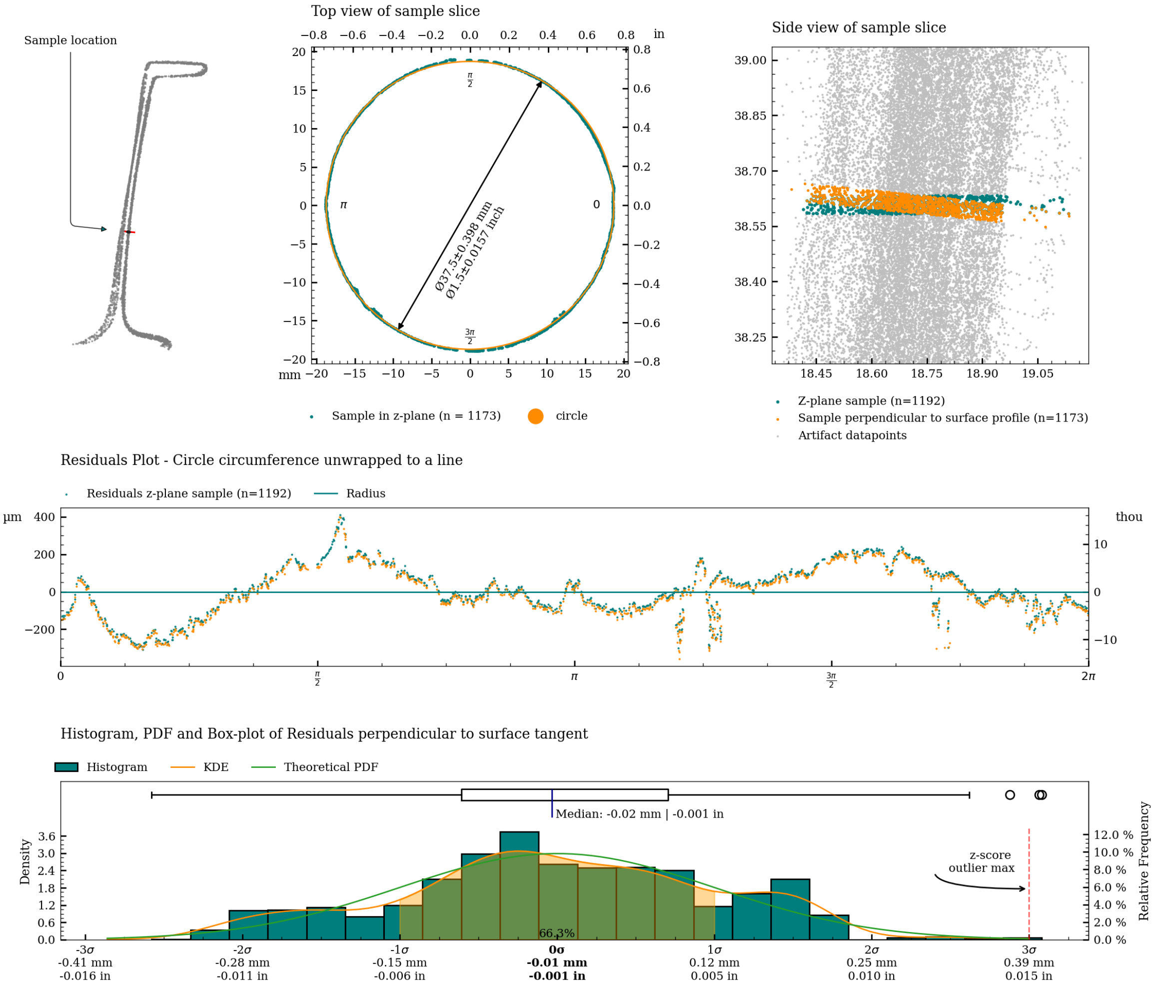


Figure 8: Charts with statistics for the measurement of c04.

Graphical overview of circularity measurement c05

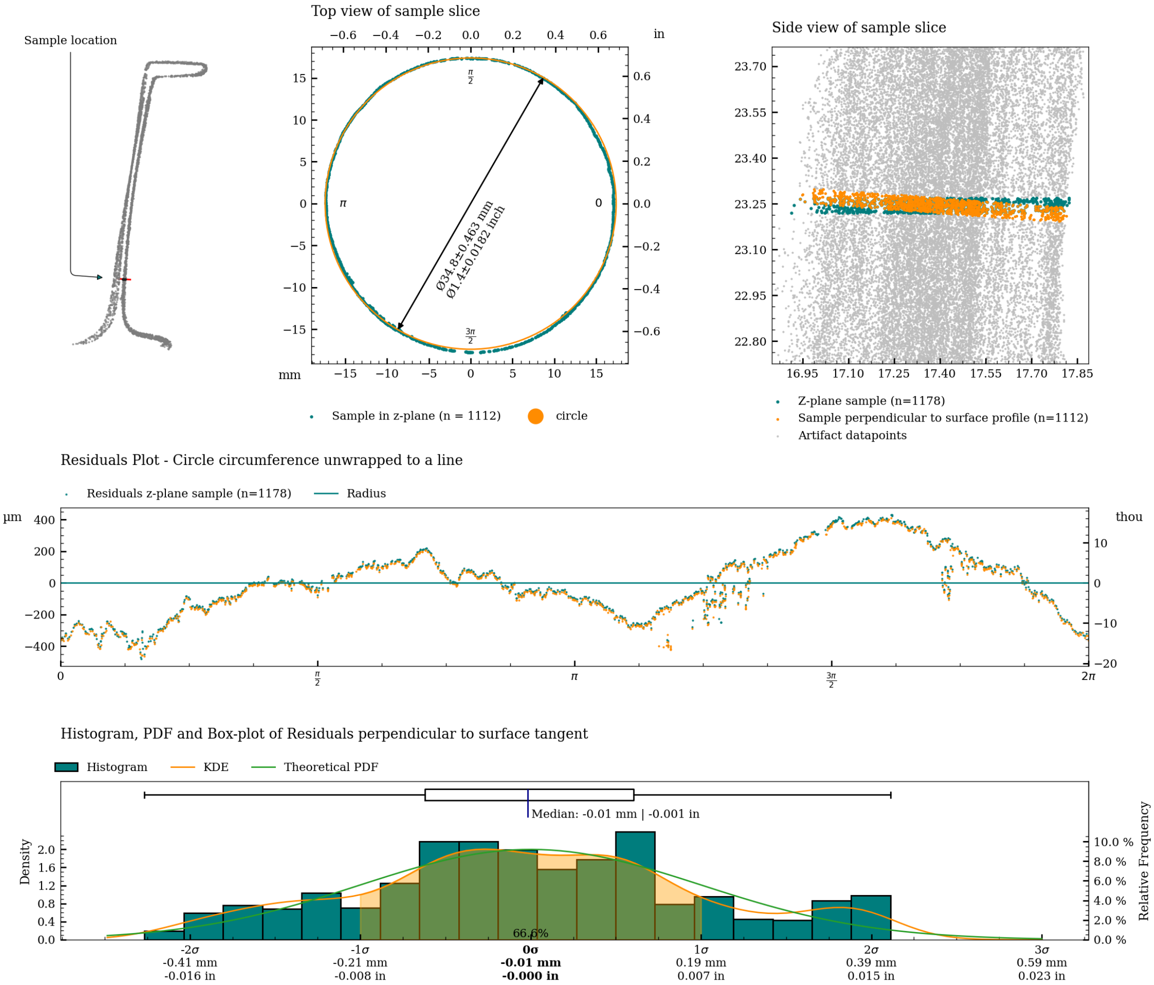


Figure 9: Charts with statistics for the measurement of c05.

Graphical overview of circularity measurement c06

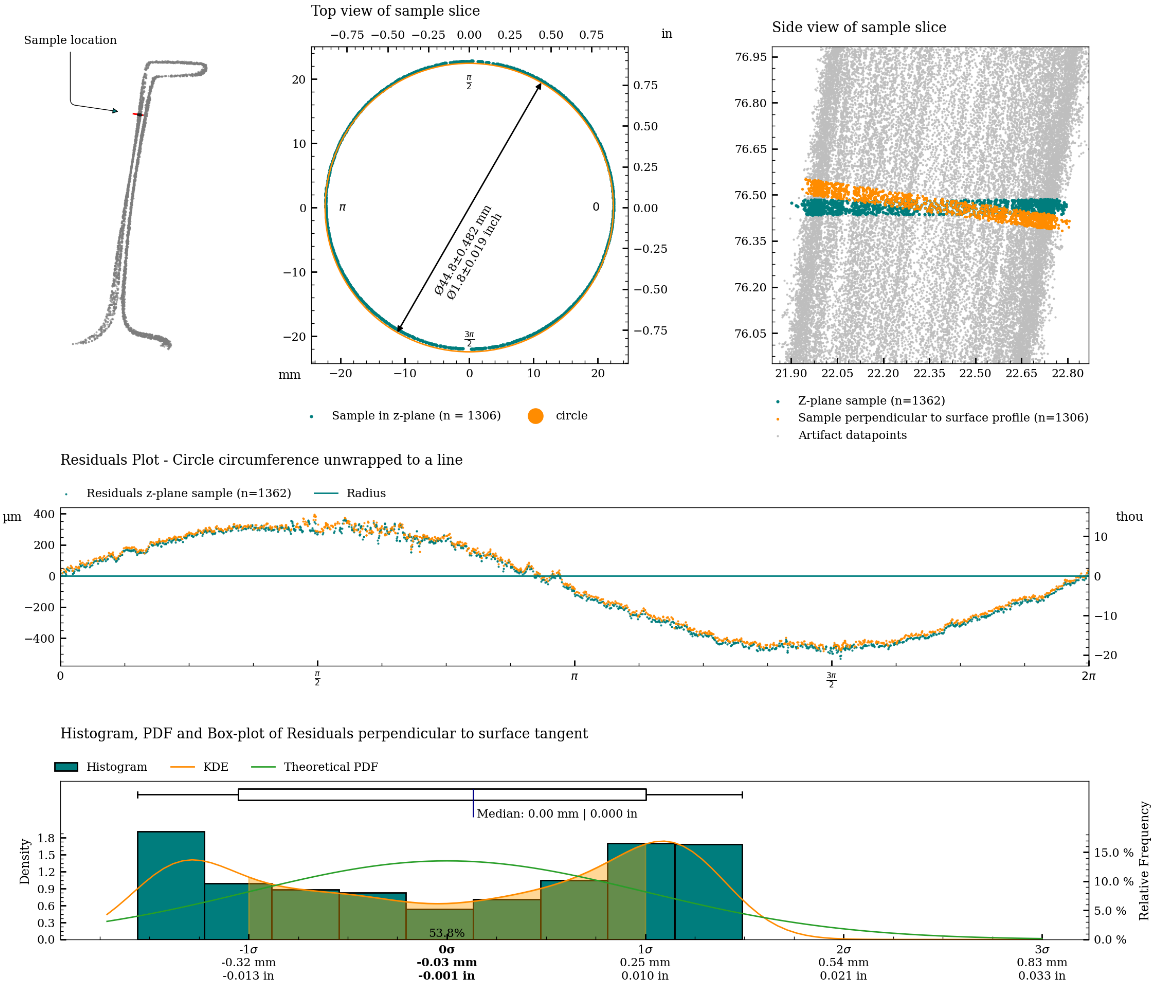


Figure 10: Charts with statistics for the measurement of c06.

Graphical overview of circularity measurement c07

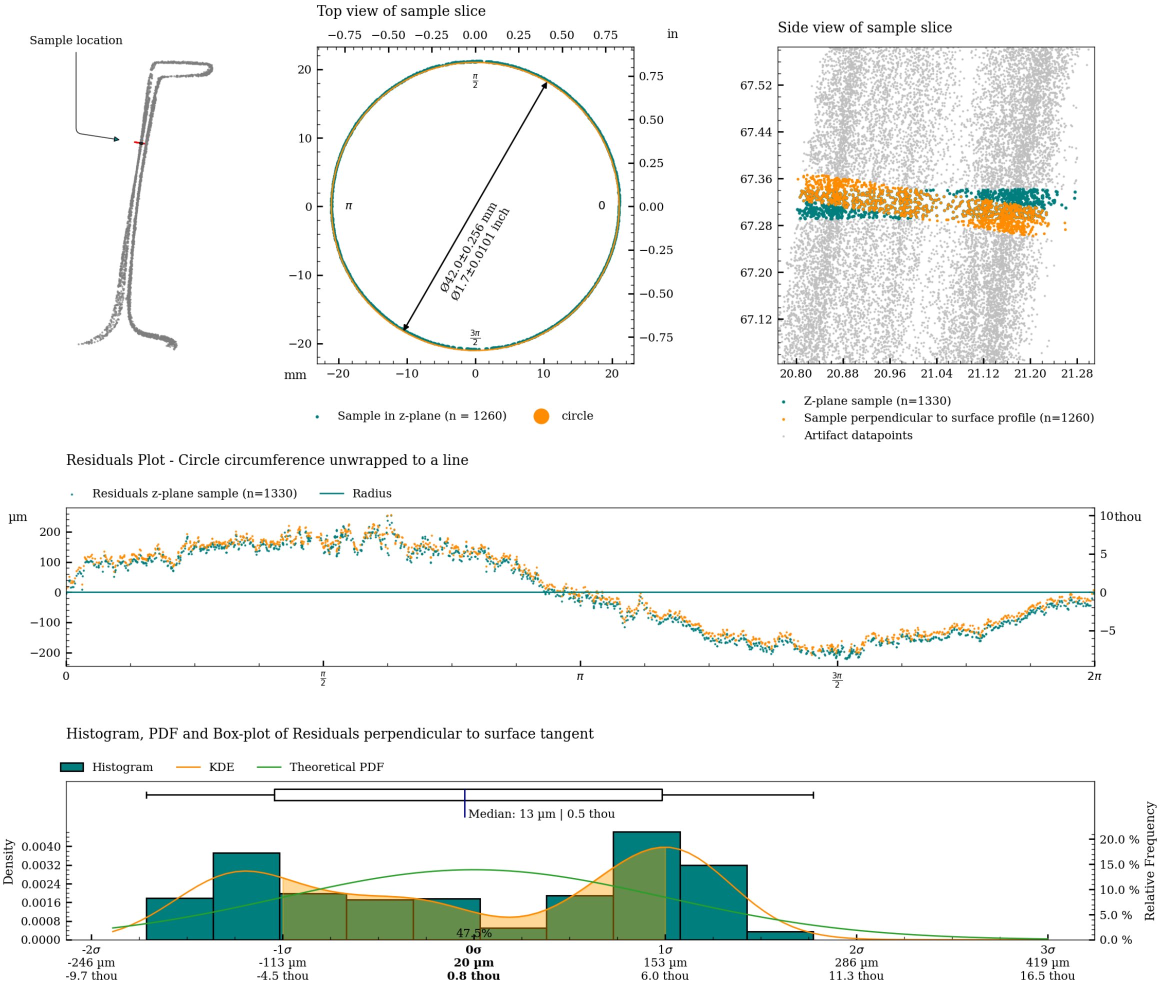


Figure 11: Charts with statistics for the measurement of c07.

Graphical overview of circularity measurement c08

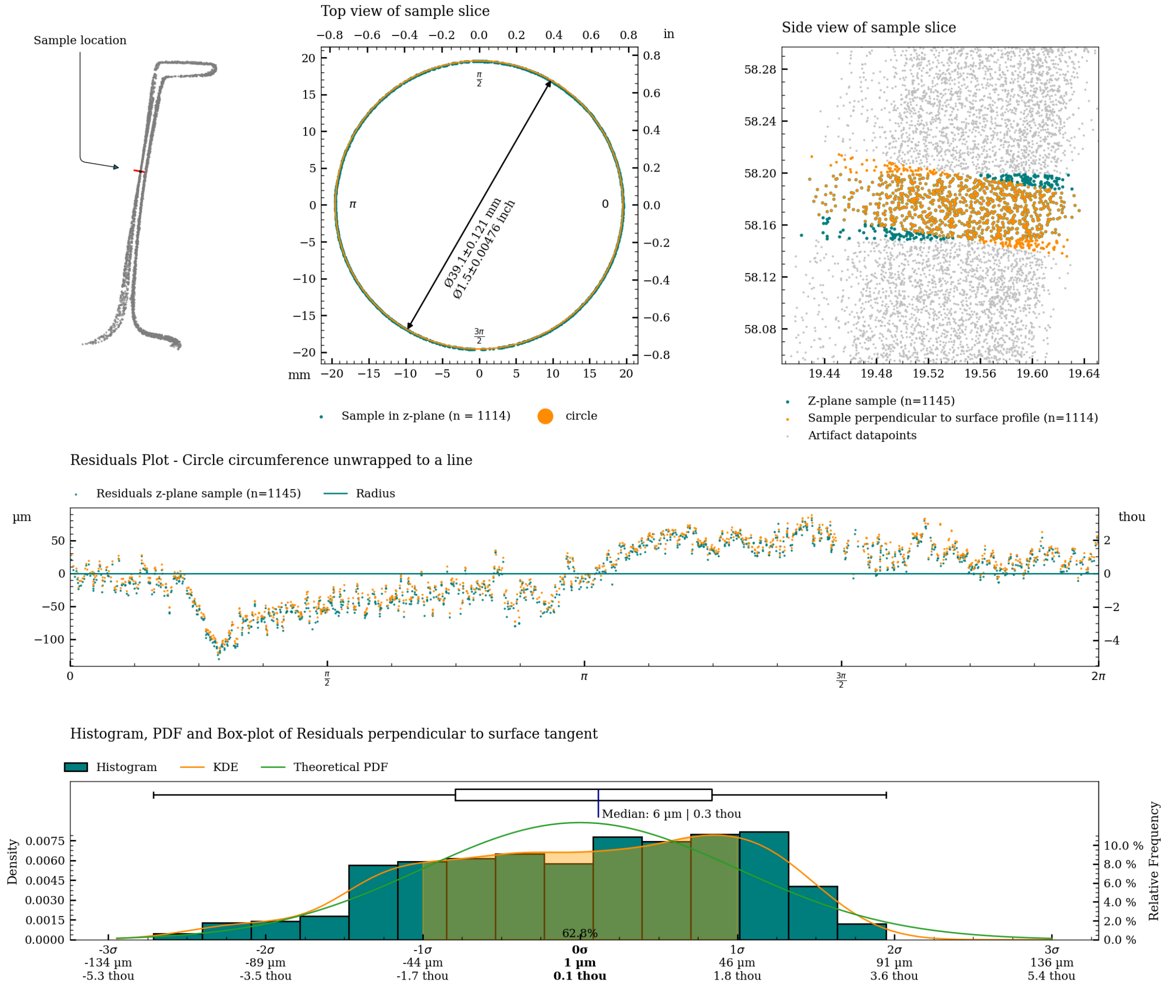


Figure 12: Charts with statistics for the measurement of c08.

Graphical overview of circularity measurement c09

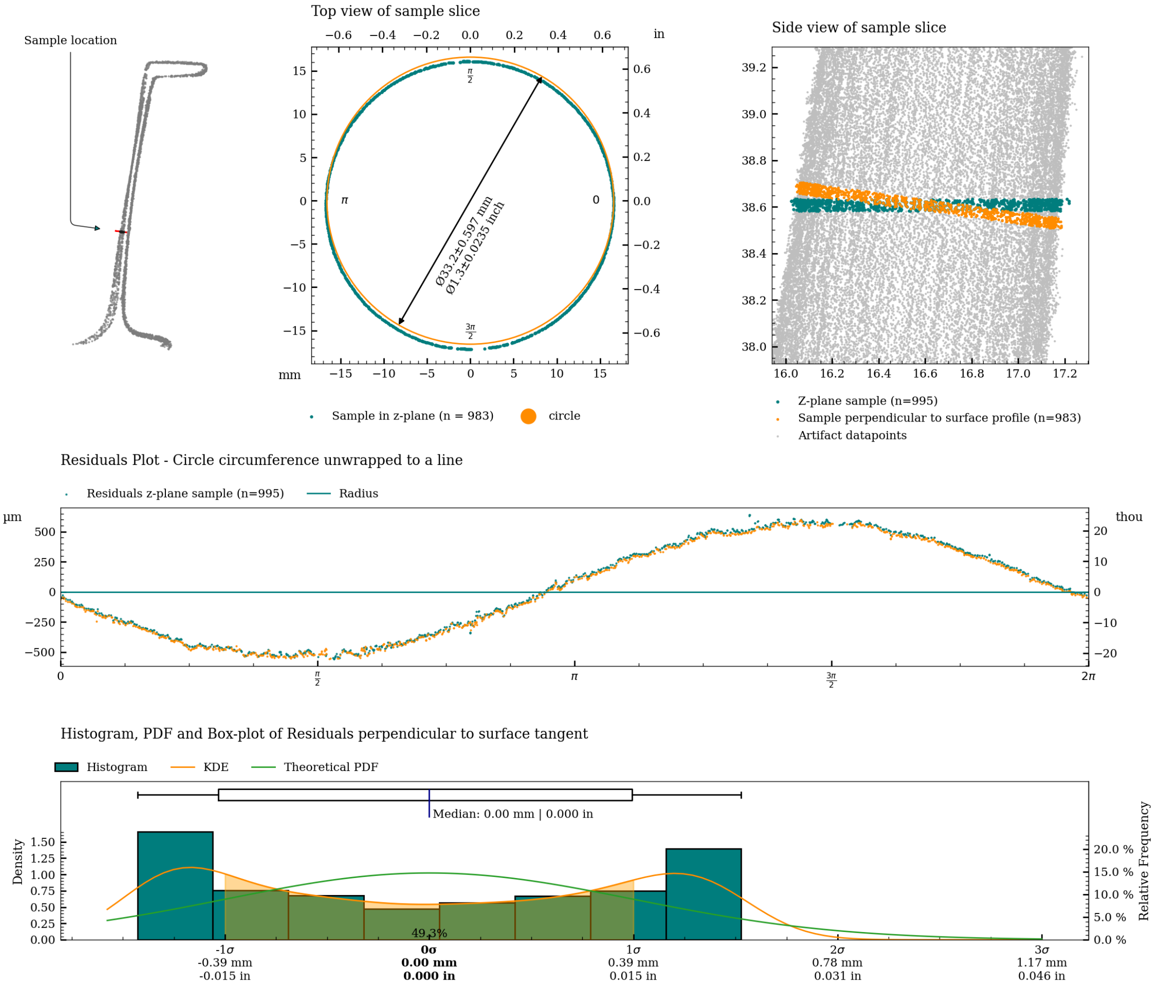


Figure 13: Charts with statistics for the measurement of c09.

Graphical overview of circularity measurement c10

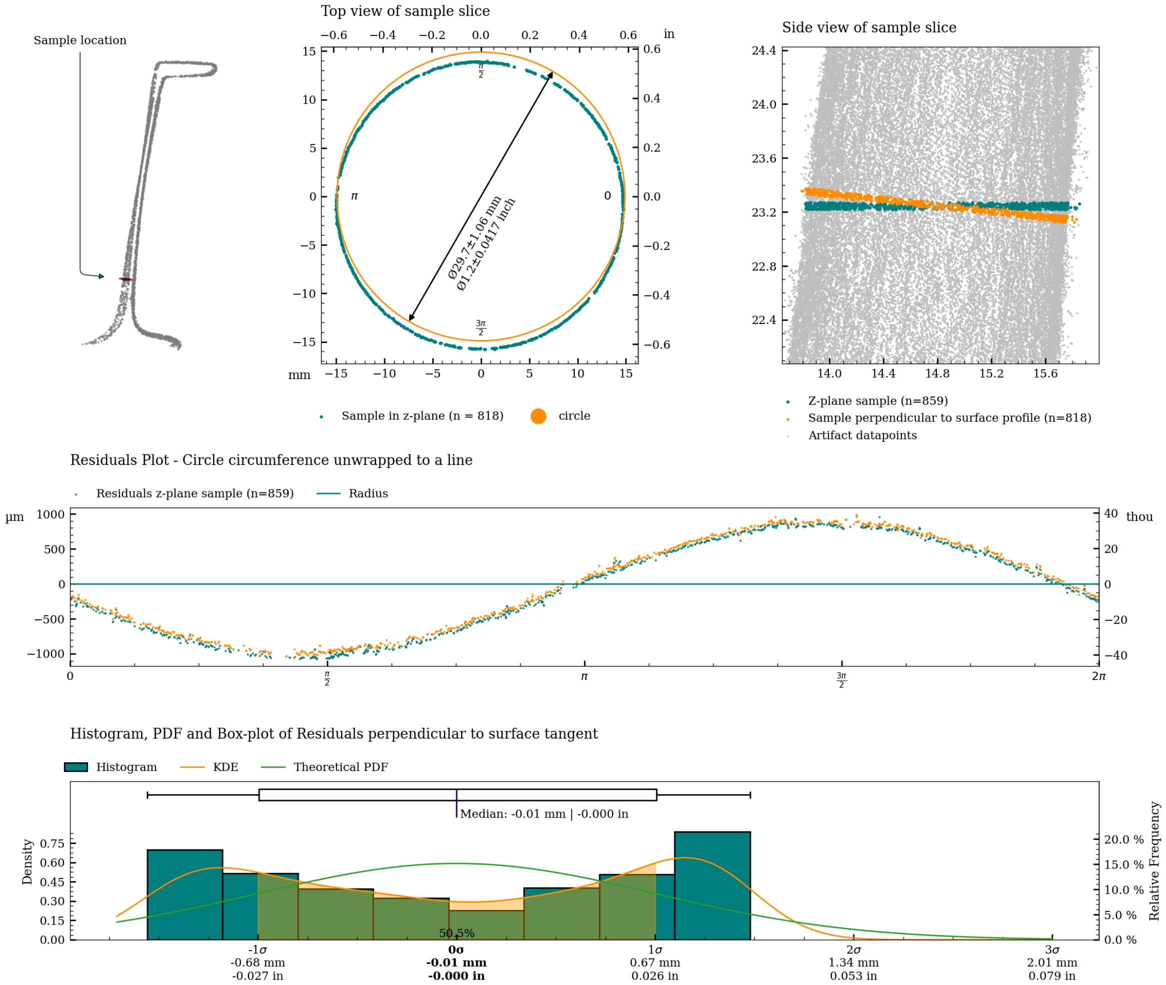


Figure 14: Charts with statistics for the measurement of c10.

Table 2 shows statistical measures of the circularity of the vessel, measured along the full height (damaged parts may reduce the measurement area).

Metric

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.871	0.444	1.865	0.174	0.098	0.335	0.026	0.049	0.327	1579	0.050
Interior	1.081	0.128	2.589	0.346	0.021	0.852	0.156	0.014	0.872	1769	0.050

Imperial

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.871	0.444	1.865	0.174	0.098	0.335	0.026	0.049	0.327	1579	0.050
Interior	1.081	0.128	2.589	0.346	0.021	0.852	0.156	0.014	0.872	1769	0.050

Table 2: Perpendicular Circularity analysis of MV013.

Circularity analysis of exterior surface

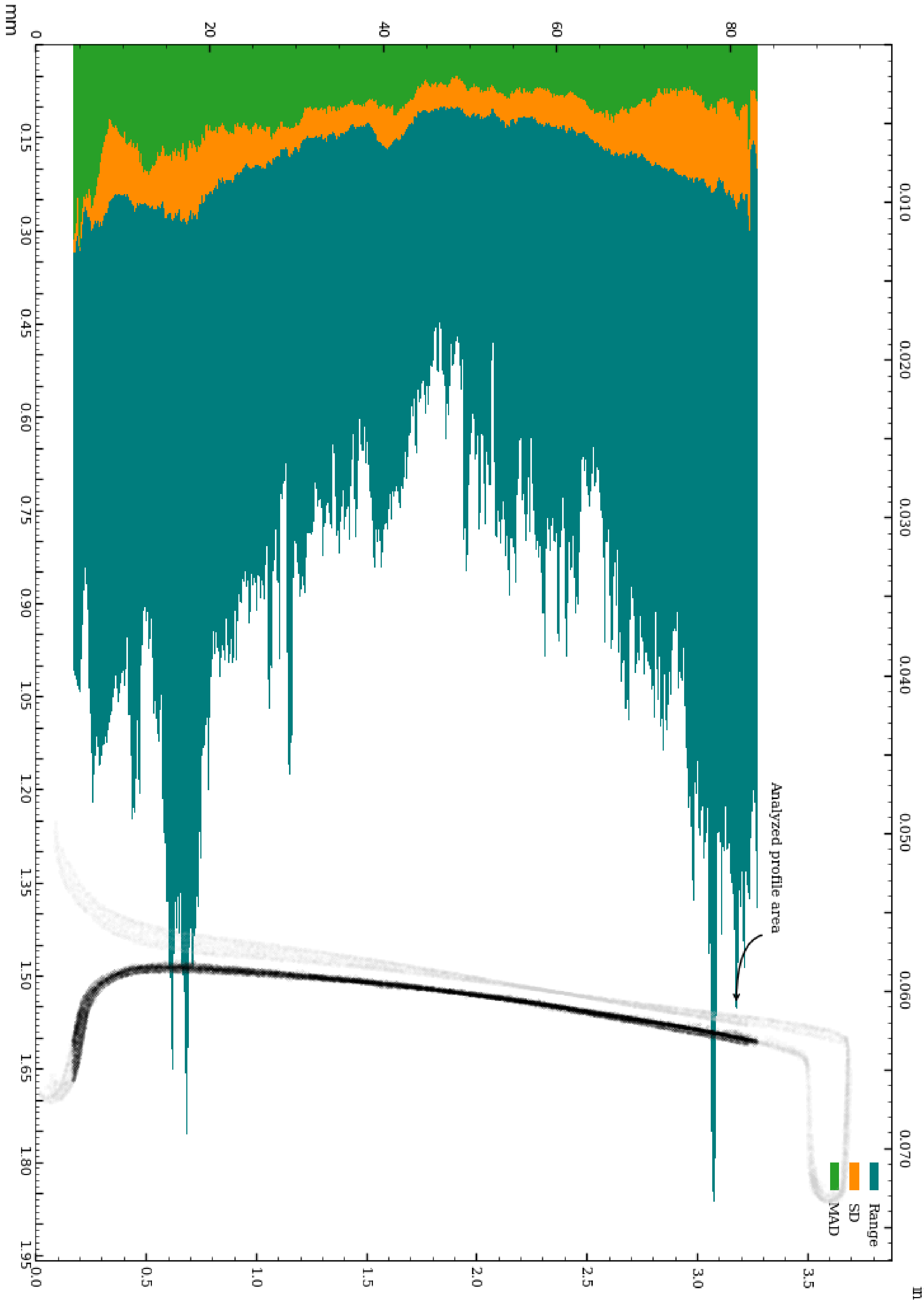


Figure 15: Circularity of exterior surface.

Circularity analysis of exterior surface, Standard Deviation and Median Absolute Deviation

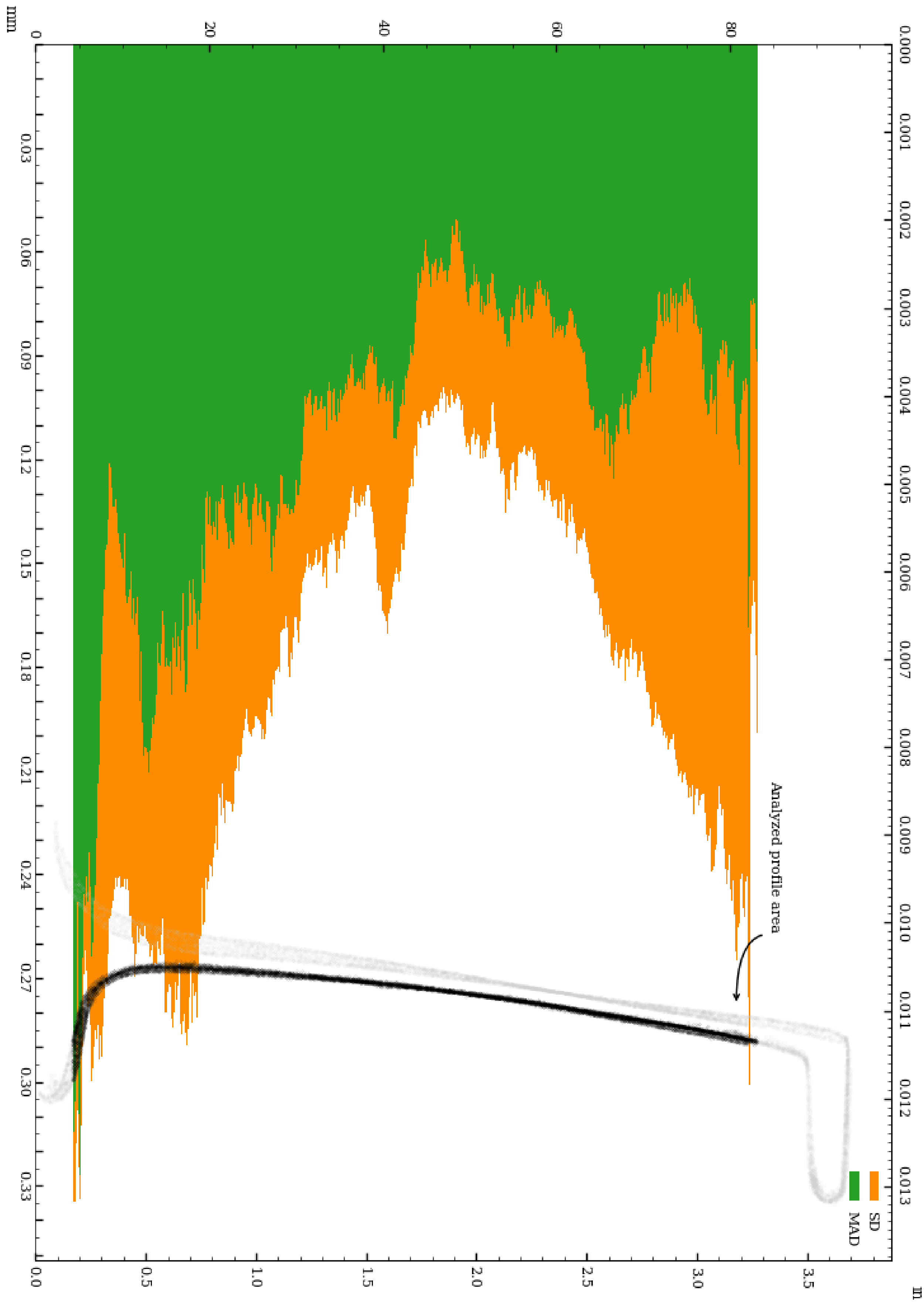


Figure 16: Vessel circularity of exterior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 1579 slices of the exterior surface are shown below.

Range measurement distribution across 1579 slices of exterior surface

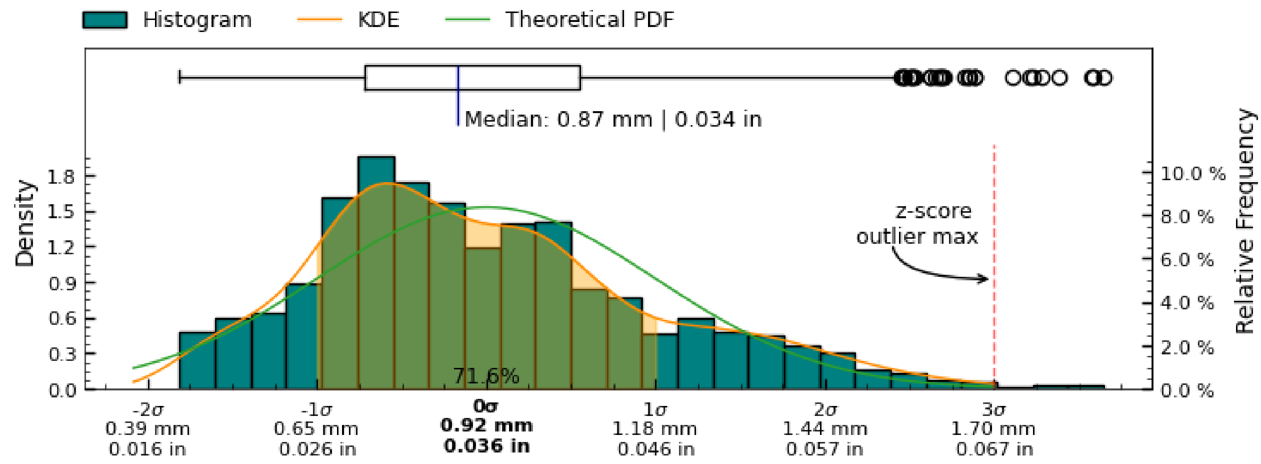


Figure 17: Range measurement distribution across measured slices of exterior surface

Standard deviation measurement distribution across 1579 slices of exterior surface

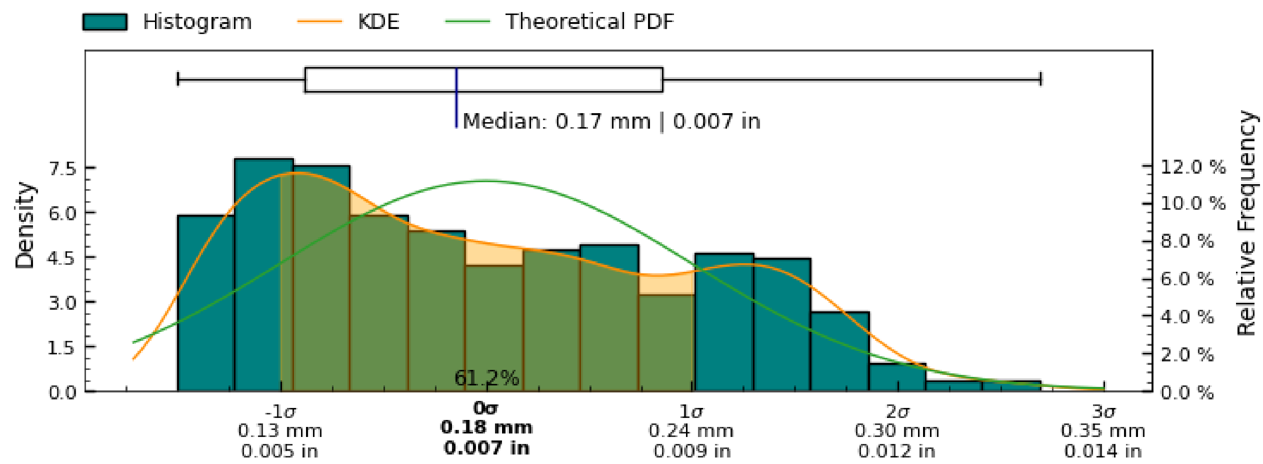


Figure 18: Standard deviation measurement distribution across measured slices of " + exterior + " surface

Median absolute deviation measurement distribution across 1579 slices of exterior surface

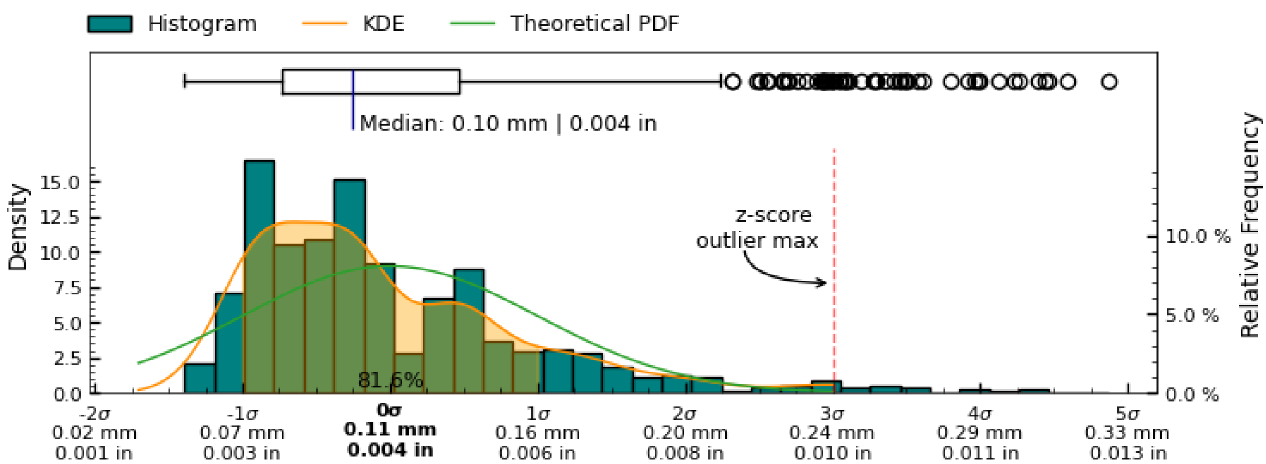


Figure 19: Median absolute deviation measurement distribution across measured slices of exterior surface

Circularity analysis of interior surface

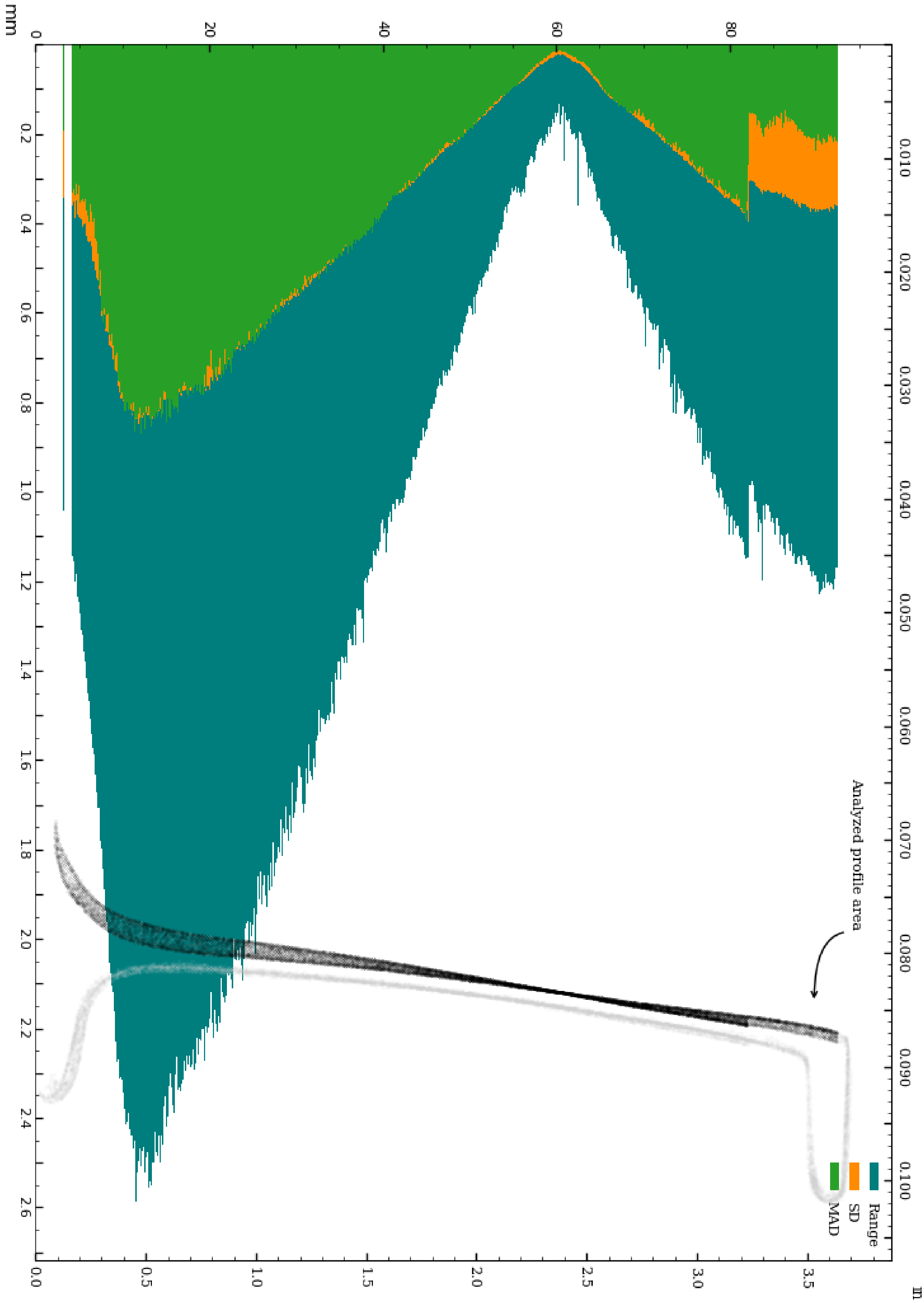


Figure 20: Circularity of interior surface.

Circularity analysis of interior surface, Standard Deviation and Median Absolute Deviation

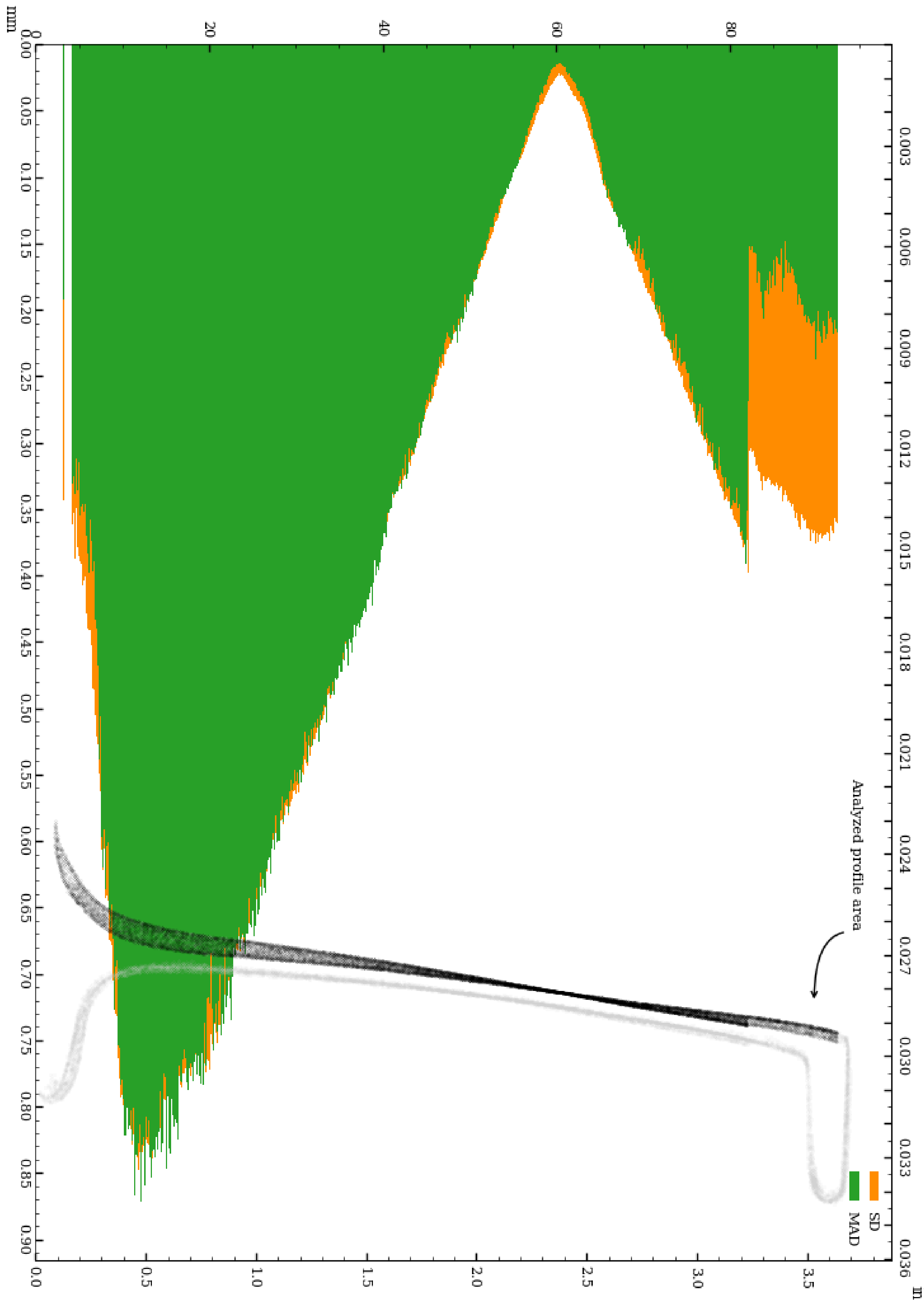


Figure 21: Vessel circularity of interior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 1769 slices of the interior surface are shown below.

Range measurement distribution across 1769 slices of interior surface

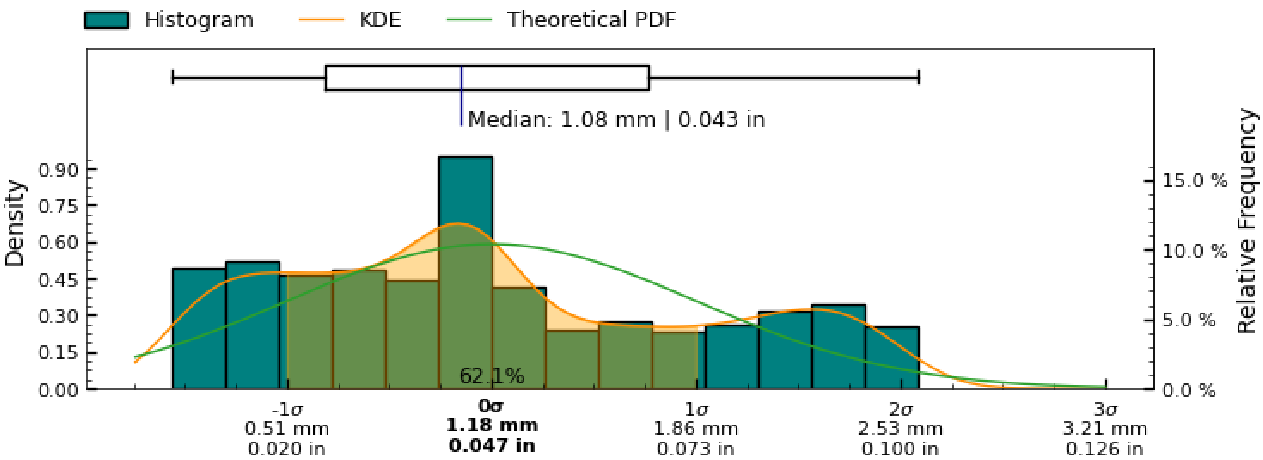


Figure 22: Range measurement distribution across measured slices of interior surface

Standard deviation measurement distribution across 1769 slices of interior surface

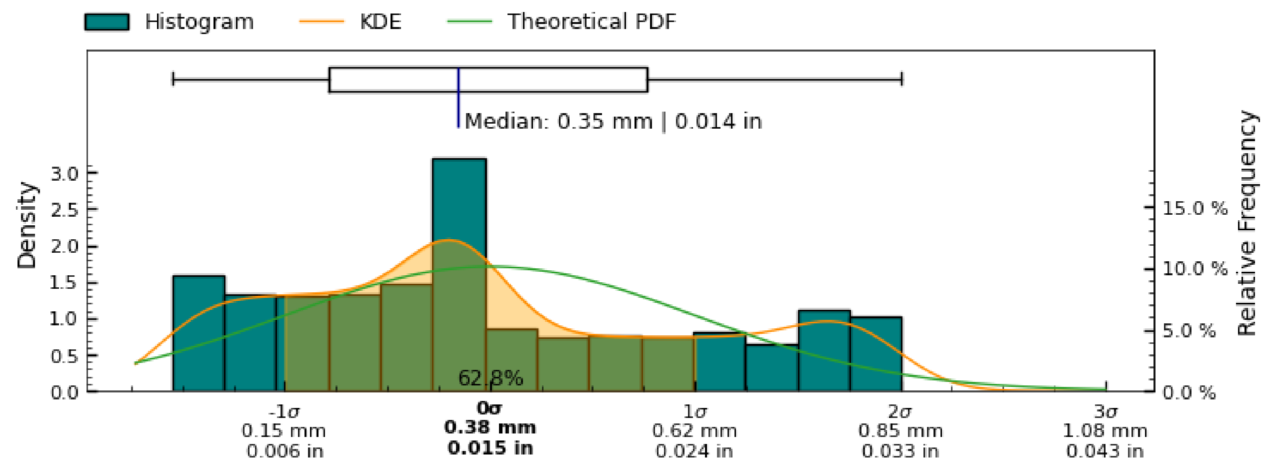


Figure 23: Standard deviation measurement distribution across measured slices of " + interior + " surface

Median absolute deviation measurement distribution across 1769 slices of interior surface

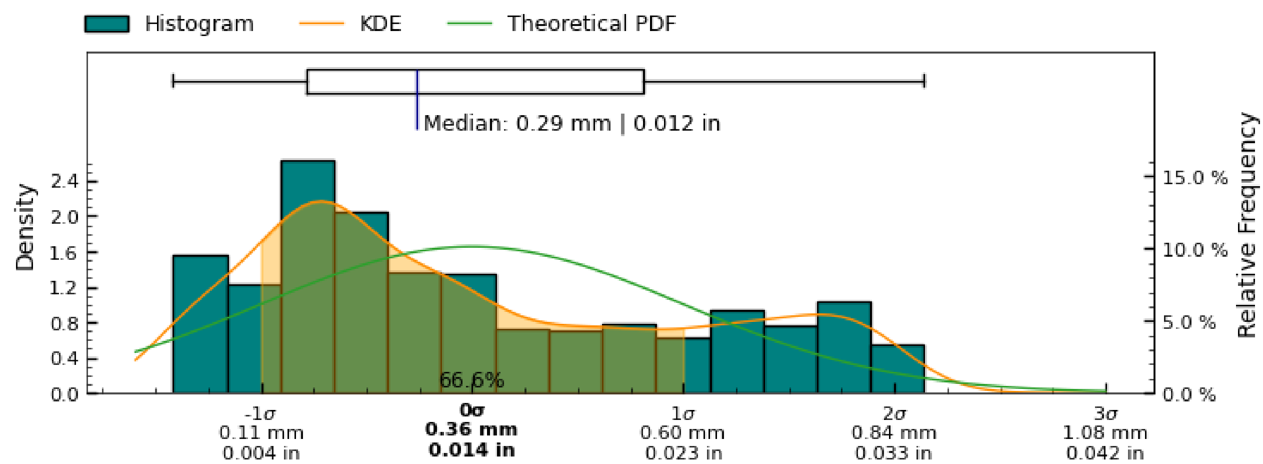


Figure 24: Median absolute deviation measurement distribution across measured slices of interior surface

Concentricity

The concentricity metric describes the deviation in the center-point of the referenced features. As such, it is a measure to determine if several features of the object share the same center point/axis, and how closely. See Figure 25 for a visual representation of this metric.

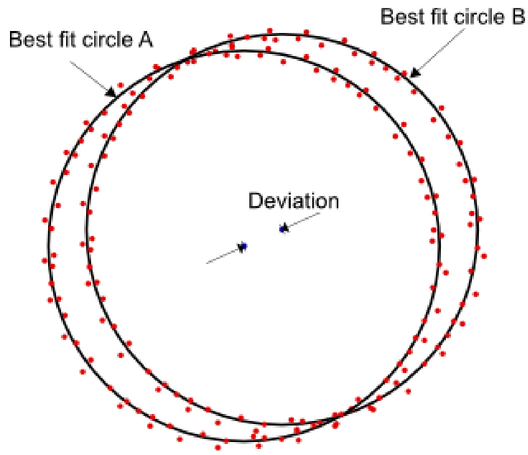


Figure 25: Concentricity measures the deviation (distance) between the center of two circles.

Determination of concentricity has been carried out by establishing the best fit circles of sample slices, using RANSAC (Random sample consensus) algorithm for outlier detection of a least squares circle regression on the scanned data-points at each cross-section, to estimate centers of each cross-section.

The concentricity between both the interior and exterior circular cross-sections is explored for cross-section measurements with the same Z-coordinates.

Additionally, the concentricity between each cross-section measurement defined in Figure 4 and the datum axis $(x, y) = (0, 0)$ has been calculated to establish the deviation of the feature center from the datum axis.

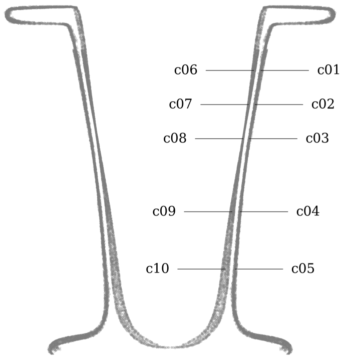


Figure 26: Concentricity measurement sample location on MV013.

Metric

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.218	1468	1.193	0.987	0.213	0.200	0.079	0.078	−104, −192
c02	z-axis	0.181	1438	1.015	0.879	0.176	0.172	0.107	0.104	−129, −127
c03	z-axis	0.109	1288	0.844	0.706	0.126	0.124	0.069	0.069	−95, −53
c04	z-axis	0.042	1172	0.756	0.646	0.134	0.131	0.088	0.087	−30, −29
c05	z-axis	0.097	1112	0.874	0.874	0.200	0.200	0.123	0.123	−6, −96
c06	z-axis	0.395	1306	0.875	0.875	0.287	0.288	0.276	0.283	41, 393
c07	z-axis	0.181	1260	0.464	0.434	0.133	0.133	0.136	0.138	6, 180
c08	z-axis	0.052	1114	0.210	0.190	0.045	0.043	0.035	0.034	−8, −51
c09	z-axis	0.543	983	1.153	1.153	0.390	0.390	0.398	0.395	−73, −538
c10	z-axis	0.934	818	2.047	1.951	0.674	0.675	0.672	0.671	−123, −926
c01	c06	0.603	1468	1.193	0.987	0.213	0.200	0.079	0.078	−145, −585
c02	c07	0.335	1438	1.015	0.879	0.176	0.172	0.107	0.104	−135, −307
c03	c08	0.087	1288	0.844	0.706	0.126	0.124	0.069	0.069	−87, −2
c04	c09	0.511	1172	0.756	0.646	0.134	0.131	0.088	0.087	44, 509
c05	c10	0.838	1112	0.874	0.874	0.200	0.200	0.123	0.123	116, 830

Imperial

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0086	1468	0.0470	0.0389	0.0084	0.0079	0.0031	0.0031	−4.1, −7.6
c02	z-axis	0.0071	1438	0.0400	0.0346	0.0069	0.0068	0.0042	0.0041	−5.1, −5.0
c03	z-axis	0.0043	1288	0.0332	0.0278	0.0049	0.0049	0.0027	0.0027	−3.7, −2.1
c04	z-axis	0.0016	1172	0.0298	0.0254	0.0053	0.0052	0.0035	0.0034	−1.2, −1.2
c05	z-axis	0.0038	1112	0.0344	0.0344	0.0079	0.0079	0.0048	0.0048	−0.2, −3.8
c06	z-axis	0.0156	1306	0.0345	0.0345	0.0113	0.0113	0.0109	0.0111	1.6, 15.5
c07	z-axis	0.0071	1260	0.0183	0.0171	0.0052	0.0052	0.0054	0.0054	0.2, 7.1
c08	z-axis	0.0020	1114	0.0083	0.0075	0.0018	0.0017	0.0014	0.0013	−0.3, −2.0
c09	z-axis	0.0214	983	0.0454	0.0454	0.0154	0.0153	0.0157	0.0155	−2.9, −21.2
c10	z-axis	0.0368	818	0.0806	0.0768	0.0266	0.0266	0.0265	0.0264	−4.8, −36.5
c01	c06	0.0237	1468	0.0470	0.0389	0.0084	0.0079	0.0031	0.0031	−5.7, −23.0
c02	c07	0.0132	1438	0.0400	0.0346	0.0069	0.0068	0.0042	0.0041	−5.3, −12.1
c03	c08	0.0034	1288	0.0332	0.0278	0.0049	0.0049	0.0027	0.0027	−3.4, −0.1
c04	c09	0.0201	1172	0.0298	0.0254	0.0053	0.0052	0.0035	0.0034	1.7, 20.0
c05	c10	0.0330	1112	0.0344	0.0344	0.0079	0.0079	0.0048	0.0048	4.6, 32.7

Table 3: Concentricity analysis of MV013.

Concentricity analysis of c01

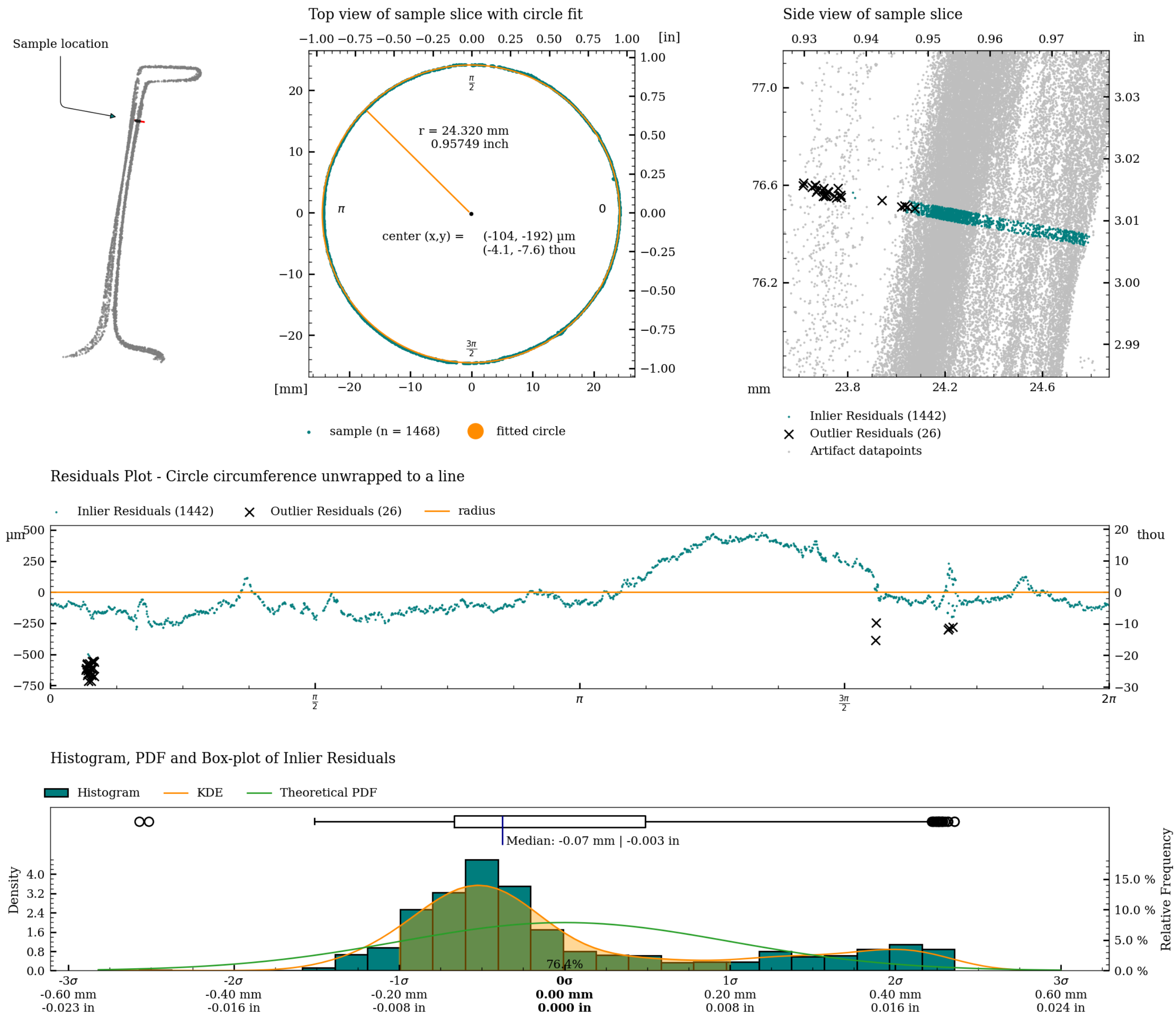


Figure 27: Detailed plot of concentricity measurement for c01.

Concentricity analysis of c02

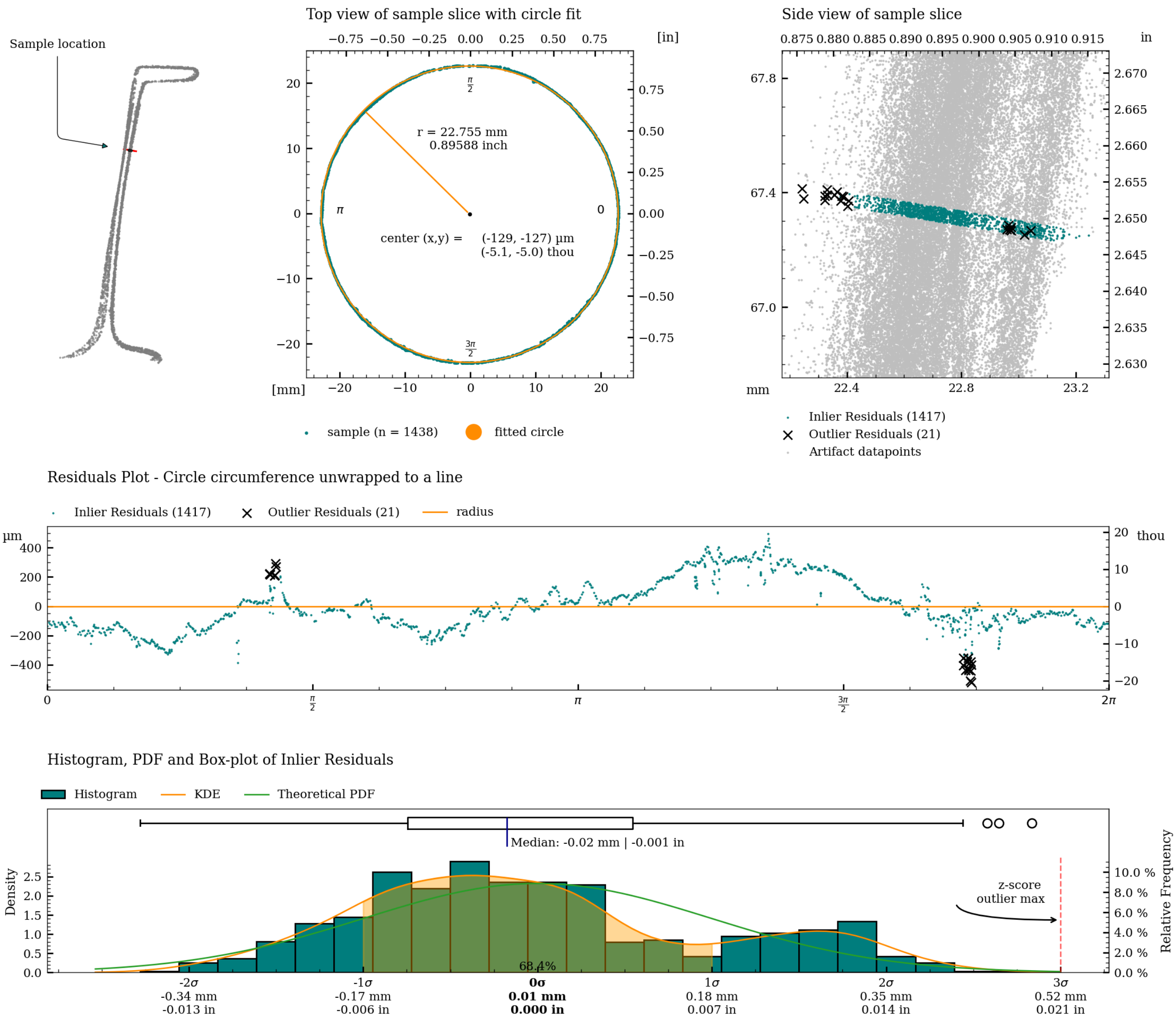


Figure 28: Detailed plot of concentricity measurement for c02.

Concentricity analysis of c03

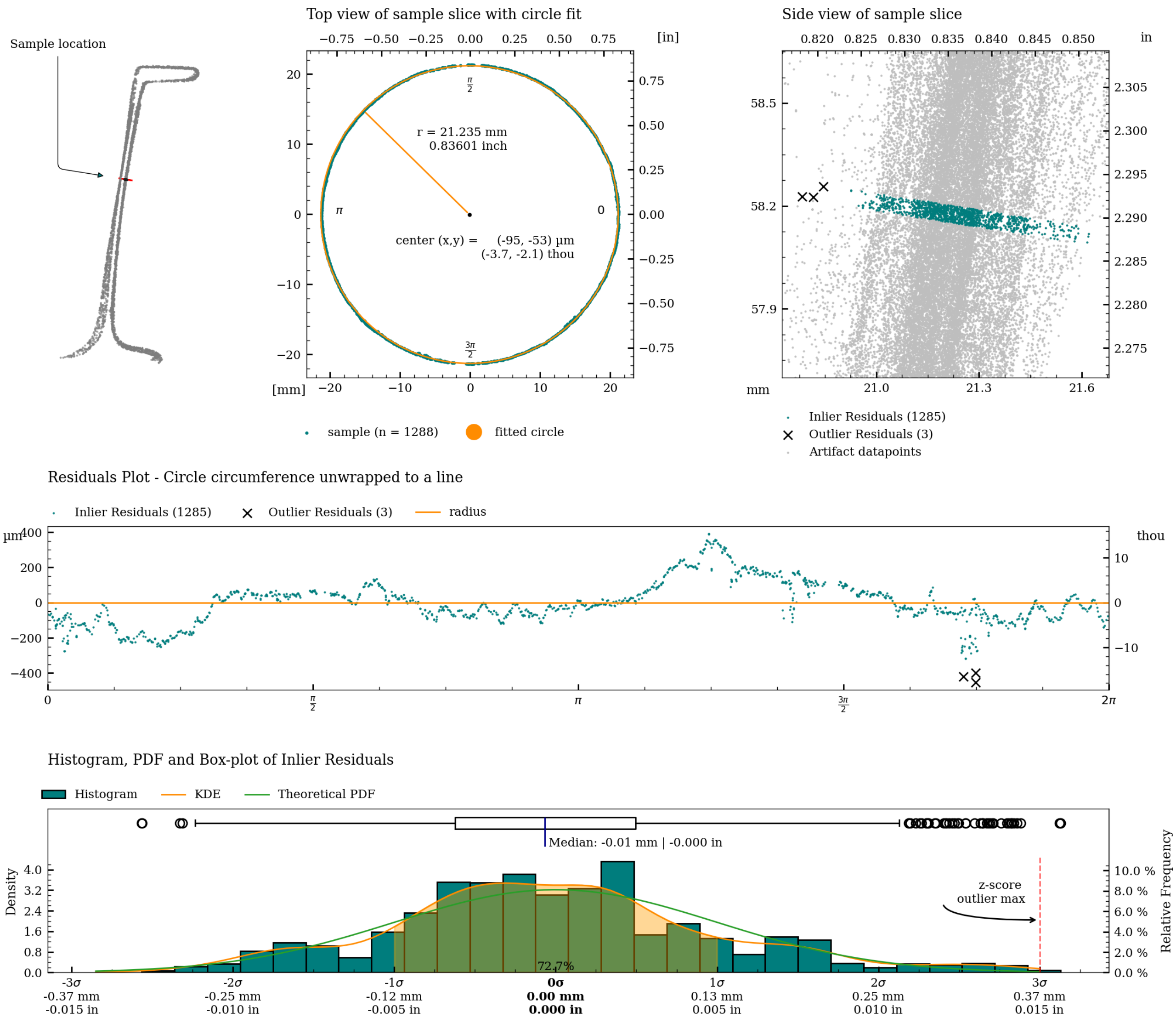


Figure 29: Detailed plot of concentricity measurement for c03.

Concentricity analysis of c04

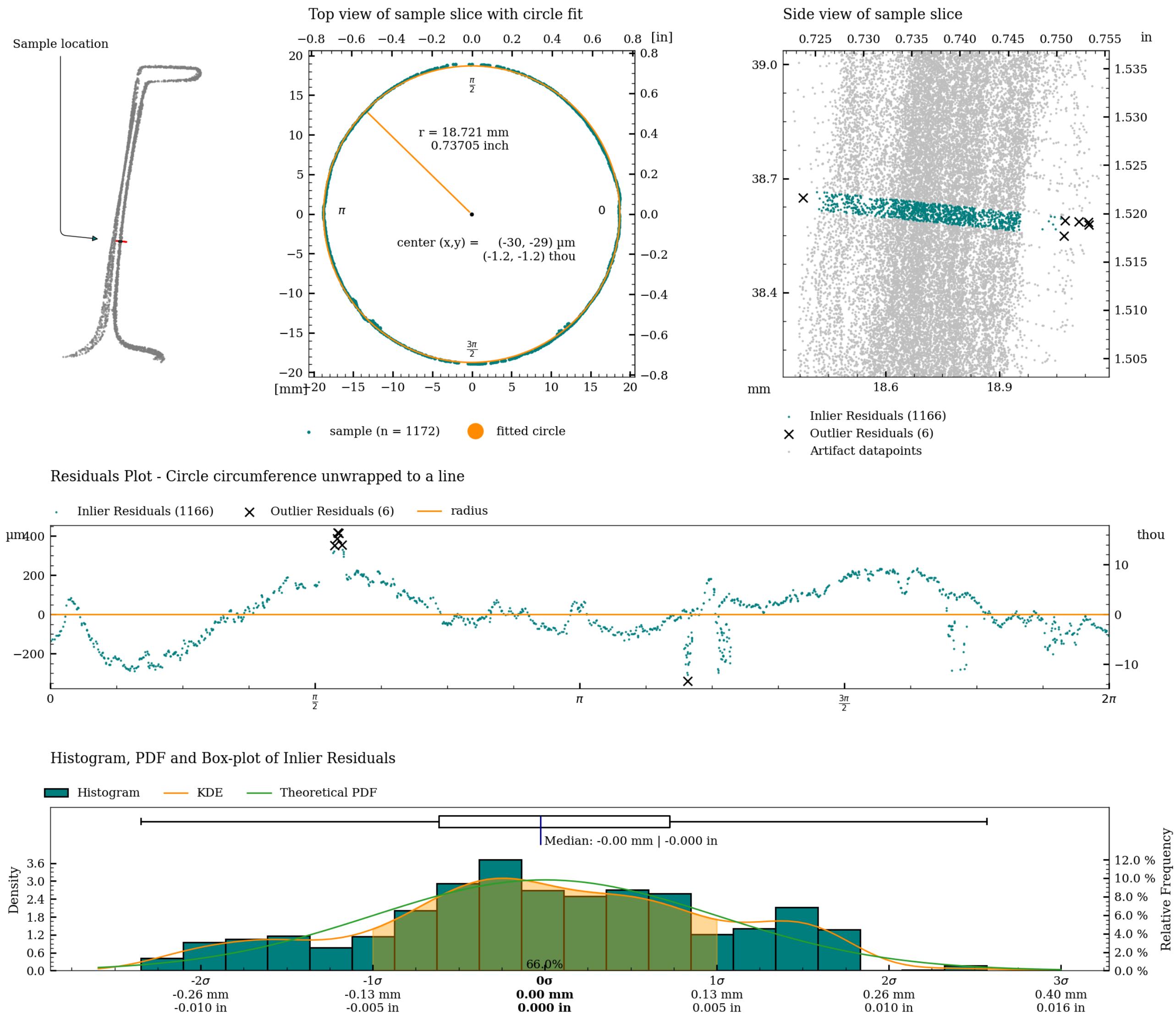


Figure 30: Detailed plot of concentricity measurement for c04.

Concentricity analysis of c05

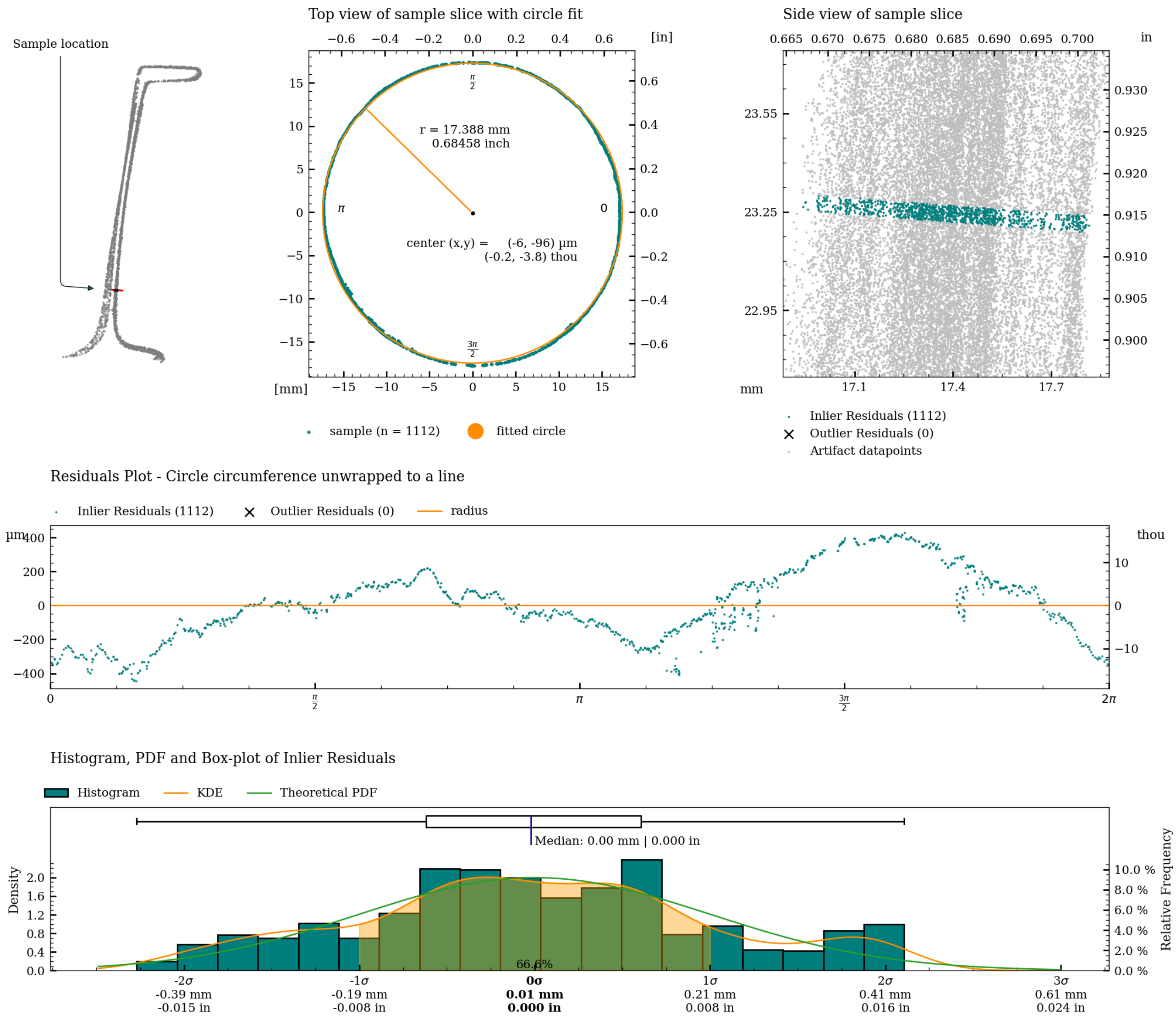


Figure 31: Detailed plot of concentricity measurement for c05.

Concentricity analysis of c06

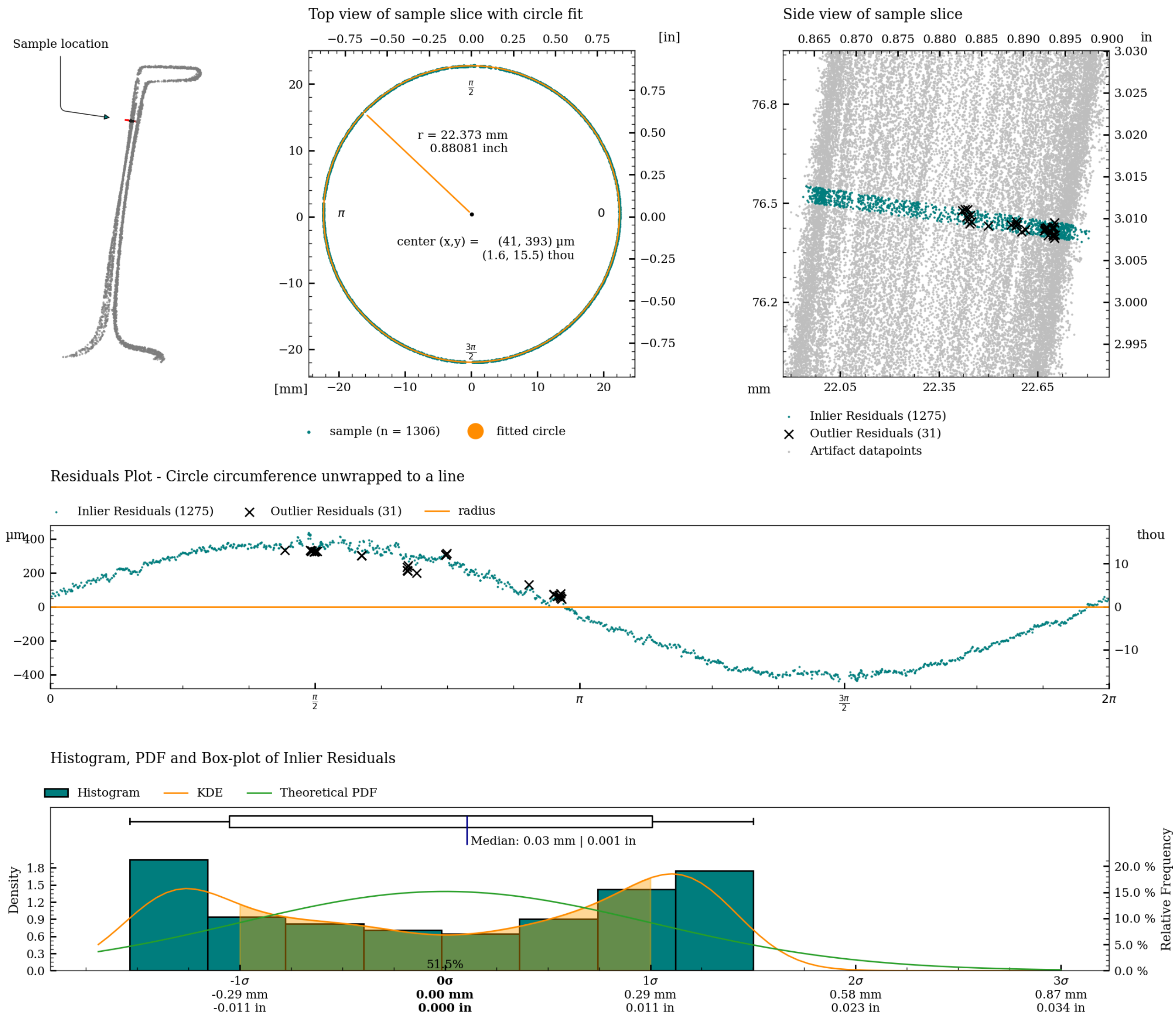


Figure 32: Detailed plot of concentricity measurement for c06.

Concentricity analysis of c07

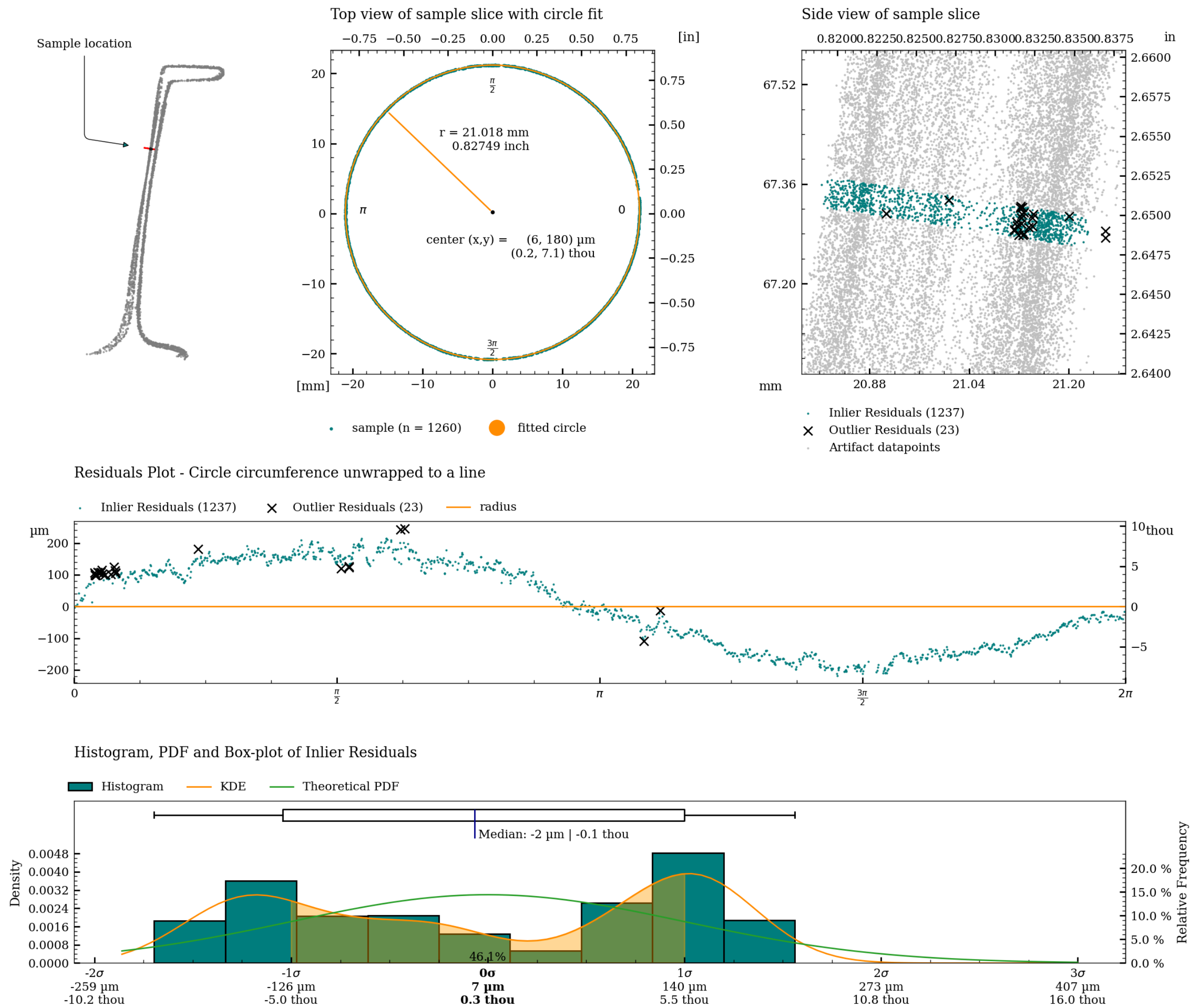


Figure 33: Detailed plot of concentricity measurement for c07.

Concentricity analysis of c08

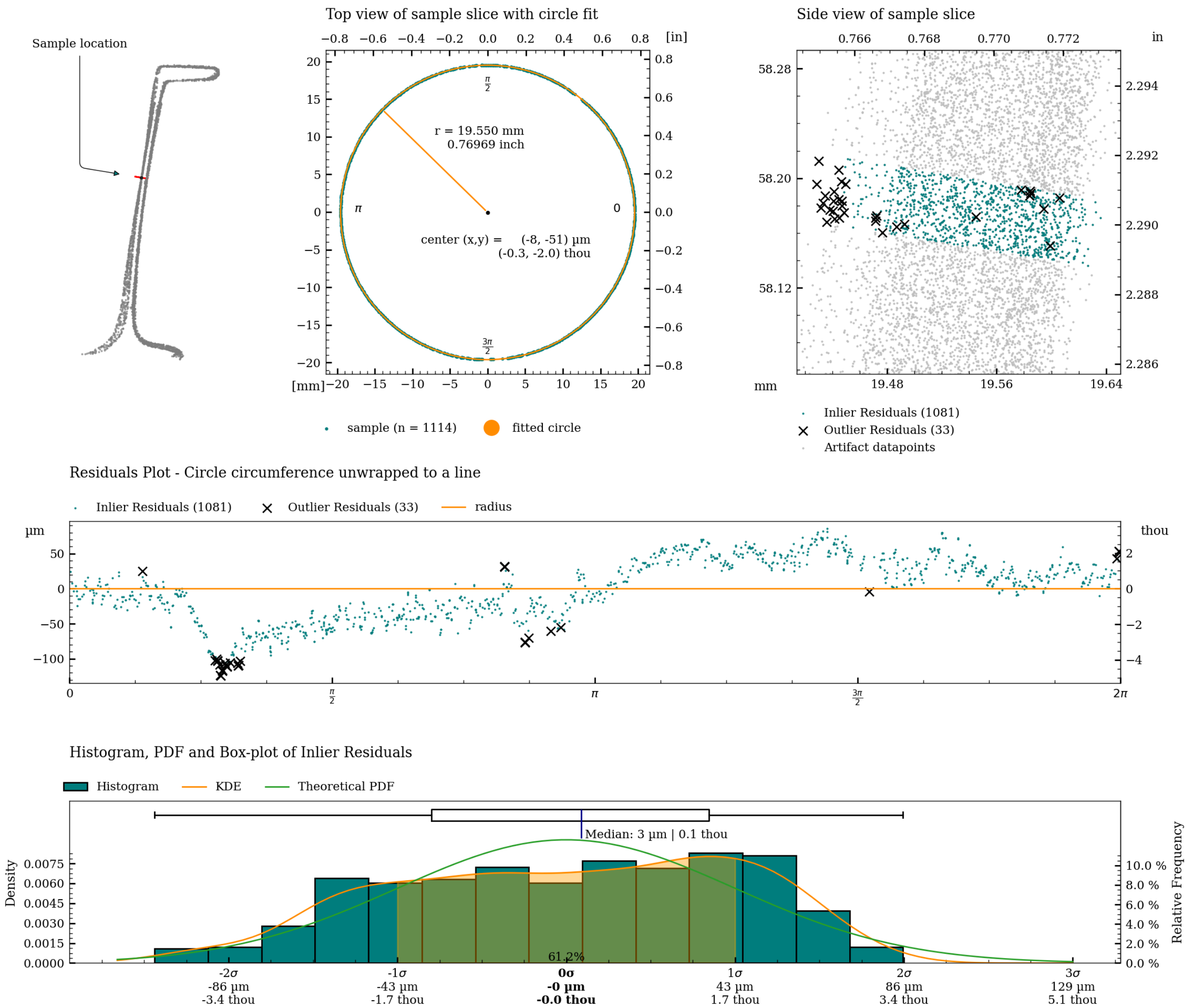


Figure 34: Detailed plot of concentricity measurement for c08.

Concentricity analysis of c09

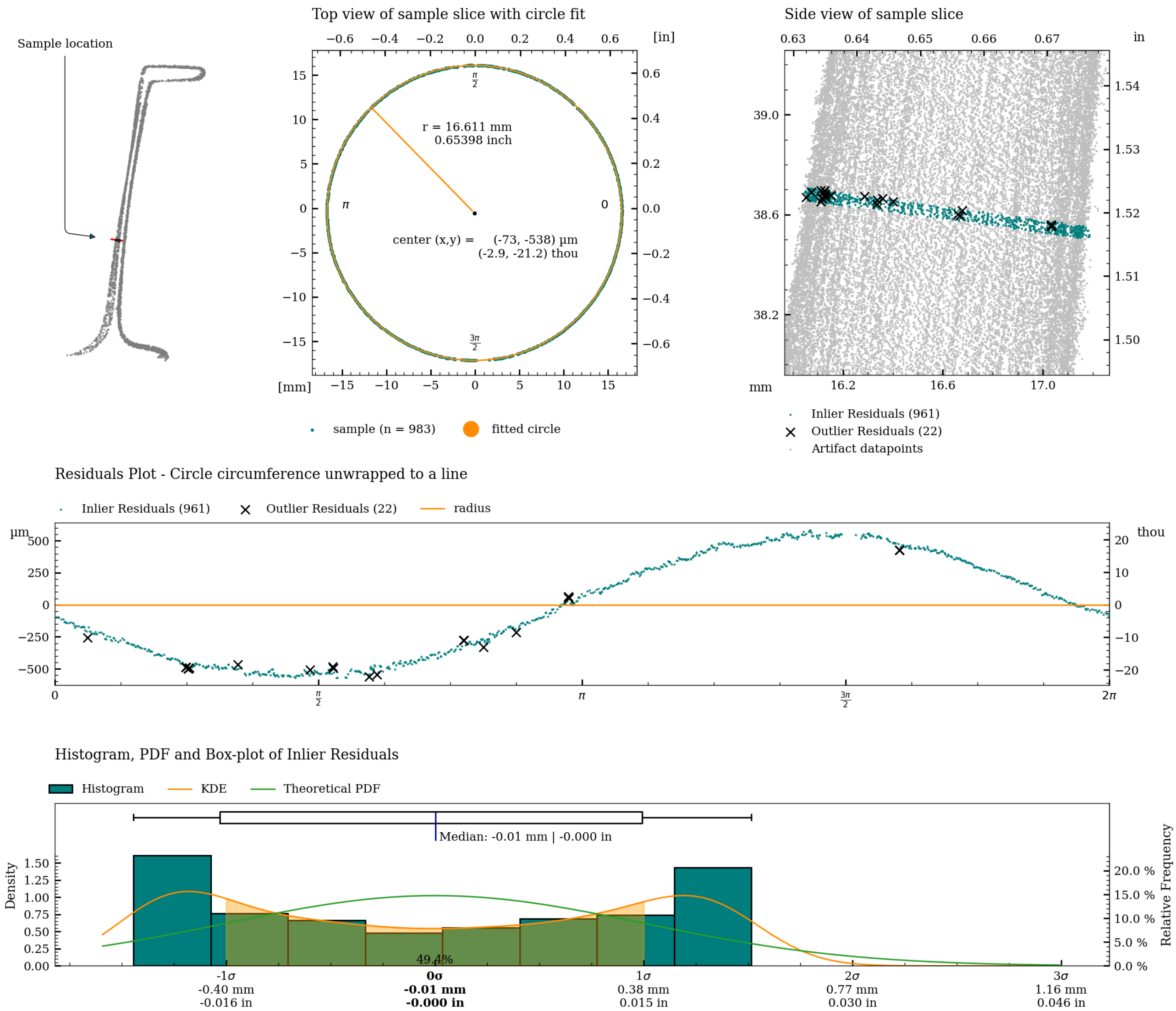


Figure 35: Detailed plot of concentricity measurement for c09.

Concentricity analysis of c10

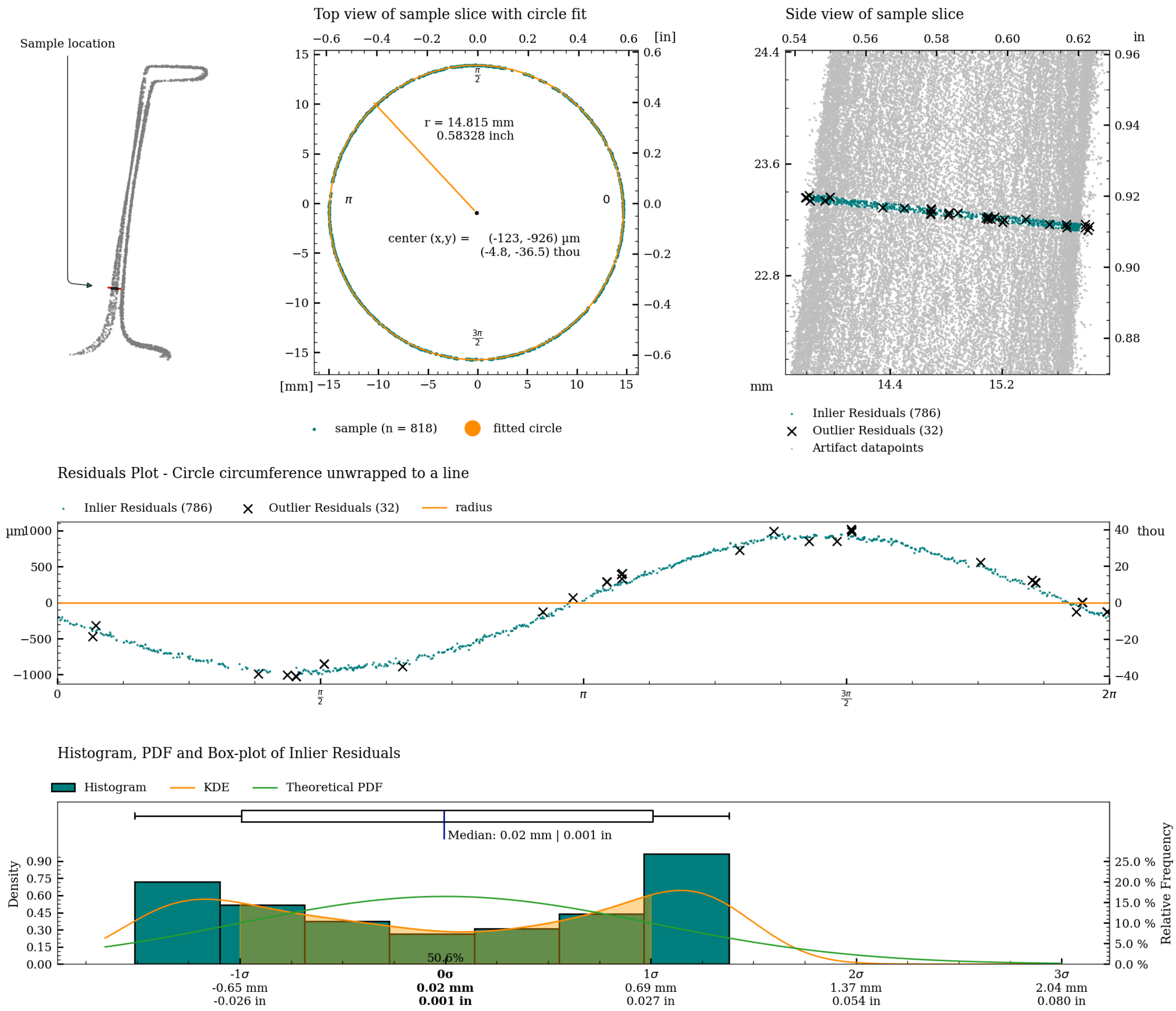


Figure 36: Detailed plot of concentricity measurement for c10.

Coaxiality

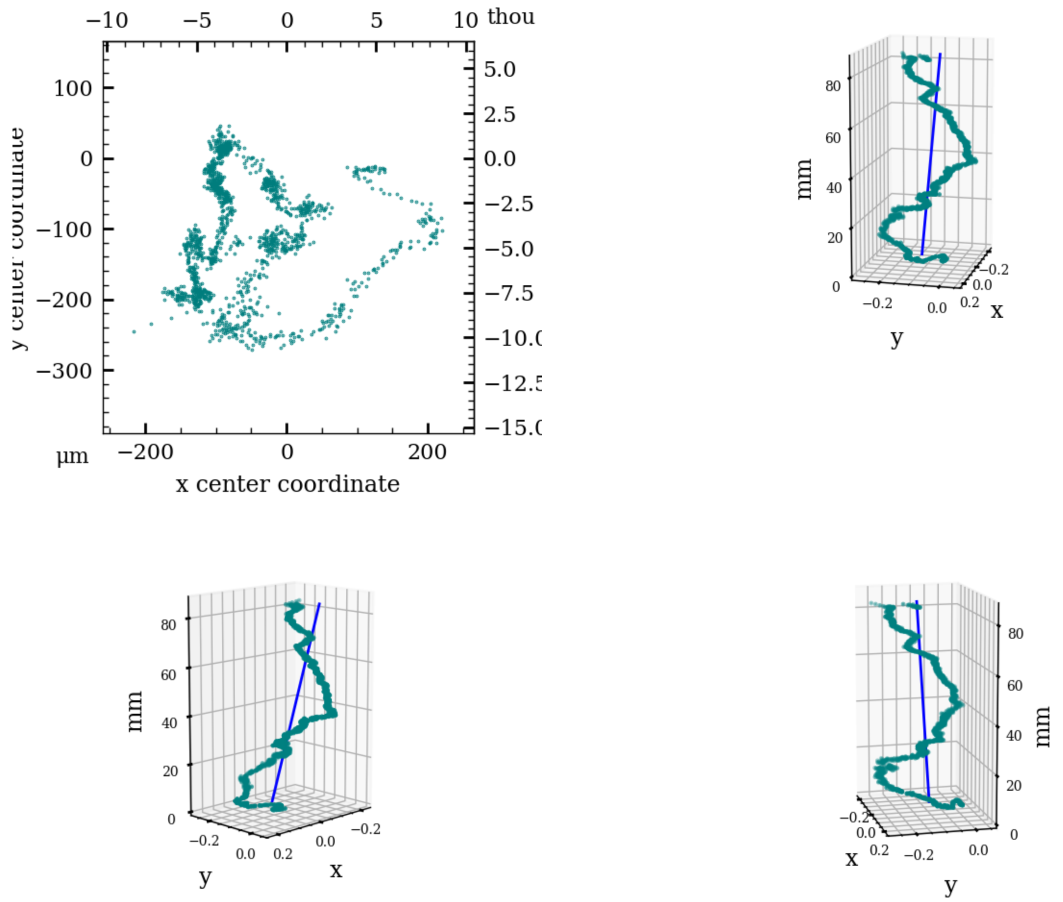
Coaxiality is a measure of the deviation in the central axis of an object. Coaxiality measurements are calculated using RANSAC (Random sample consensus) algorithm for outlier detection of a least squares circle regression on cross-sections of the vessel (excluding potential handles) to estimate the best fit circle centers for each slice of the vessel.

Coaxiality is measured for:
- The exterior surface (excluding handles)
- The interior surface

	Exterior		Interior	
Analyzed Slices		1579		1769
Median sample size		1200		982
Slice Height	50 µm	2.0 thou	50 µm	2.0 thou
Statistics with Z-axis as Reference				
Median Absolute Deviation (MAD)	124 µm	4.9 thou	527 µm	20.8 thou
Standard Deviation (SD)	66 µm	2.6 thou	325 µm	12.8 thou
Root Mean Square Deviation (RMSD)	162 µm	6.4 thou	636 µm	25.0 thou
Statistics with Best Fit Central Axis as Reference				
Best fit Central Axis Equation (in metric coordinate system with unit [mm])	x = 0.063 + t0.00273 y = -0.108 + t-0.00013 z = 0.000 + t-1.00000		x = -0.173 + t0.00297 y = -1.436 + t0.02383 z = 0.000 + t0.99971	
Axis tilt		0.156°		0.166°
Median Absolute Deviation (MAD)	83 µm	3.3 thou	26 µm	1.0 thou
Standard Deviation (SD)	44 µm	1.7 thou	153 µm	6.0 thou
Root Mean Square Deviation (RMSD)	94 µm	3.7 thou	167 µm	6.6 thou

Table 4: Coaxiality analysis of vessel MV013.

Coaxiality plots, exterior surface



Coaxiality residuals from fitted axis, exterior surface

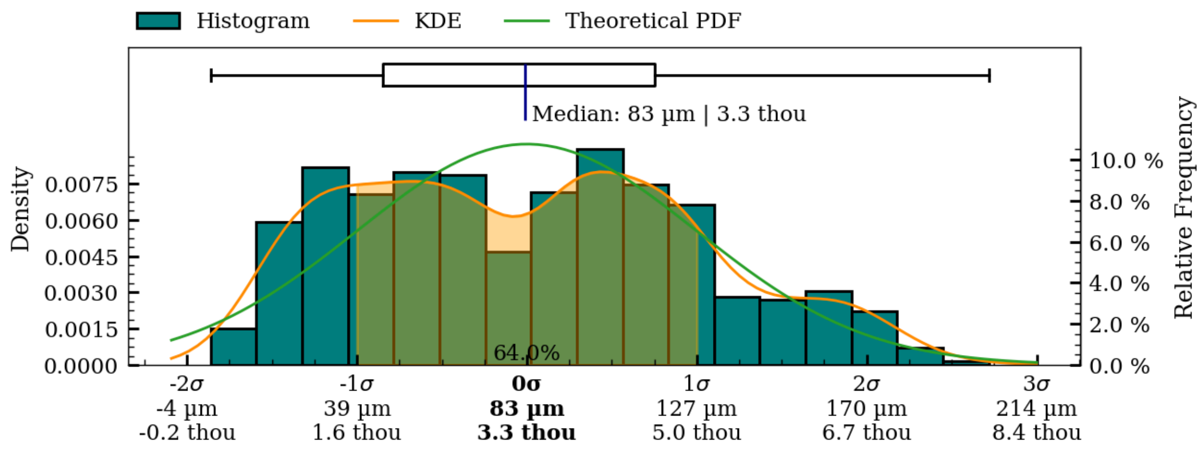
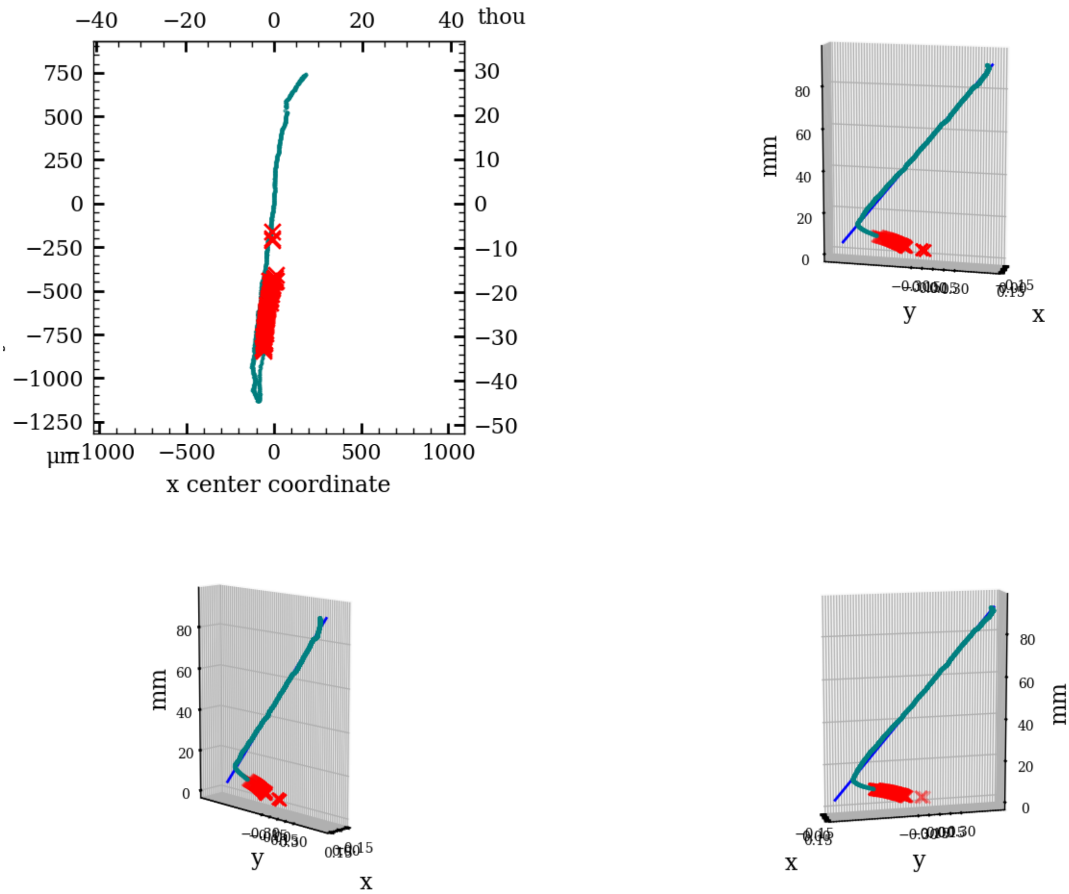


Figure 37: Coaxiality residual plots of exterior surface, MV013.

Coaxiality plots, interior surface



Coaxiality residuals from fitted axis, interior surface

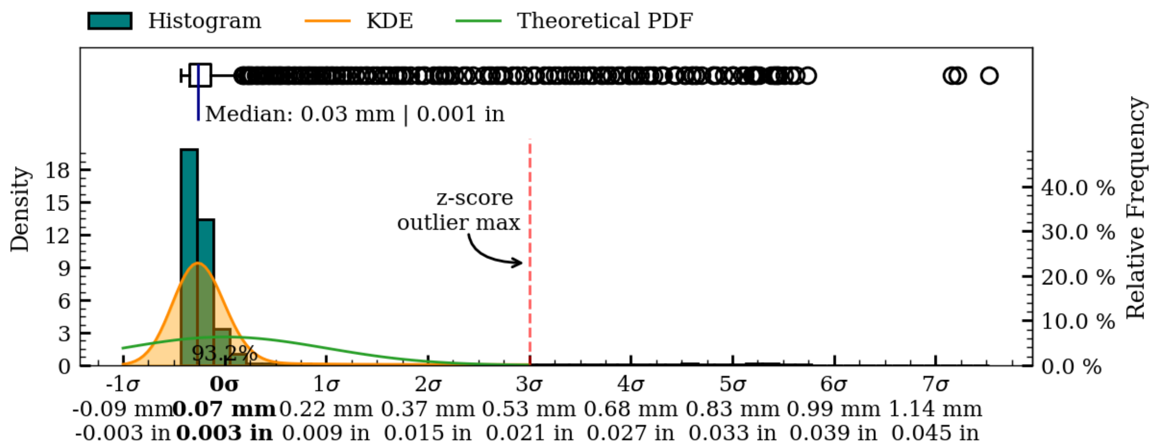


Figure 38: Coaxiality residual plots of interior surface, MV013.

Surface Variability

To illustrate the overall surface deviations of the object, a surface variability heatmap has been created. This heatmap provides an accessible overview of the topography of the manufacturing precision and surface structure of the object.

The surface variability measurements are created by fitting a number of higher-order polynomials to the two-dimensional folded profile of the scan data. This process creates an idealized mathematical representation of actual surface curvature of object, and as such provides a continuous model representation of the actual object. It is important to note that only such a non-discretized representation is sufficient to avoid introducing inconsistently varying errors in the mapping of the final surface deviation results, that the rendered heatmaps are based on.

To produce the final surface variability map, the distance from each scanned vertex to the fitted polynomial is calculated and used as the mapping function input, for applying colours to the surface of the object.

It is important to note that this variability map does not describe deviations from the original *intended* shape of the artifact (if any), as this shape (the *intended design*, so to speak) will have been lost to time. It does however provide a very informative visualization of the texture and structure of the surface and very importantly, *does* hightlight potential manufacturing-relevant patterns in the surface texture (if present). Such patterns are, as an example, clearly evident on the interior surface of artifact PV001.

Exterior surface

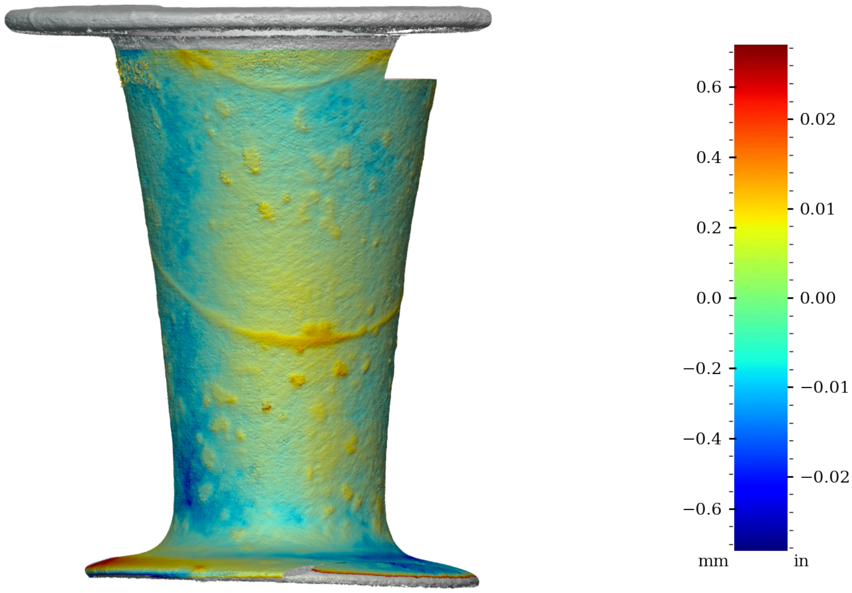


Figure 39: Surface variability heatmap of MV013, front view

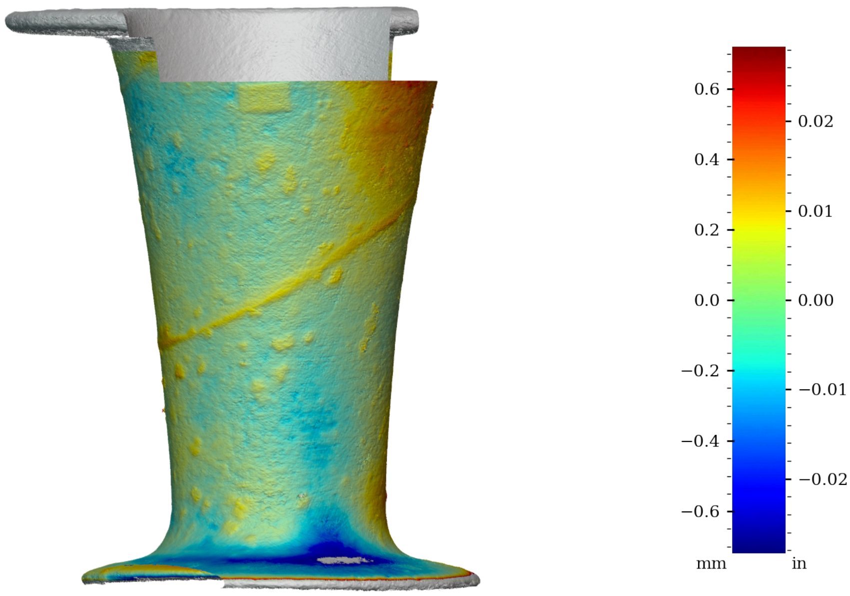


Figure 40: Surface variability heatmap of MV013, rotated 90°

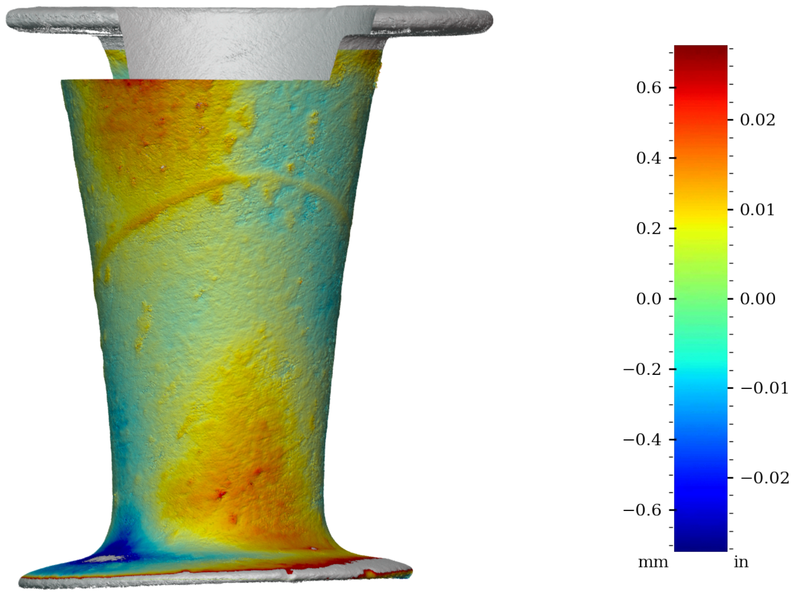


Figure 41: Surface variability heatmap of MV013, rotated 180°

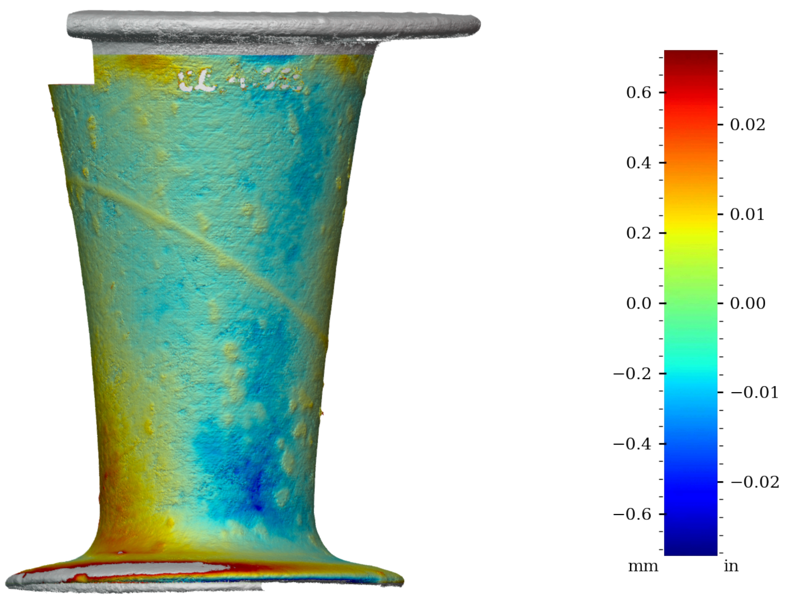


Figure 42: Surface variability heatmap of MV013, rotated 270°

Interior surface

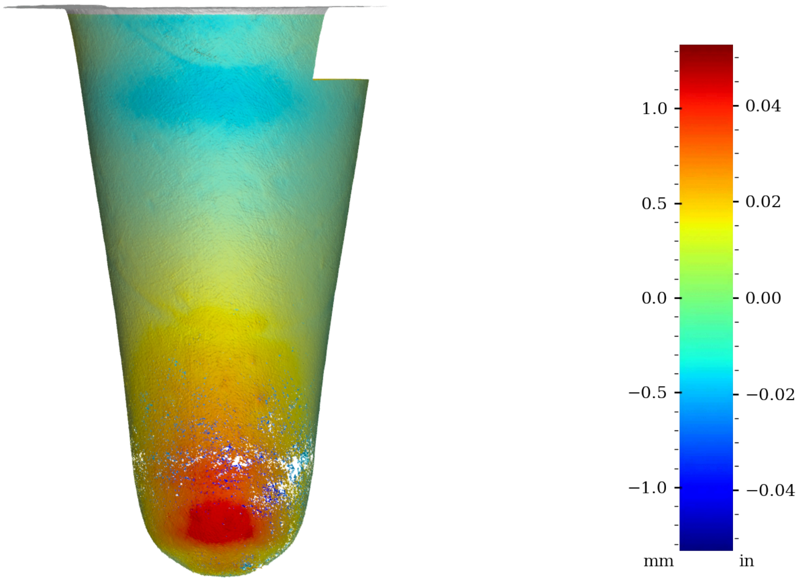


Figure 43: Surface variability heatmap of MV013, front view

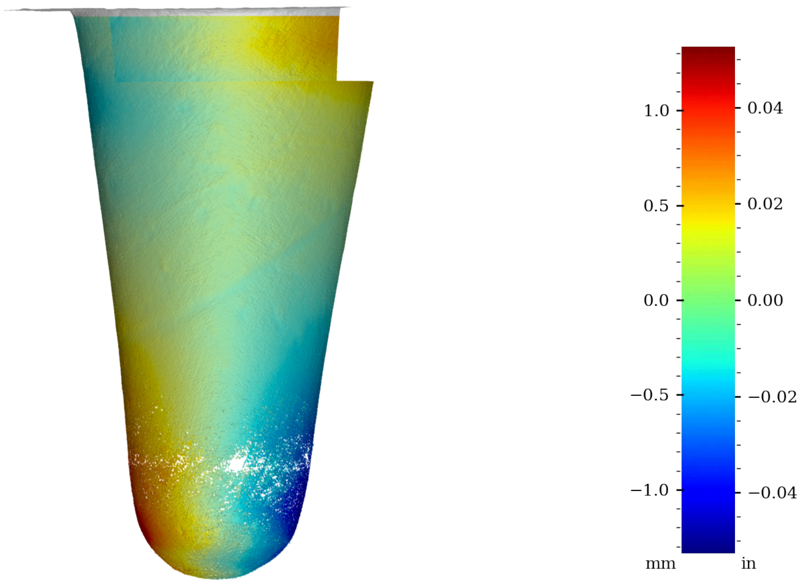


Figure 44: Surface variability heatmap of MV013, rotated 90°

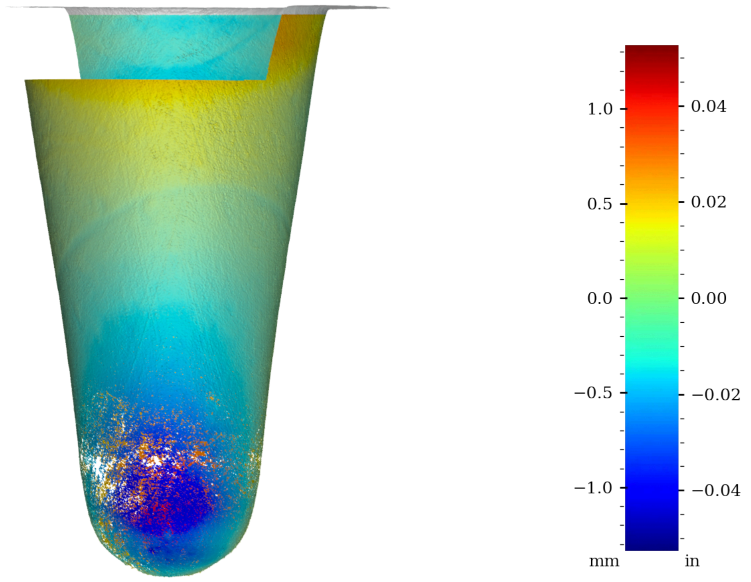


Figure 45: Surface variability heatmap of MV013, rotated 180°

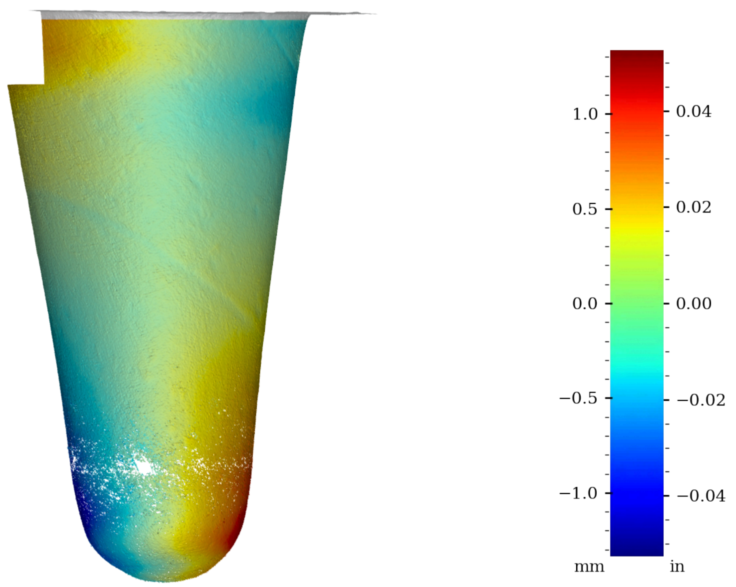


Figure 46: Surface variability heatmap of MV013, rotated 270°

Surface variability statistics

Area	MSD	RMSD	SD	Mean AD	Median AD	Range	Min	Max	Sample size
	mm^2	mm	mm	mm	mm	mm	mm	mm	
Exterior	0.0501	0.224	0.224	0.119	0.167	1.963	−1.097	0.865	2233412
Interior	0.1846	0.430	0.430	0.253	0.327	2.663	−1.332	1.330	1826963
	in^2	in	in	in	in	in	in	in	
Exterior	0.000078	0.0088	0.0088	0.0047	0.0066	0.0773	−0.0432	0.0341	2233412
Interior	0.000286	0.0169	0.0169	0.0100	0.0129	0.1048	−0.0525	0.0524	1826963

Table 5: Surface variability statistics, MV013

Table 5 shows the statistics of the distance from the scan vertices to the best fit object model. These statistics are briefly explained below.

Mean Squared Deviation (MSD), also known as Mean Squared Error (MSE).

$$\text{MSD} = \frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}$$

The MSD metric shows the the average squared difference between the scanned points and the fitted composite polynomial model (a value of 0 would be a perfect match). This metric emphasizes imperfections in the surface of the artifact. Outliers will negatively influence this metric, raising the value of the MSE.

Root Mean Squared Deviation (RMSD), also known as Root Mean Squared Error (RMSE).

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}}$$

Measures the dispersion of the measured surface variability y_i around a model predictor ($\hat{y}$). By obtaining the root of the MSD, the exponent will be removed from the measurement, enabling comparisons with other statistics of the same unit and making it more accessible to those familiar with the RMSD metric. This measure is used to assess the fit of a regression model to a dataset, in this case our best fit composite polynomial model. The lower the RMSD metric, the better the fit.

Standard Deviation (SD)

$$s = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}}$$

Measures the dispersion of the measured surface variability y_i around the mean ($\bar{y}$). If the residuals are normally distributed around the mean ($\bar{y} \approx 0$), the SD will be equal to the RMSD. See Figure 47 and Figure 48

Mean Absolute Deviation (MeanAD)

$$\text{MeanAD} = \frac{\sum_{i=1}^{n} |y_i - \bar{y}|}{n}$$

This metric is similar to the SD, but the difference between the residuals and the mean is *not* squared. Instead of indicating the spread of the data, we look at the average distance between each data point and the mean. The Mean Absolute Deviation is affected less by outliers than the Standard Deviation.

Median Absolute Deviation (MedianAD)

$$\text{MedianAD} = \text{median}(|y_i - \text{median}(y)|)$$

The Median Absolute Deviation is measure of the dispersion of the data around the median.

Range

$$\max(y_i) - \min(y_i)$$

Range is a measure of the total spread of the residuals

Histogram, KDE and Box-plot of measured surface variability - exterior surface

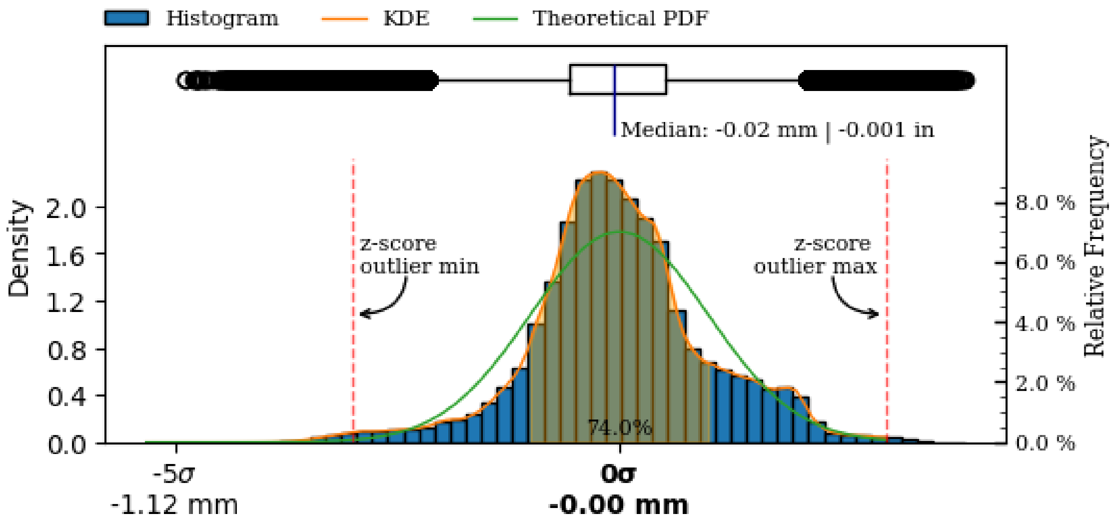


Figure 47: Exterior surface variability boxplot, kds and histogram.

Histogram, KDE and Box-plot of measured surface variability - interior surface

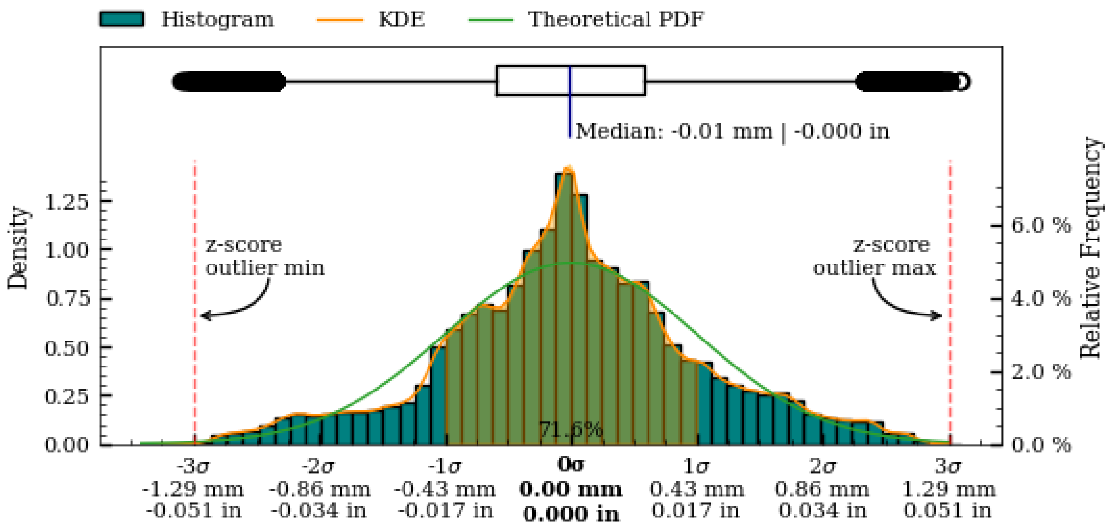


Figure 48: Interior surface variability boxplot, kds and histogram.

Precision Score Of The Artifact

To enable valid comparison of the manufacturing precision of different artifacts, a metric that robustly quantifies the overall precision of the object is required. The considerations for such a metric will be explored in this section.

Based on these considerations, a *Precision Score* metric will be defined.

For an object to be described as having been manufactured with high precision, several qualities must be present *concurrently*, and throughout the *entire* geometry of the final object. A given object may exhibit high levels of one or more *components* of precision, but be lacking in others. For example:

- An object may present high levels of coaxialility, but lack circularity.
- An object may exhibit good circularity, but show imperfections in the surface structure.
- An object may be smoothed to perfection *without* any circularity or coaxiality.
- An object may exhibit high levels of all of the above metrics in *some* areas, but not in others.

Therefore, a precision score metric **must** account for *all* aspects of the individual, underlying precision metrics (circularity, concentricity, coaxiality and surface variability) throughout the *entire* surface area of the object.

The composite high order polynomial model, used to generate the surface variability map (described in Surface Variability, p. 41) is the best continuous mathematical representation of the object available to us (lacking any original design plans, as would normally be available in metrological analysis). This idealized model encompasses all of the above component metrics.

In the creation of the model, all scan data-points are taken into account (excluding areas with extensive damage), making it the best possible idealized representation we can achieve. When this model has been accurately created, the deviation between the model and the scanned data-points can be calculated over the non-discretized polynomials, *without* the need for an "original" CAD model (and importantly, unless such a CAD model *actually* corresponded to the original design intent, it would be an insufficient comparison basis).

Within the context of defining a valid, overall precision metric, this approach satisfies the incorporation of all of the necessary metrics:

- **Circularity**: Because the reconstructed polynomial model is revolved around the Z-plane, the idealized representation is perfectly circular, and thus incorporates the circularity component.
- **Concentricity and coaxiality**: Because the Z-axis (datum axis) is the center axis of the model, it incorporates the concentricity and coaxiality components.
- **Surface variability**: Because the model is continuous and non-discretized, it can be used accurately for all points of the scan data, and incorporates the surface variability component.

The level of precision ultimately achieved in a physical object does not share a linear relationship with its manufacturing requirements. Since continuously higher levels of final precision becomes progressively harder to achieve, an overall precision metric must take this relationship into account.

A robust statistical metric that satisfies this requirement is the *Mean Squared Deviation* (MSD or MSE). Here specifically, we can utilize the mean square of the deviations between the model ($\hat{y}$) and the data-points (y_i).

Combining all of the above considerations, we can express a well-defined *Precision Score* metric, that provides an immediately accessible way to understand the overall precision of an object, while being statistically valid. Since the Mean Squared Deviation tends towards zero as the overall precision increases, the inverse of the Mean Squared Deviation is taken to obtain a precision score metric that increases as precision increases[12]:

$$\text{Precision Score} = \frac{n}{\sum_{i=1}^{n} \left(y_i - \hat{y}\right)^2}$$

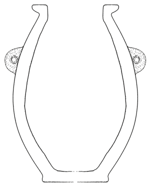
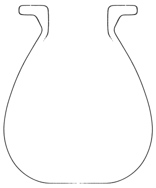
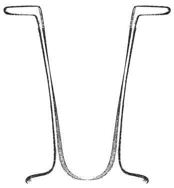
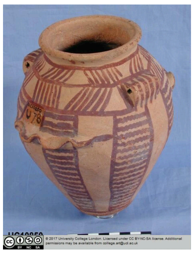
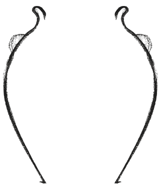
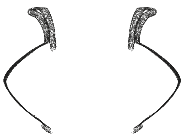
[12] The precision score unit is $\frac{1}{\text{mm}^2}$

A precision score will be calculated seperately for:

- The exterior surface
- The interior surface
- The full surface

As most scans do not include sufficient scan data for the interior surface, the exterior surface will be used for calculating the precision score in most cases. In the rare case that the scan data is more complete for the *interior* surface, this will be used instead.

Table 6 shows the precision score of this artifact (MV013), compared to the two most precise, and the two least precise vessels currently analyzed.

Artifact			Material	Precision Score	Link to Report
		PV001	Red Granite	**1905** Full: 980 Exterior: 1905 Interior: 705	Report Publication
		PV006	Dark grey granite	**621** Full: 521 Exterior: 621 Interior: 152	Report Publication
		MV013	Diorite	**20** Full: 9 Exterior: 20 Interior: 5	Report Publication
		MV001	Pottery	**1.93** Full: 1.92 Exterior: 1.93 Interior: 1.85	Report Publication
		MV010	Calcite (Egyptian Alabaster)	**1.12** Full: 0.64 Exterior: 1.12 Interior: 0.20	Report Publication

Analysis Roadmap

While the current iteration of this work already provides valuable results, continued future additions and improvements will enhance their utility further. This section details planned iterative updates and improvements, to both the reports themselves, and to the underlying methodology and software they are created with.

Alignment Section

- Detailed exploration of different circle regression algorithms
- If handles are present on the vessel, exploring alignment of the vessels so the handle positions match each other
- Add optimization of the perpendicular surface deviation, with the best results of the coaxial alignment
- Align by minimizing circularity results (of rotated sample slice, to compensate for sample height distortions)

Measurements of Precision

- Section detailing how measurements perpendicular to the surface curvature are obtained
- Detailed surface area analysis, exploring the residual patterns throughout subsequent sample slices of the artifact surface
- Wall thickness deviation color map
- Robust outlier identification on circularity, to better handle analysis of damaged areas of the artifacts in addition to removal of interior crystalline structure points present in CT scans
- Layout updates to the charts and tables

Visibility of Outliers and Damaged Sections

- Identification and marking of damaged parts
- Visualization of outliers on the artifact surface

Exploration of Mathematical Primitives

- Analysis of selected curvatures and flat surfaces on the vessel in both the horizontal and vertical planes
  - Circles
  - Parabolas
  - Ellipsoids
  - Hyperbolas
  - Cones
- Implementation of robust regressions models suitable for this domain, based on RANSAC.

Metrics on Primary Features

- Measurements of features in the horizontal plane
- Measurements of features in the vertical plane
- Measurements of angles
- Measurements of volume

Exploration of Potential Design Ratios

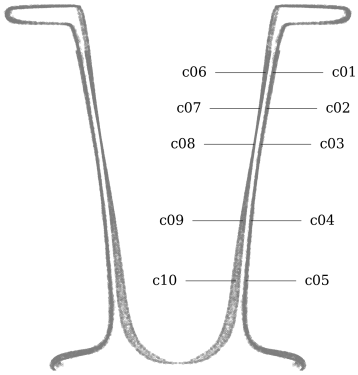
- π, φ, e, 1, 2, 3, 4 etc.

Raw Dataset Attachments

- Including all measurement and sample coordinates as CSV-files embedded in the report
- Including an STL file of the aligned object alongside the report, for easier external replication and validation of the research results

Appendix A - Comparison Of Circularity Measurements (Z-plane vs. surface-perpendicular)

Comparison of circularity samples



Samples perpendicular to the surface curvature

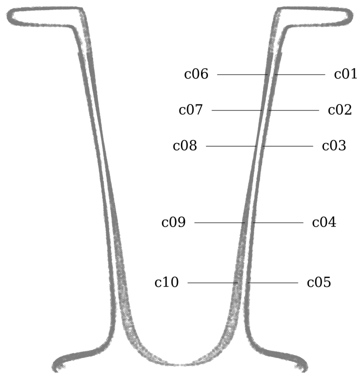
Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø48.636±0.715	1.209	0.214	0.127	0.214	1458	0.050	76.460	24.318
c02	exterior	Ø45.516±0.525	1.015	0.176	0.122	0.176	1441	0.050	67.317	22.758
c03	exterior	Ø42.470±0.455	0.844	0.126	0.071	0.126	1287	0.050	58.174	21.235
c04	exterior	Ø37.476±0.398	0.756	0.134	0.089	0.134	1173	0.050	38.608	18.738
c05	exterior	Ø34.809±0.463	0.874	0.200	0.122	0.200	1112	0.050	23.244	17.405
c06	interior	Ø44.829±0.482	0.875	0.289	0.278	0.287	1306	0.050	76.460	22.415
c07	interior	Ø42.014±0.256	0.464	0.135	0.136	0.133	1260	0.050	67.317	21.007
c08	interior	Ø39.093±0.121	0.210	0.045	0.037	0.045	1114	0.050	58.174	19.546
c09	interior	Ø33.191±0.597	1.153	0.390	0.397	0.390	983	0.050	38.608	16.596
c10	interior	Ø29.691±1.060	2.047	0.674	0.674	0.674	818	0.050	23.244	14.846

Table 7: Detailed circularity measurements at selected samples in z-plane, vessel MV013.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø48.492±0.748	1.316	0.230	0.082	0.219	1540	0.050	76.460	24.246
c02	exterior	Ø45.451±0.498	0.992	0.179	0.106	0.177	1544	0.050	67.317	22.725
c03	exterior	Ø42.444±0.514	0.967	0.136	0.073	0.136	1361	0.050	58.174	21.222
c04	exterior	Ø37.448±0.410	0.720	0.137	0.088	0.136	1192	0.050	38.608	18.724
c05	exterior	Ø34.787±0.480	0.910	0.205	0.128	0.205	1178	0.050	23.244	17.394
c06	interior	Ø44.864±0.531	0.895	0.298	0.283	0.294	1362	0.050	76.460	22.432
c07	interior	Ø42.046±0.254	0.475	0.139	0.140	0.138	1330	0.050	67.317	21.023
c08	interior	Ø39.104±0.130	0.214	0.046	0.036	0.045	1145	0.050	58.174	19.552
c09	interior	Ø33.159±0.640	1.195	0.397	0.396	0.397	995	0.050	38.608	16.579
c10	interior	Ø29.787±1.074	2.034	0.679	0.670	0.678	859	0.050	23.244	14.894

Table 8: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel MV013.



Samples perpendicular to the surface curvature

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø1.9148±0.0281	0.0476	0.0084	0.0050	0.0084	1458	0.0020	3.0102	0.9574
c02	exterior	Ø1.7920±0.0207	0.0400	0.0069	0.0048	0.0069	1441	0.0020	2.6503	0.8960
c03	exterior	Ø1.6721±0.0179	0.0332	0.0049	0.0028	0.0050	1287	0.0020	2.2903	0.8360
c04	exterior	Ø1.4754±0.0157	0.0298	0.0053	0.0035	0.0053	1173	0.0020	1.5200	0.7377
c05	exterior	Ø1.3704±0.0182	0.0344	0.0079	0.0048	0.0079	1112	0.0020	0.9151	0.6852
c06	interior	Ø1.7649±0.0190	0.0345	0.0114	0.0109	0.0113	1306	0.0020	3.0102	0.8825
c07	interior	Ø1.6541±0.0101	0.0183	0.0053	0.0054	0.0052	1260	0.0020	2.6503	0.8271
c08	interior	Ø1.5391±0.0048	0.0083	0.0018	0.0015	0.0018	1114	0.0020	2.2903	0.7695
c09	interior	Ø1.3067±0.0235	0.0454	0.0153	0.0156	0.0154	983	0.0020	1.5200	0.6534
c10	interior	Ø1.1689±0.0417	0.0806	0.0265	0.0265	0.0266	818	0.0020	0.9151	0.5845

Table 9: Detailed circularity measurements at selected samples in z-plane, vessel MV013.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø1.9091±0.0295	0.0518	0.0090	0.0032	0.0086	1540	0.0020	3.0102	0.9546
c02	exterior	Ø1.7894±0.0196	0.0391	0.0071	0.0042	0.0070	1544	0.0020	2.6503	0.8947
c03	exterior	Ø1.6710±0.0202	0.0381	0.0053	0.0029	0.0053	1361	0.0020	2.2903	0.8355
c04	exterior	Ø1.4743±0.0161	0.0283	0.0054	0.0035	0.0054	1192	0.0020	1.5200	0.7372
c05	exterior	Ø1.3696±0.0189	0.0358	0.0081	0.0050	0.0081	1178	0.0020	0.9151	0.6848
c06	interior	Ø1.7663±0.0209	0.0352	0.0117	0.0111	0.0116	1362	0.0020	3.0102	0.8832
c07	interior	Ø1.6553±0.0100	0.0187	0.0055	0.0055	0.0055	1330	0.0020	2.6503	0.8277
c08	interior	Ø1.5395±0.0051	0.0084	0.0018	0.0014	0.0018	1145	0.0020	2.2903	0.7698
c09	interior	Ø1.3055±0.0252	0.0470	0.0156	0.0156	0.0156	995	0.0020	1.5200	0.6527
c10	interior	Ø1.1727±0.0423	0.0801	0.0267	0.0264	0.0267	859	0.0020	0.9151	0.5864

Table 10: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel MV013.

Comparison of circularity on the full vessel surface

Metric

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.871	0.444	1.865	0.174	0.098	0.335	0.026	0.049	0.327	1579	0.050
Interior	1.081	0.128	2.589	0.346	0.021	0.852	0.156	0.014	0.872	1769	0.050

Table 11: Detailed circularity measurements at selected samples in z-plane, vessel MV013.

Samples in the z-plane

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.881	0.450	6.981	0.176	0.098	1.959	0.025	0.047	1.791	1577	0.050
Interior	1.110	0.129	4.637	0.357	0.022	1.269	0.182	0.014	1.292	1805	0.050

Table 12: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel MV013.

Imperial

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.871	0.444	1.865	0.174	0.098	0.335	0.026	0.049	0.327	1579	0.050
Interior	1.081	0.128	2.589	0.346	0.021	0.852	0.156	0.014	0.872	1769	0.050

Table 13: Detailed circularity measurements at selected samples in z-plane, vessel MV013.

Samples in the z-plane

Area	Range			Standard Deviation			Medan Absolute Deviation			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.881	0.450	6.981	0.176	0.098	1.959	0.025	0.047	1.791	1577	0.050
Interior	1.110	0.129	4.637	0.357	0.022	1.269	0.182	0.014	1.292	1805	0.050

Table 14: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel MV013.

Circularity analysis of exterior samples perpendicular to surface curvature

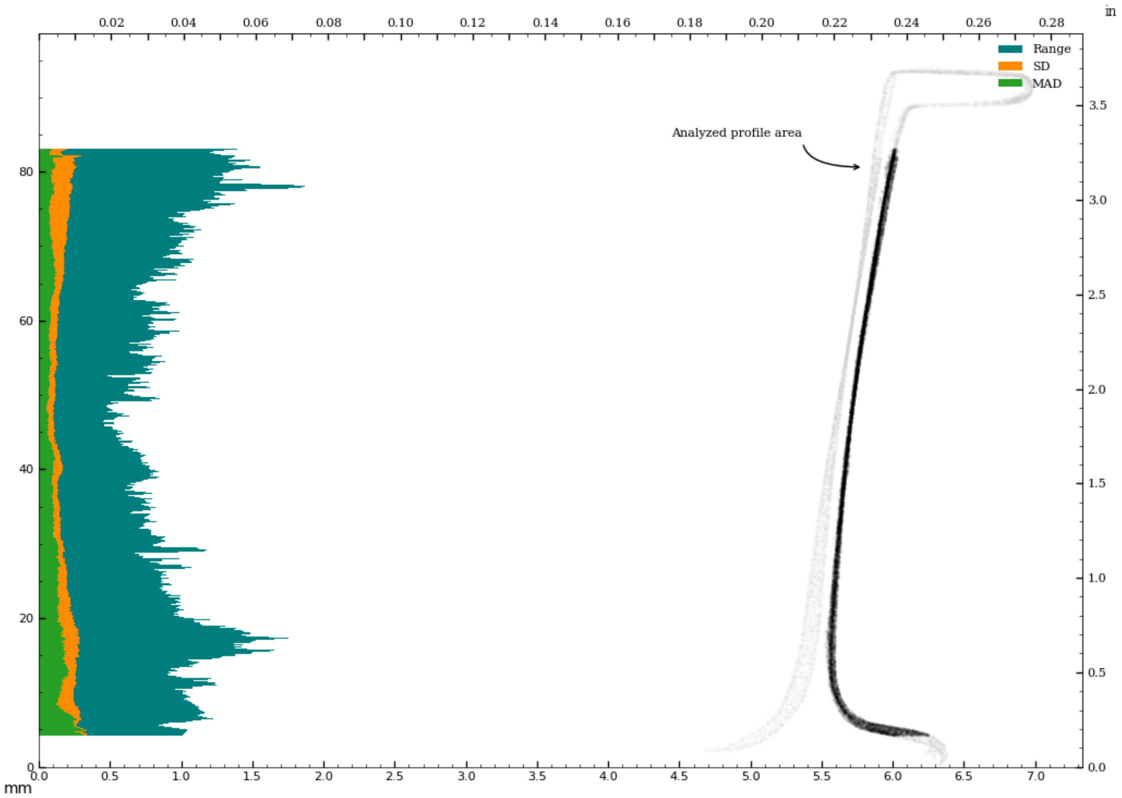


Figure 49: Circularity analysis of exterior samples perpendicular to surface curvature

Circularity analysis of exterior surface - in z-plane

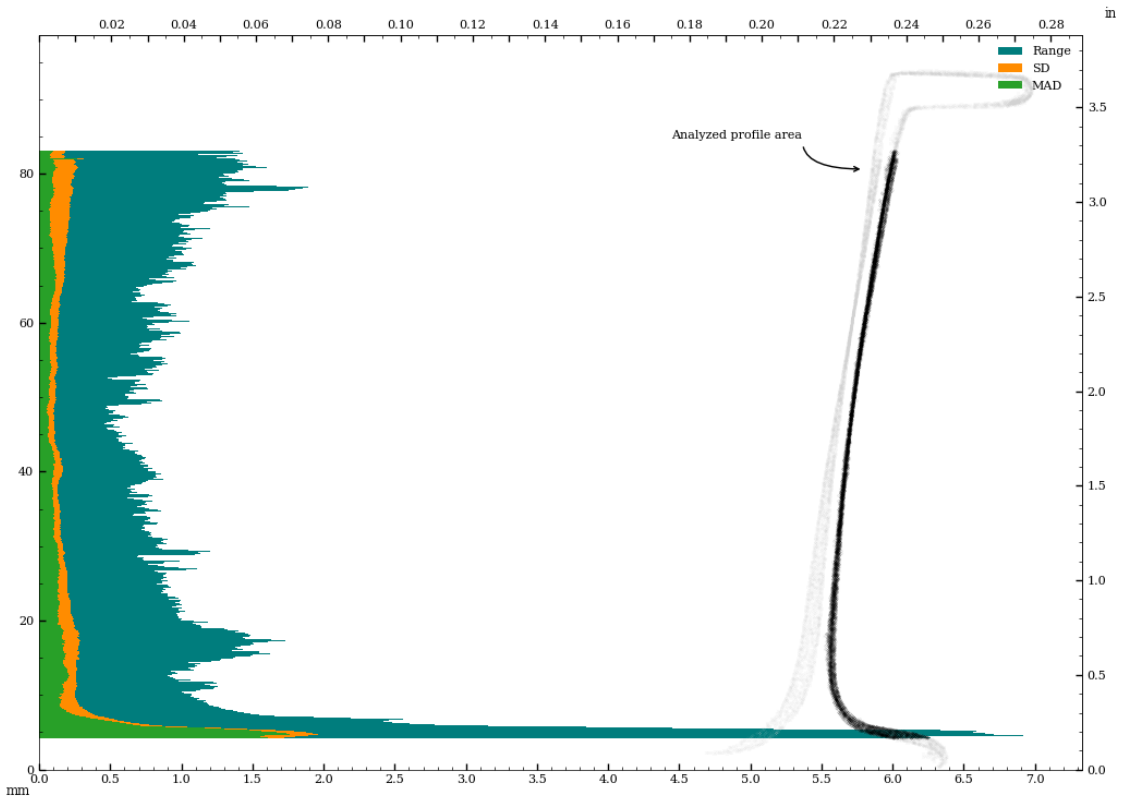


Figure 50: Circularity analysis of exterior surface - in z-plane

Circularity analysis of exterior samples perpendicular to surface curvature

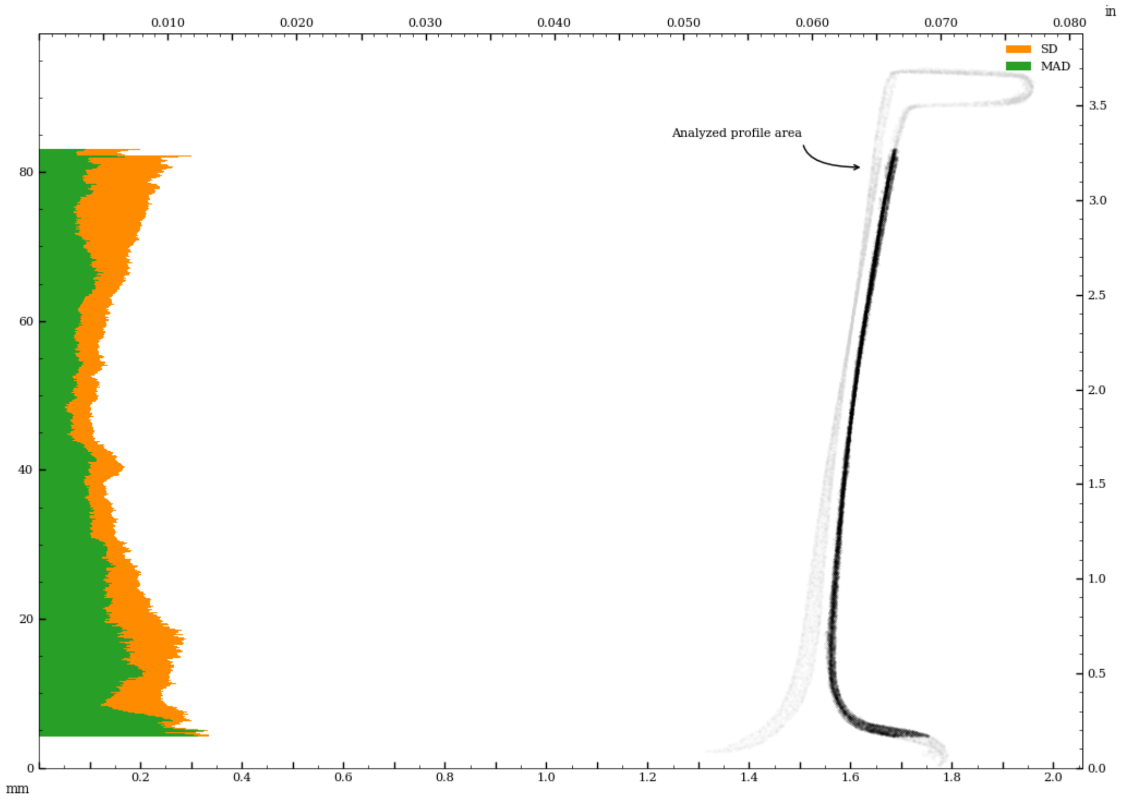


Figure 51: Circularity analysis of exterior samples perpendicular to surface curvature

Circularity analysis of exterior surface - in z-plane

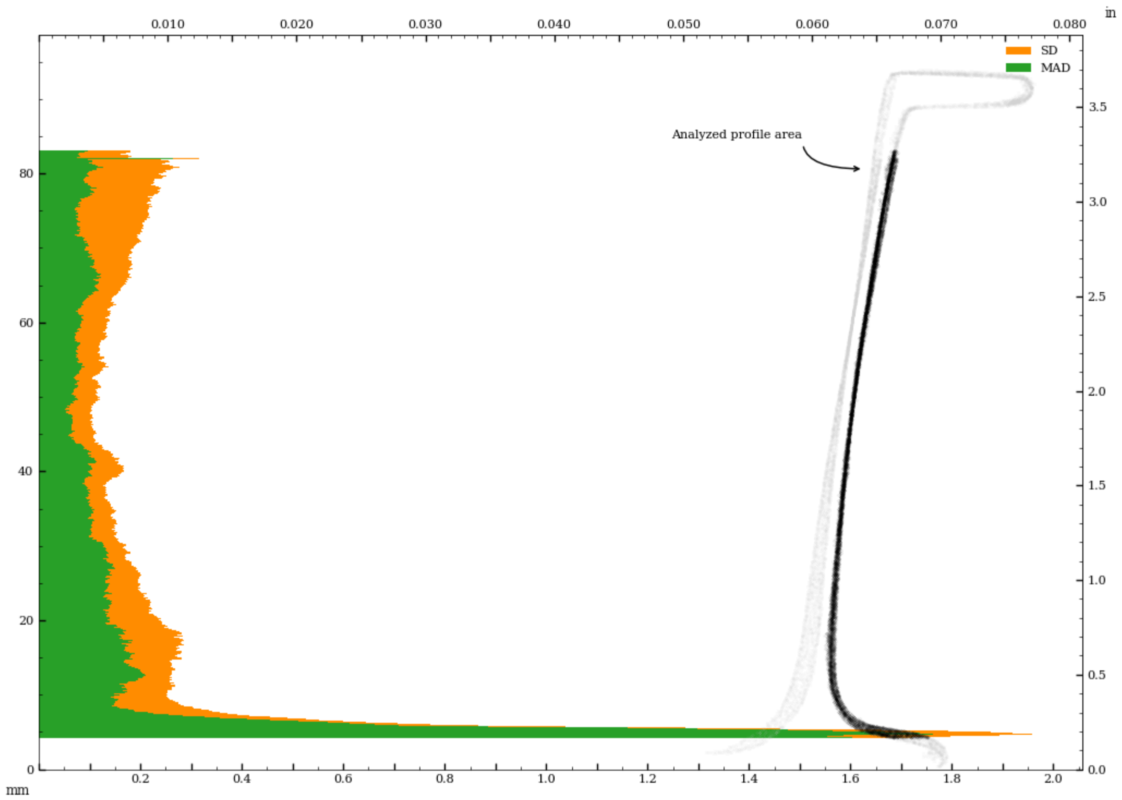


Figure 52: Circularity analysis of exterior surface - in z-plane

Circularity analysis of interior samples perpendicular to surface curvature

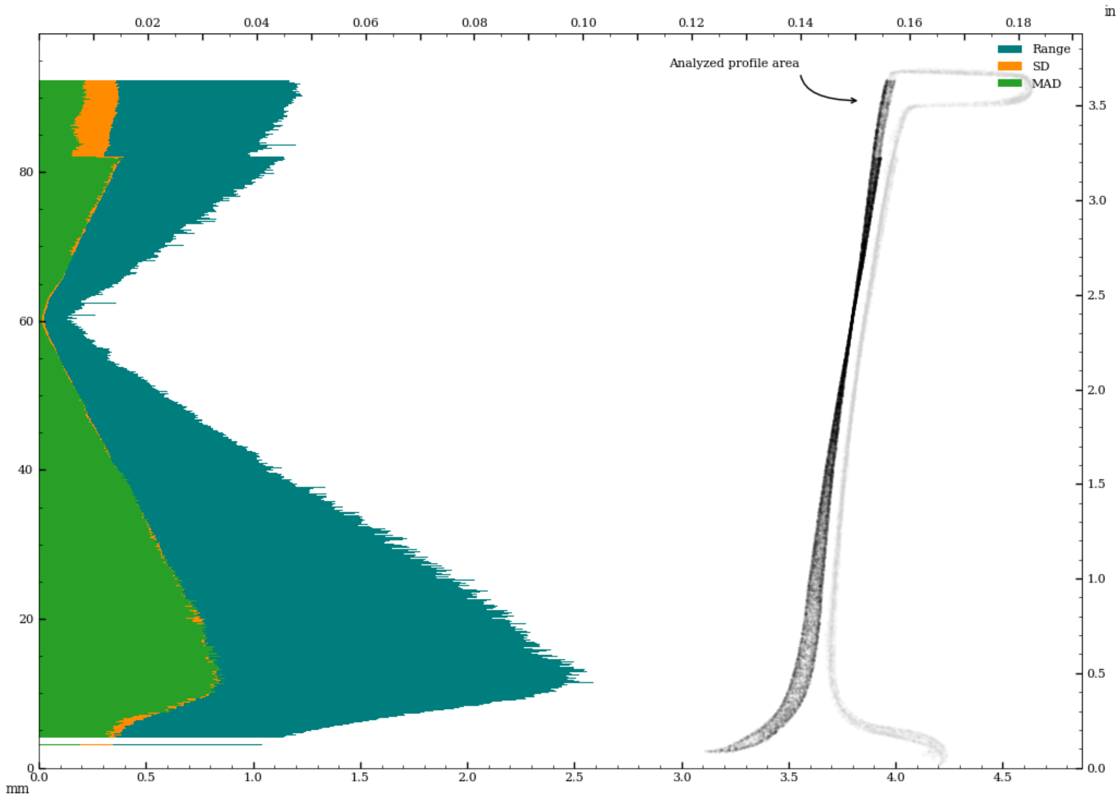


Figure 53: Circularity analysis of interior samples perpendicular to surface curvature

Circularity analysis of interior surface - in z-plane

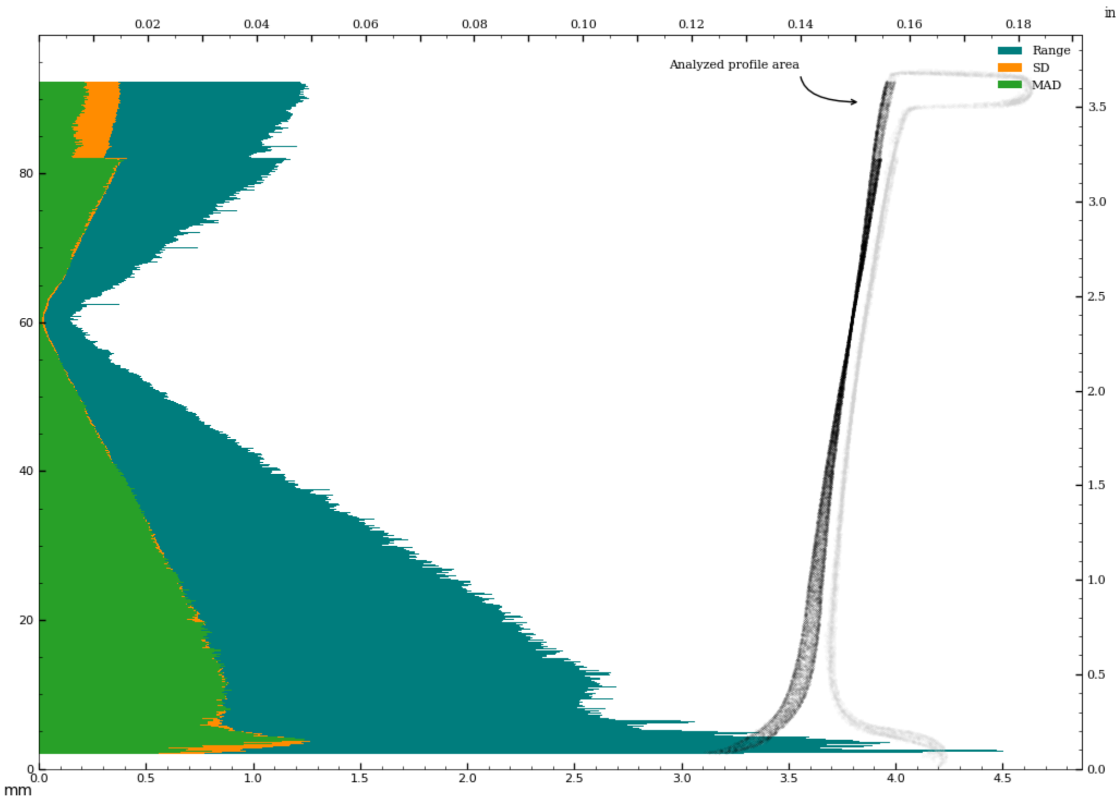


Figure 54: Circularity analysis of interior surface - in z-plane

Circularity analysis of interior samples perpendicular to surface curvature

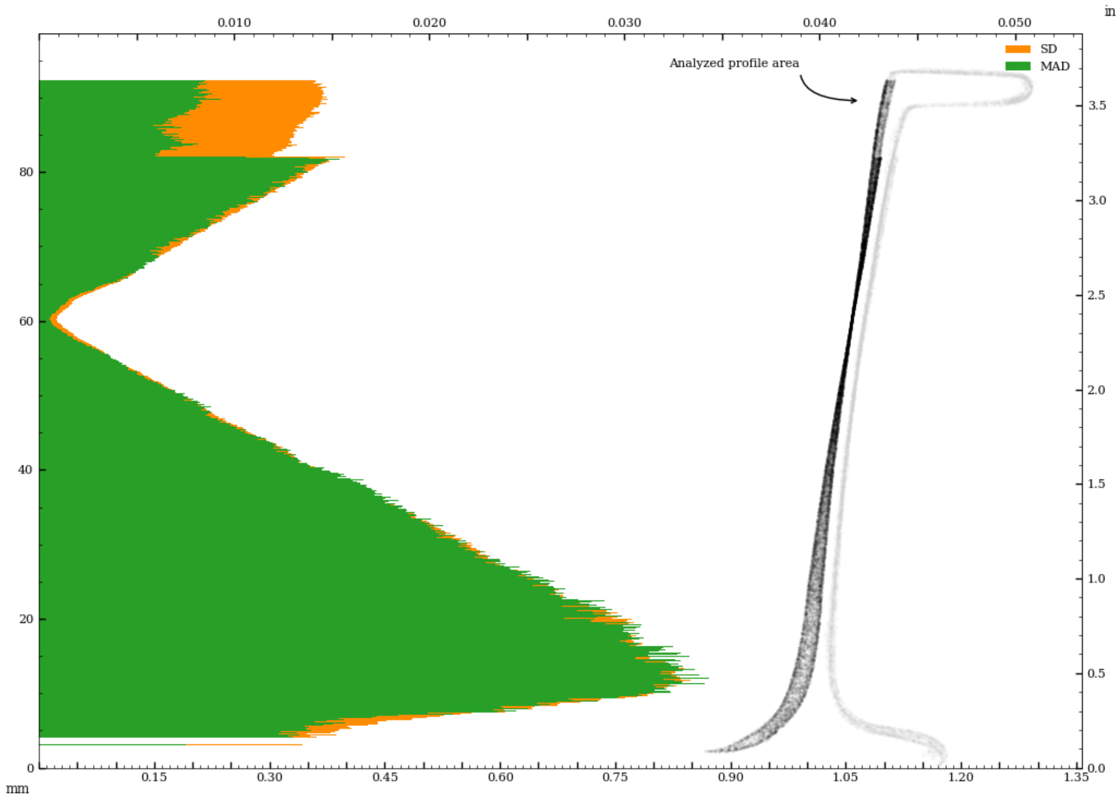


Figure 55: Circularity analysis of interior samples perpendicular to surface curvature

Circularity analysis of interior surface - in z-plane

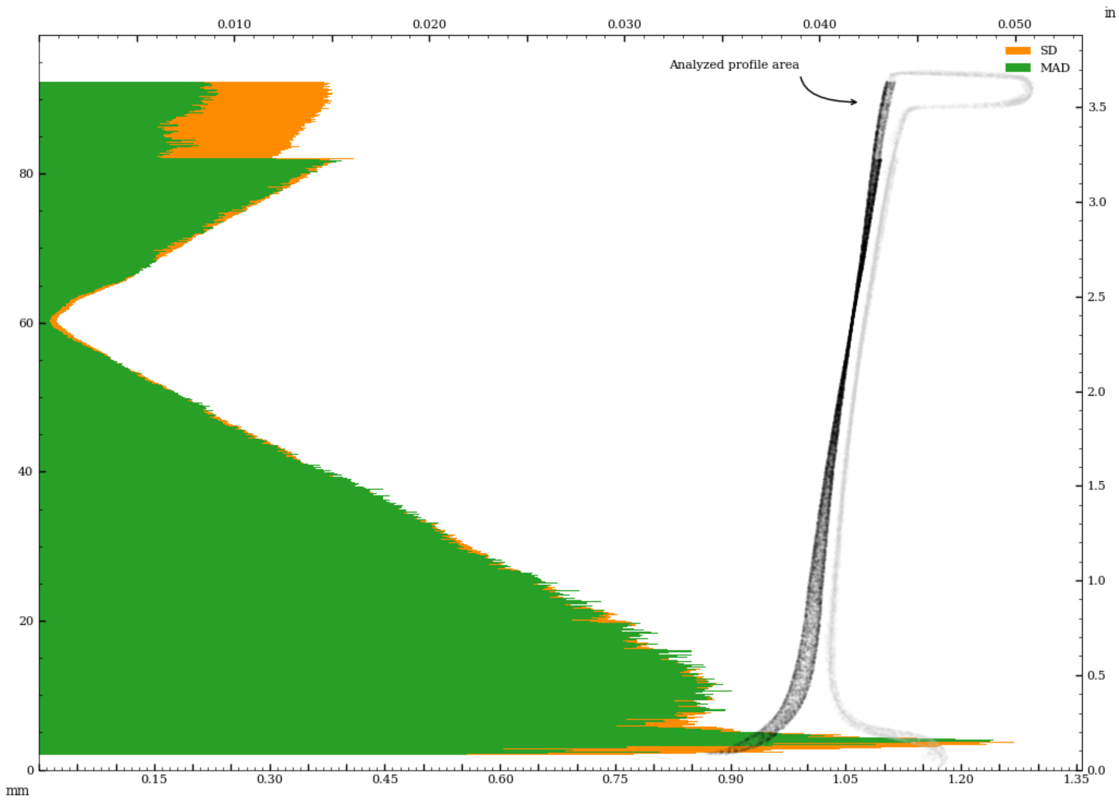


Figure 56: Circularity analysis of interior surface - in z-plane

Appendix B - Comparison Of Concentricity Measurements (Z-plane vs. surface-perpendicular)

Metric

Concentricity measurements perpendicular to surface curvature

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.218	1468	1.193	0.987	0.213	0.200	0.079	0.078	−104, −192
c02	z-axis	0.181	1438	1.015	0.879	0.176	0.172	0.107	0.104	−129, −127
c03	z-axis	0.109	1288	0.844	0.706	0.126	0.124	0.069	0.069	−95, −53
c04	z-axis	0.042	1172	0.756	0.646	0.134	0.131	0.088	0.087	−30, −29
c05	z-axis	0.097	1112	0.874	0.874	0.200	0.200	0.123	0.123	−6, −96
c06	z-axis	0.395	1306	0.875	0.875	0.287	0.288	0.276	0.283	41, 393
c07	z-axis	0.181	1260	0.464	0.434	0.133	0.133	0.136	0.138	6, 180
c08	z-axis	0.052	1114	0.210	0.190	0.045	0.043	0.035	0.034	−8, −51
c09	z-axis	0.543	983	1.153	1.153	0.390	0.390	0.398	0.395	−73, −538
c10	z-axis	0.934	818	2.047	1.951	0.674	0.675	0.672	0.671	−123, −926
c01	c06	0.603	1468	1.193	0.987	0.213	0.200	0.079	0.078	−145, −585
c02	c07	0.335	1438	1.015	0.879	0.176	0.172	0.107	0.104	−135, −307
c03	c08	0.087	1288	0.844	0.706	0.126	0.124	0.069	0.069	−87, −2
c04	c09	0.511	1172	0.756	0.646	0.134	0.131	0.088	0.087	44, 509
c05	c10	0.838	1112	0.874	0.874	0.200	0.200	0.123	0.123	116, 830

Concentricity measurements in z-plane

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.229	1540	1.821	1.550	0.387	0.377	0.261	0.263	−117, −198
c02	z-axis	0.183	1544	1.198	1.198	0.279	0.277	0.192	0.192	−134, −124
c03	z-axis	0.113	1361	1.056	0.973	0.171	0.166	0.090	0.089	−100, −53
c04	z-axis	0.038	1192	0.730	0.689	0.141	0.139	0.091	0.090	−32, −20
c05	z-axis	0.109	1178	1.148	1.148	0.261	0.261	0.159	0.159	−14, −108
c06	z-axis	0.403	1362	1.881	1.881	0.656	0.660	0.663	0.669	43, 401
c07	z-axis	0.187	1330	0.898	0.897	0.303	0.303	0.299	0.306	6, 187
c08	z-axis	0.052	1145	0.347	0.320	0.090	0.087	0.075	0.076	−8, −52
c09	z-axis	0.554	995	2.649	2.585	0.897	0.899	0.921	0.911	−75, −549
c10	z-axis	0.944	859	4.391	4.391	1.508	1.517	1.509	1.522	−124, −936
c01	c06	0.619	1540	1.821	1.550	0.387	0.377	0.261	0.263	−159, −599
c02	c07	0.341	1544	1.198	1.198	0.279	0.277	0.192	0.192	−140, −311
c03	c08	0.093	1361	1.056	0.973	0.171	0.166	0.090	0.089	−93, −1
c04	c09	0.531	1192	0.730	0.689	0.141	0.139	0.091	0.090	43, 529
c05	c10	0.835	1178	1.148	1.148	0.261	0.261	0.159	0.159	110, 828

Concentricity measurements perpendicular to surface curvature

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0086	1468	0.0470	0.0389	0.0084	0.0079	0.0031	0.0031	−4.1, −7.6
c02	z-axis	0.0071	1438	0.0400	0.0346	0.0069	0.0068	0.0042	0.0041	−5.1, −5.0
c03	z-axis	0.0043	1288	0.0332	0.0278	0.0049	0.0049	0.0027	0.0027	−3.7, −2.1
c04	z-axis	0.0016	1172	0.0298	0.0254	0.0053	0.0052	0.0035	0.0034	−1.2, −1.2
c05	z-axis	0.0038	1112	0.0344	0.0344	0.0079	0.0079	0.0048	0.0048	−0.2, −3.8
c06	z-axis	0.0156	1306	0.0345	0.0345	0.0113	0.0113	0.0109	0.0111	1.6, 15.5
c07	z-axis	0.0071	1260	0.0183	0.0171	0.0052	0.0052	0.0054	0.0054	0.2, 7.1
c08	z-axis	0.0020	1114	0.0083	0.0075	0.0018	0.0017	0.0014	0.0013	−0.3, −2.0
c09	z-axis	0.0214	983	0.0454	0.0454	0.0154	0.0153	0.0157	0.0155	−2.9, −21.2
c10	z-axis	0.0368	818	0.0806	0.0768	0.0266	0.0266	0.0265	0.0264	−4.8, −36.5
c01	c06	0.0237	1468	0.0470	0.0389	0.0084	0.0079	0.0031	0.0031	−5.7, −23.0
c02	c07	0.0132	1438	0.0400	0.0346	0.0069	0.0068	0.0042	0.0041	−5.3, −12.1
c03	c08	0.0034	1288	0.0332	0.0278	0.0049	0.0049	0.0027	0.0027	−3.4, −0.1
c04	c09	0.0201	1172	0.0298	0.0254	0.0053	0.0052	0.0035	0.0034	1.7, 20.0
c05	c10	0.0330	1112	0.0344	0.0344	0.0079	0.0079	0.0048	0.0048	4.6, 32.7

Concentricity measurements in z-plane

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	SD full	SD inliers	MAD full	MAD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0090	1540	0.0717	0.0610	0.0152	0.0148	0.0103	0.0104	−4.6, −7.8
c02	z-axis	0.0072	1544	0.0472	0.0472	0.0110	0.0109	0.0076	0.0076	−5.3, −4.9
c03	z-axis	0.0045	1361	0.0416	0.0383	0.0067	0.0065	0.0035	0.0035	−4.0, −2.1
c04	z-axis	0.0015	1192	0.0287	0.0271	0.0056	0.0055	0.0036	0.0036	−1.3, −0.8
c05	z-axis	0.0043	1178	0.0452	0.0452	0.0103	0.0103	0.0063	0.0063	−0.5, −4.3
c06	z-axis	0.0159	1362	0.0740	0.0740	0.0258	0.0260	0.0261	0.0264	1.7, 15.8
c07	z-axis	0.0074	1330	0.0354	0.0353	0.0119	0.0119	0.0118	0.0120	0.2, 7.4
c08	z-axis	0.0021	1145	0.0137	0.0126	0.0035	0.0034	0.0030	0.0030	−0.3, −2.0
c09	z-axis	0.0218	995	0.1043	0.1018	0.0353	0.0354	0.0362	0.0359	−3.0, −21.6
c10	z-axis	0.0372	859	0.1729	0.1729	0.0594	0.0597	0.0594	0.0599	−4.9, −36.9
c01	c06	0.0244	1540	0.0717	0.0610	0.0152	0.0148	0.0103	0.0104	−6.3, −23.6
c02	c07	0.0134	1544	0.0472	0.0472	0.0110	0.0109	0.0076	0.0076	−5.5, −12.3
c03	c08	0.0037	1361	0.0416	0.0383	0.0067	0.0065	0.0035	0.0035	−3.7, −0.0
c04	c09	0.0209	1192	0.0287	0.0271	0.0056	0.0055	0.0036	0.0036	1.7, 20.8
c05	c10	0.0329	1178	0.0452	0.0452	0.0103	0.0103	0.0063	0.0063	4.3, 32.6