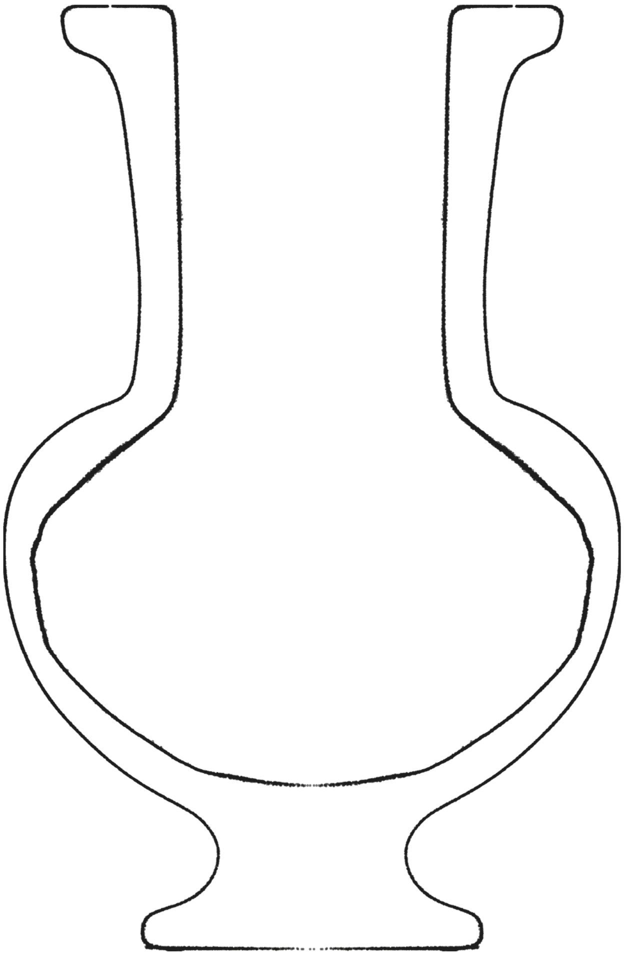
PV003 - Tall Bag Shaped Jar

An Exploration of Precision



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Date: 2025-09-17
Version: 01.20


Image courtesy Max Fomitchec-Zamilov

Contents

Artifact Information

Artifact Data

Collection	
Provenance[1]	
Provenience[2]	
Attribution	

Art dealer information

Ref.	
Description	
URL	

Maijers vessel classification[3]

Short classification	Tall Bag Shaped Jar
Long classification	The vessel is created in a closed form classified as a tall jar with a bag shaped shape, it has a footed base and a raised blunt rim.

Physical properties

Precision score[4]	1092	
Height (approximate)	110 mm	4.33 in
Width (approximate)	74 mm	2.91 in
Material		
Mohs Hardness[5]	Unknown (Unknown)	
Weight		

Scan information

Source	Artifact Foundation
Source file name	vase3.stl
Scan method	CT
Scanner	Zeiss
Rated scan accuracy	Not specified
Scan date	
Scanned by	Zeiss

Mesh decimation	Unknown
Number of vertices	2 498 975
Mesh density[6]	86 µm | 3.39 thou
Max vertex distance	221 µm | 8.691 thou
Min vertex distance	2 µm | 0.096 thou
Vertices per cm2	6840 (approximated)
Vertices per in2	44 128 (approximated)

[1] The verifiable chain of custody of an artifact
[2] The location or site where an artifact was recovered
[3] Vessel artifact classification developed by W. Arnold Maijer and described in his publication Masters of Stone, ISBN 978-90-829212-0-5
[4] The precision score metric is described in Precision Score Of The Artifact, p. 68
[5] The Mohs scale is an ordinal scale, from 1 to 10, describing the materials resistance to abrasion (the ability of harder material to scratch softer material)
[6] Median distance between vertices

Alignment In The Cartesian Coordinate System

For precise and valid measurements of the vessel's geometry to be possible, the points of the scanned dataset must first and foremost be placed optimally in a Cartesian coordinate system. Several alignment methods and algorithms have been tested on a number of different vessels to determine the best way to achieve optimal alignment.

Any misalignment of the artifact will increase the error of the precision measurements, due to the distortion/wobble effect caused by the misaligned object. To visualize this distortion, we can consider a representation of the three-dimensional point cloud data, folded to a two-dimensional plane. This folded representation is obtained by rotating all scanned points around an assumed center axis to y = 0, x > 0, thus resulting in a two-dimensional profile representation of all scanned vertices in the object.

Figure 1 illustrates this effect on a ideal ellipsoid. In the first image, the ellipsoid is perfectly aligned, resulting in a narrow and precise two-dimensional folded profile. As misalignments are introduced, the two-dimensional profile increases in width, visually showing the distortion, causing the error in the precision measurements to increase. While easy to understand visually, this distortion can also be objectively quantified, and as such used to compare the fitness of different assumed center axes against each other, and further to create an automated and solid process for optimal Cartesian alignment of the scan data.

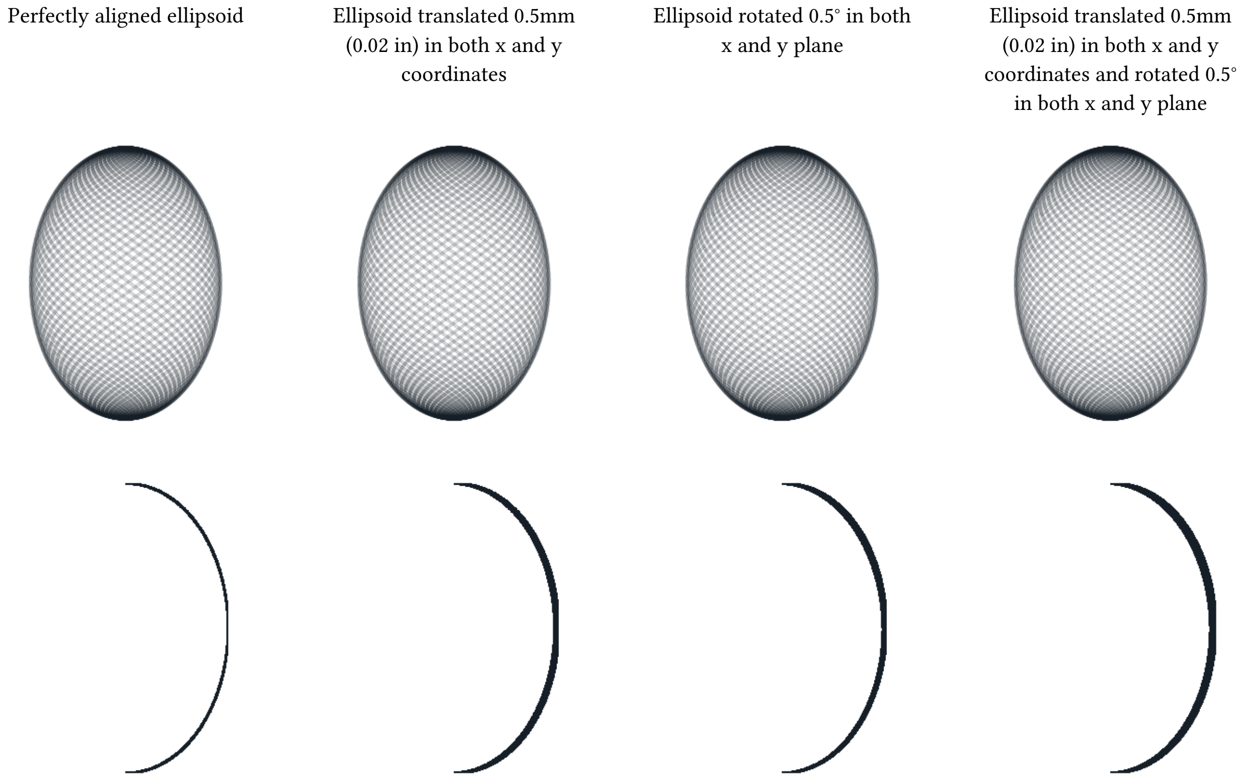


Figure 1: Distortion caused by a misalignment of the artifact

In contemporary metrology analysis of modern production objects, it is common to align the object in a Cartesian coordinate system by fitting a flat surface of the object to a reference plane in the coordinate system, cylindrical features to an ideal cylinder etc., or by using specific markers placed on the object in the design process. This methodology, however, is inadequate for the ancient objects in question. Most scanned artifacts, do not have a valid flat surface which could be aligned to a plane in the Cartesian coordinate system; most surfaces seem to be curved. Some artifacts do have a flat base, however this is often a worn area of the artifact and practical tests have shown that alignment to such surfaces will not produce optimal alignment of the scan data.

As conventional methods of alignment do not always yield good results with these types of artifacts, a more adequate method of alignment has been developed to enable precise measurements and statistical analysis of the scan data.

To find the optimal position of the vessel in the coordinate system, a range of rotation and translation tests are carried out to find the best fit of the central axis.

Based on the assumption that the analyzed object was created using a rotational process, and thus have symmetry around a central axis, the alignment of the artifact is carried out in a two-step process. An overview of this process is given below.

The artifact is placed in a Cartesian coordinate system, in an initially unaligned state. The first step in the alignment process estimates the central rotational axis of the vessel, by analyzing the coaxiality of thin cross-section slices of the vessel. The slices will be as thin as possible based on the mesh density of the scan, while still ensuring enough data points in each slice to be statistically valid.

For each slice, circular regression[7] (estimate of best fit circle) is used to estimate the center point of this slice. Combined over the total Z-axis range of the vessel, these center points provide us with an indicator of the incline and position of the vessel's central axis.

The next step will optimize the center axis alignment by progressively minimizing the deviation (perpendicular to the surface curvature) of the two-dimensional profile, see Figure 1. By ascertaining and comparing the resulting fit of many thousands of different potential rotations, the best fit alignment of the scan data can be estimated, and an optimal center axis (in relation to the data points) can be reconstructed. The actual three-dimensional point-cloud is then aligned to this axis, by rotating and translating the scanned data points to match the Z-axis of the Cartesian coordinate system.

To enable extensive analysis of the full surface of the artifact, the mesh is split into exterior and interior surfaces. The exterior surface is aligned independently of interior data points, providing a baseline for exterior quality assessment. The interior surface is represented by two alignments:

- Aligned with the exterior mesh to analyze concentricity, and
- Aligned separately to assess its precision and compare the true tilt/displacement between interior and exterior surfaces.

[7] Circle regression algorithm used: Kenichi Kanatani, Prasanna Rangarajan, "Hyper least squares fitting of circles and ellipses" Computational Statistics & Data Analysis, Vol. 55, pages 2197-2208, (2011)

Statistics used throughout the report

This section provides an overview of the key statistical and model-evaluation metrics employed throughout the report to analyze dataset variability, model fit, and predictive accuracy.

Each measure is introduced with its mathematical formulation, practical interpretation, and explicit reference to how it is calculated in the context of the evaluated models and residuals. Together, these metrics quantify:

- Data variability (e.g., MAD, Standard Deviation, Range).
- Model accuracy (e.g., MSD, RMSD).
- Robustness vs. sensitivity to extreme values and central tendencies.

Mean Squared Deviation (MSD), also known as Mean Squared Error (MSE).

$$\text{MSD} = \frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}$$

The Mean Squared Deviation (MSD) measures the average magnitude of squared differences between observed (y_i) and predicted ($\hat{y}$) values, calculated as the mean of squared residuals, and is a used as a measure of discrepancy in regression and model-fitting contexts.

This measure amplifies the influence of larger deviations through squaring, emphasizes imperfections in the observed data, but retains sensitivity to outliers.

This CT scan contains outliers in the form of scan points from the internal crystalline structures in the walls of the object, which will raise the MSE metric.

Root Mean Squared Deviation (RMSD), also known as Root Mean Squared Error (RMSE).

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}}$$

The Root Mean Square Deviation (RMSD) measures the magnitude of differences between observed (y_i) and predicted ($\hat{y}$) values by calculating the square root of the average of squared residuals.

RMSD is a commonly used measure of discrepancy in regression and model-fitting contexts. It quantifies the average magnitude of residuals while retaining sensitivity to larger deviations (via squaring), making it particularly useful for evaluating model accuracy.

Standard Deviation (SD)

$$s = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}}$$

The Standard Deviation measures the spread of data (y_i) around the mean ($\bar{y}$) by calculating the square root of the average of squared differences between each value and the mean.

It is sensitive to outliers as it amplifies their influence through squaring, in contrast to MAD.

Throughout this report, the Standard Deviation is calculated using the absolute residuals from regression models.

Median Absolute Deviation (MedianAD)

$$\text{MedianAD} = \text{median}(|y_i - \text{median}(y)|)$$

The Median Absolute Deviation (MAD) measures the spread of data around the median by calculating the median of absolute differences between each value and the median.

MAD is a robust measure of spread, analogous to the interquartile range (a robust measure centered on the middle 50% of data), and differs from the standard deviation in that it minimizes the impact of outliers.

Throughout this report, the MAD is calculated using the absolute values of residuals from regression models.

Range

$$\max(y_i) - \min(y_i)$$

The Range measures the spread of a dataset by calculating the difference between the maximum and minimum values.

The Range is a simple measure of spread, capturing the full extent of variability. Range is very sensitive to extreme values, as it is entirely determined by the two most extreme data points.

Throughout this report, the Range is calculated using the full range of residuals from regression models.

Precision

To explore the manufacturing precision of the artifact in depth, the following analysis have been carried out:

- Circularity around the axis of symmetry is examined in detail at selected cross-sections.
- Overall circularity around the axis of symmetry is measured for the full height of the vessel (areas of the vessel with extensive damage are not taken into account for this metric).
- Concentricity of the vessel between selected cross-sections are examined in detail to determine if the existence of an axis of rotation in the manufacture of the object can be established.
- The coaxiality of the vessel is analyzed to explore the precision of the central axis of the object.
- The surface variability is analyzed and visualized on through a heatmap.

Circularity

Circularity is the measurement of how round the surface of an object is, optionally in reference to a datum axis. The *circularity tolerance* is the radial distance of two circles, each with their centers in the datum axis, and each of them conforming, respectively, to the minimum and maximum deviations of the data-set to a true circle, see Figure 2.

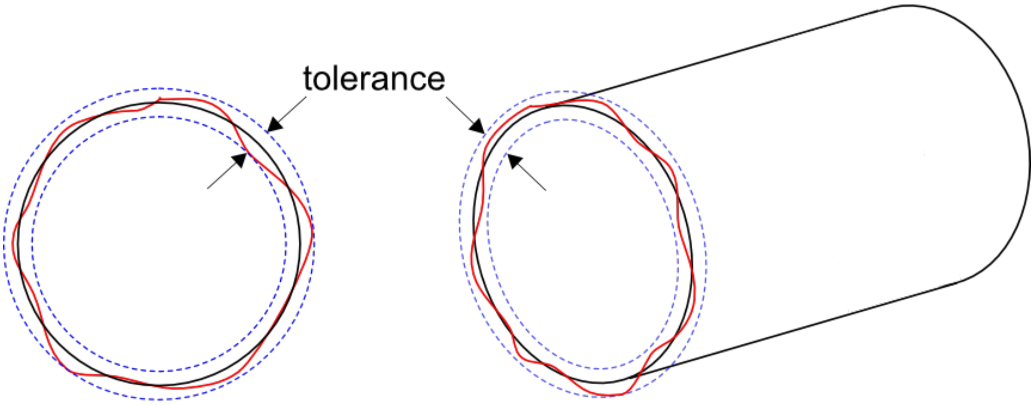


Figure 2: Circularity tolerance.

Circularity is examined at different cross-sections of the vessel, using the established Z-axis as the datum axis (axis of symmetry). The distance between the scanned points in the local datum plane is measured to determine the range between the two concentric circles encompassing the measured points, see Figure 3.

Referencing all of the individual circularity measurements to the global (reconstructed) axis of symmetry of the object, allows us to ascertain not only circularity of local features of the object, but how well circularity was *maintained* over the entire manufacturing process. This is an important distinction, which may be able to provide valuable insights into requirements of the construction methods. For reference, and seeing that the variance in local circularity also holds interest, measurements of circularity of the vessel without reference to the axis of symmetry can additionally be found in the Concentricity, p. 38.

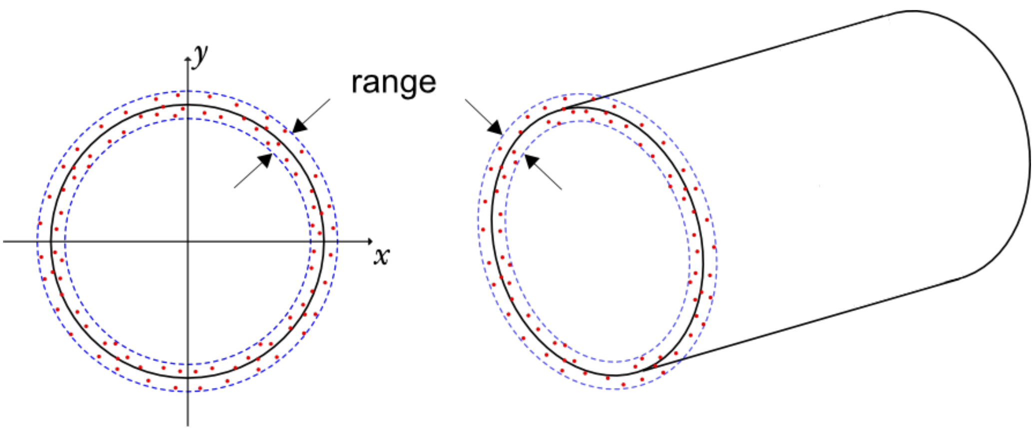


Figure 3: Circularity measurements.

If the circularity is determined from slices of the vessel exclusively in the *Z-plane* (actually measuring the cylindricity of a very thin slices of the vessel, in an attempt to approximate circularity), this would - in some areas - introduce significant distortion (increasing measurement errors) in the samples, due to the curvature of the vessel's surface.

Each sample slice of the vessel is therefore obtained perpendicular to the surface curvature, see Figure 6 to Figure 20. The measurements are taken conservatively without filtration of potential outliers.

To explore the potential distortion caused by obtaining samples in the Z-plane only, please refer to Appendix A, where measurements in the Z-plane and measurements perpendicular to surface curvature are compared side by side.

Detailed circularity measurements of selected points

Circularity measurements across a range of selected slices of the vessel (see Table 1) have been analyzed in-depth, and detailed plots of each measurement is provided. Furthermore, full circularity measurements are shown for each available scanned surface including a detailed plot to visualize the circularity of all areas of the vessel.

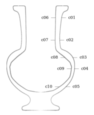


Figure 4: Circularity measurement sample locations, full mesh aligned with exterior surface

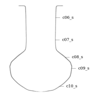


Figure 5: Circularity measurement sample location, separately aligned interior mesh

Metric

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø43.913±0.085	0.161	0.030	0.012	0.017	759	0.100	97.044	21.957
c02	exterior	Ø40.017±0.073	0.133	0.025	0.010	0.015	598	0.100	73.618	20.008
c03	exterior	Ø67.658±0.100	0.199	0.042	0.018	0.022	1042	0.100	55.408	33.829
c04	exterior	Ø71.077±0.073	0.131	0.024	0.010	0.015	1055	0.100	43.717	35.538
c05	exterior	Ø52.922±0.068	0.116	0.023	0.009	0.013	627	0.100	24.855	26.461
c06	interior	Ø31.236±0.150	0.274	0.064	0.026	0.036	759	0.100	97.044	15.618
c06_s	interior sep.	Ø31.236±0.134	0.251	0.056	0.024	0.030	749	0.100	97.044	15.618
c07	interior	Ø30.436±0.138	0.271	0.052	0.018	0.035	623	0.100	73.618	15.218
c07_s	interior sep.	Ø30.438±0.116	0.232	0.047	0.018	0.028	623	0.100	73.618	15.219
c08	interior	Ø51.056±0.352	0.697	0.122	0.049	0.079	1096	0.100	55.408	25.528
c08_s	interior sep.	Ø51.056±0.346	0.690	0.120	0.049	0.077	1081	0.100	55.408	25.528
c09	interior	Ø64.270±0.221	0.396	0.081	0.031	0.046	1389	0.100	43.717	32.135
c09_s	interior sep.	Ø64.271±0.228	0.416	0.082	0.033	0.047	1402	0.100	43.717	32.135
c10	interior	Ø39.230±0.122	0.206	0.037	0.014	0.022	610	0.100	24.855	19.615
c10_s	interior sep.	Ø39.231±0.118	0.194	0.037	0.015	0.023	609	0.100	24.855	19.616

Imperial

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø1.7289±0.0033	0.0064	0.0012	0.0005	0.0007	759	0.0039	3.8206	0.8644
c02	exterior	Ø1.5755±0.0029	0.0052	0.0010	0.0004	0.0006	598	0.0039	2.8983	0.7877
c03	exterior	Ø2.6637±0.0039	0.0078	0.0016	0.0007	0.0009	1042	0.0039	2.1814	1.3319
c04	exterior	Ø2.7983±0.0029	0.0052	0.0009	0.0004	0.0006	1055	0.0039	1.7211	1.3991
c05	exterior	Ø2.0835±0.0027	0.0046	0.0009	0.0004	0.0005	627	0.0039	0.9785	1.0418
c06	interior	Ø1.2297±0.0059	0.0108	0.0025	0.0010	0.0014	759	0.0039	3.8206	0.6149
c06_s	interior sep.	Ø1.2298±0.0053	0.0099	0.0022	0.0010	0.0012	749	0.0039	3.8206	0.6149
c07	interior	Ø1.1983±0.0054	0.0107	0.0020	0.0007	0.0014	623	0.0039	2.8983	0.5991
c07_s	interior sep.	Ø1.1984±0.0046	0.0091	0.0018	0.0007	0.0011	623	0.0039	2.8983	0.5992
c08	interior	Ø2.0101±0.0138	0.0274	0.0048	0.0019	0.0031	1096	0.0039	2.1814	1.0050
c08_s	interior sep.	Ø2.0101±0.0136	0.0272	0.0047	0.0019	0.0030	1081	0.0039	2.1814	1.0050
c09	interior	Ø2.5303±0.0087	0.0156	0.0032	0.0012	0.0018	1389	0.0039	1.7211	1.2652
c09_s	interior sep.	Ø2.5303±0.0090	0.0164	0.0032	0.0013	0.0018	1402	0.0039	1.7211	1.2652
c10	interior	Ø1.5445±0.0048	0.0081	0.0014	0.0006	0.0009	610	0.0039	0.9785	0.7723
c10_s	interior sep.	Ø1.5445±0.0047	0.0076	0.0015	0.0006	0.0009	609	0.0039	0.9785	0.7723

Table 1: Detailed circularity measurements at selected samples of PV003.

Figure 6 to Figure 20 shows a detailed plots of each circularity measurement.

[8] Sample diameter Ø± maximum measured deviation from measured radius
[9] Root mean square deviation (RMSD) also called Root mean square error (RMSE)
[10] Median absolute deviation
[11] Median sample radius from z-axis

Graphical overview of circularity measurement c01

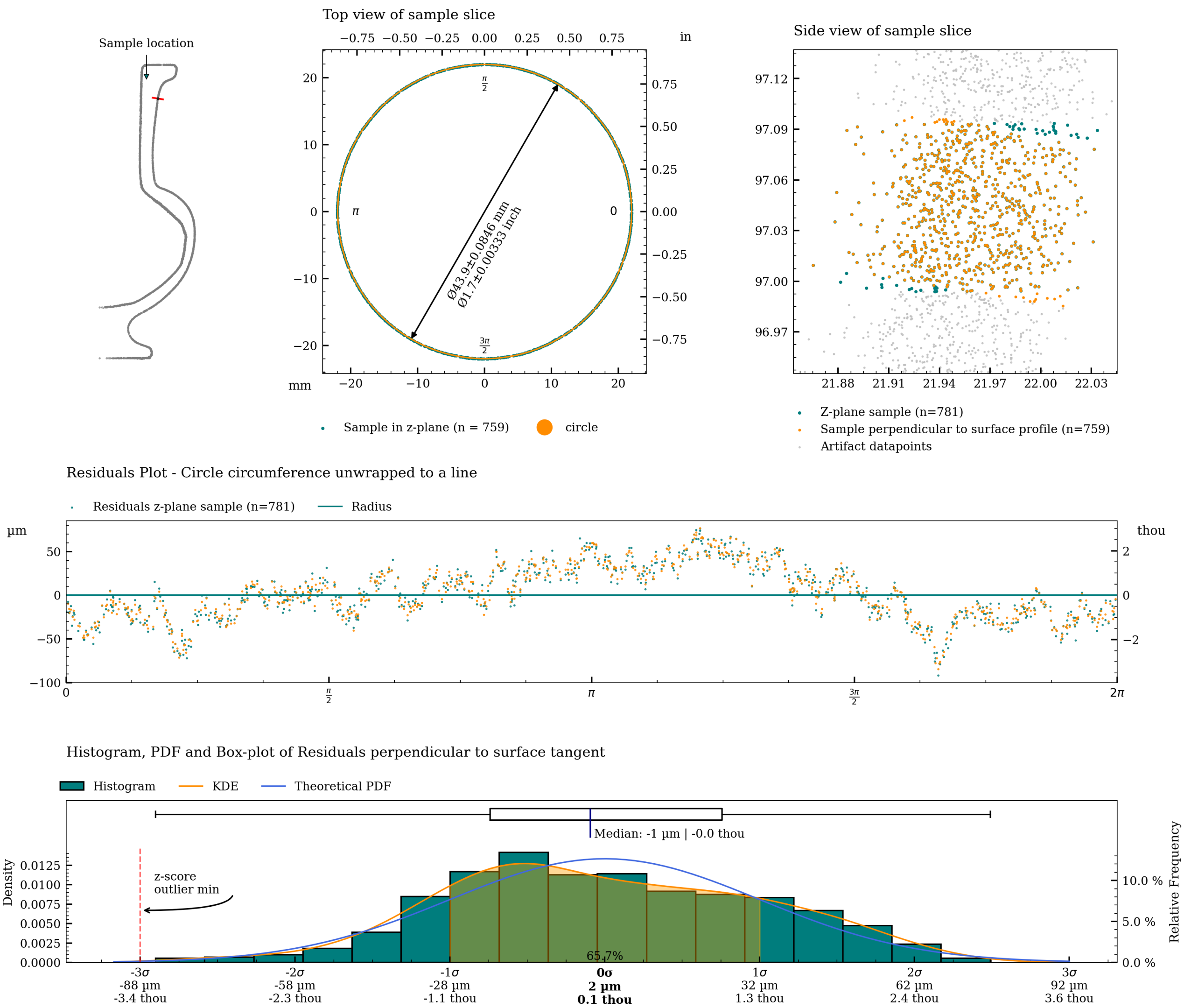


Figure 6: Charts with statistics for the measurement of c01.

Graphical overview of circularity measurement c02

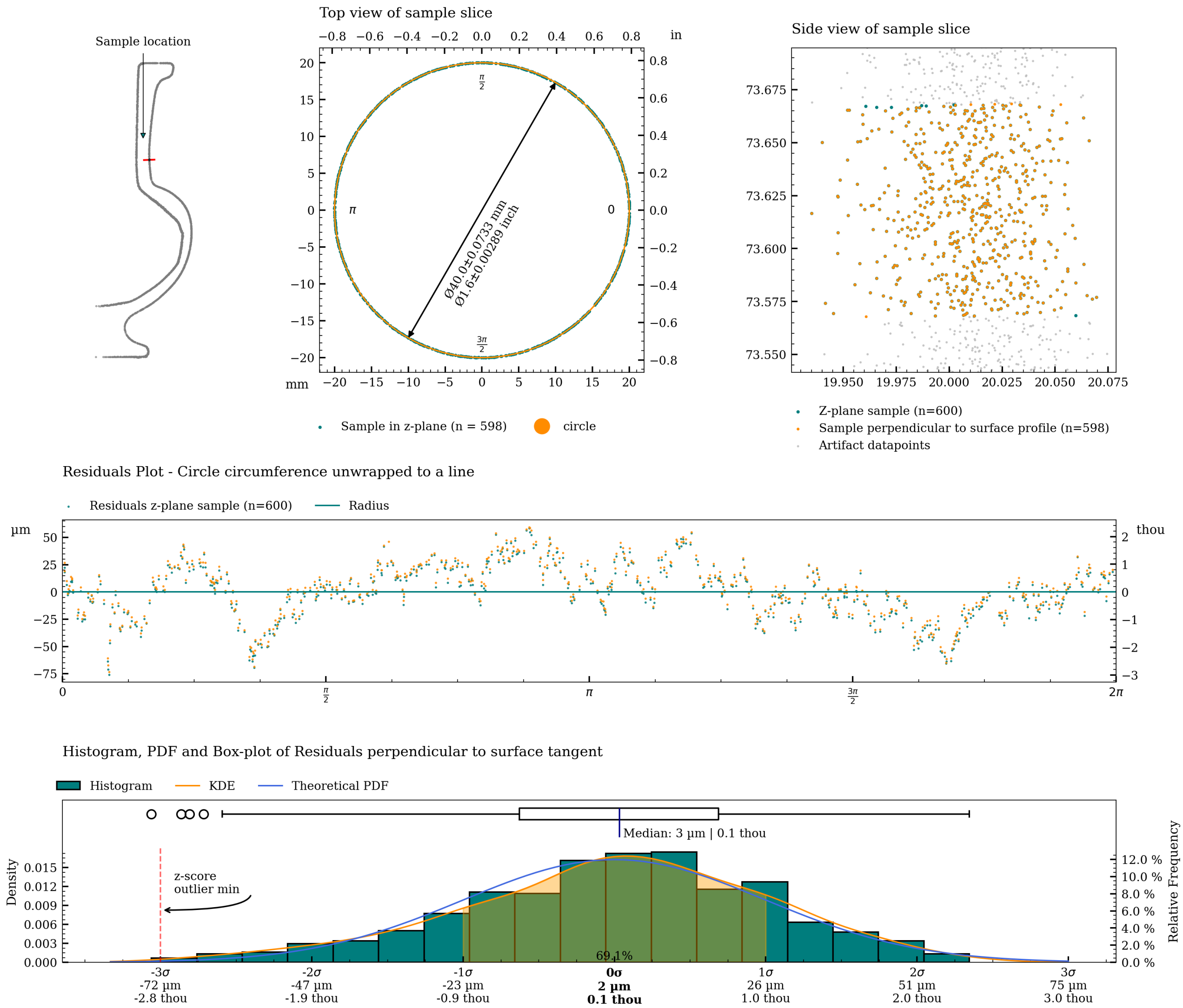


Figure 7: Charts with statistics for the measurement of c02.

Graphical overview of circularity measurement c03

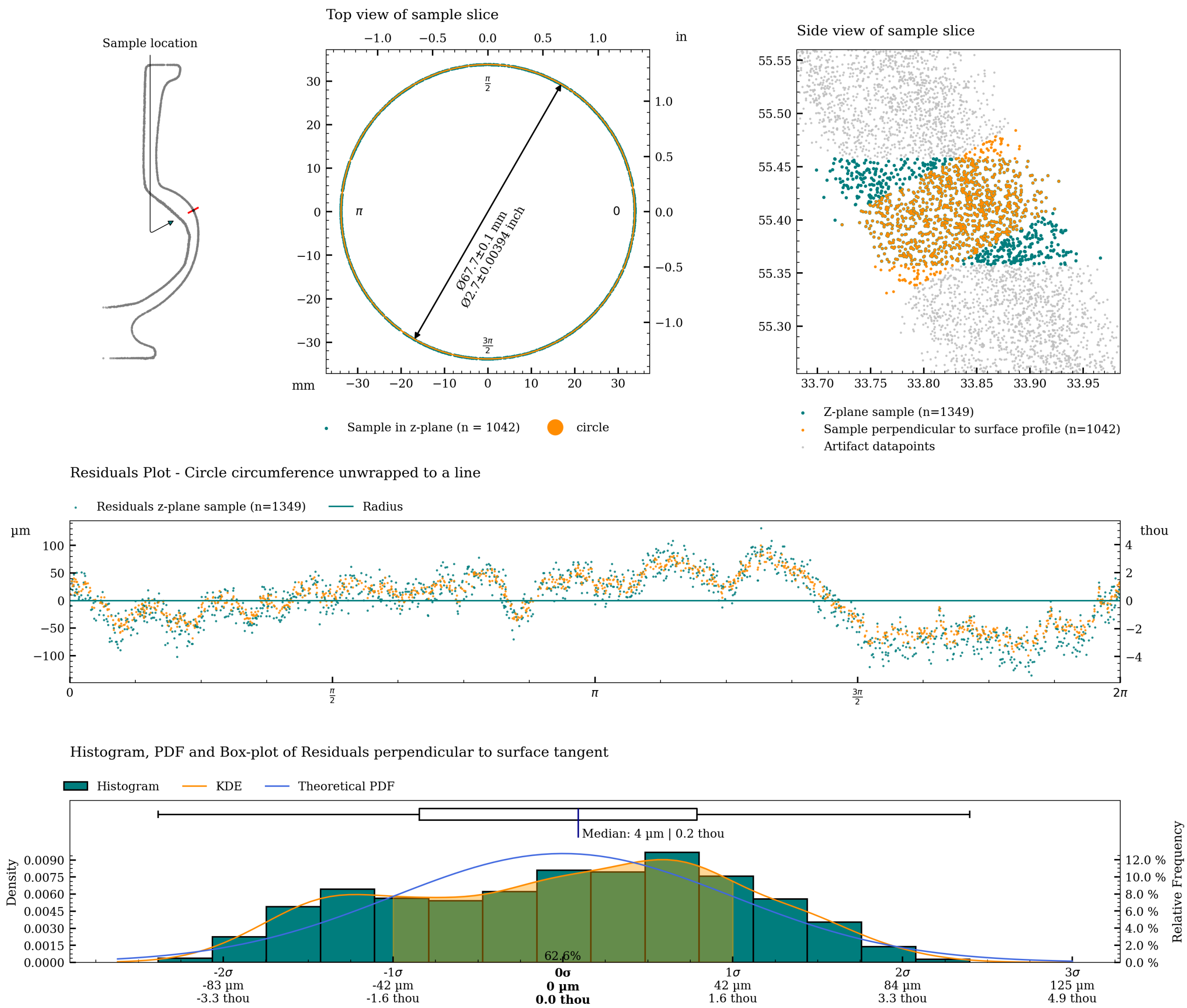


Figure 8: Charts with statistics for the measurement of c03.

Graphical overview of circularity measurement c04

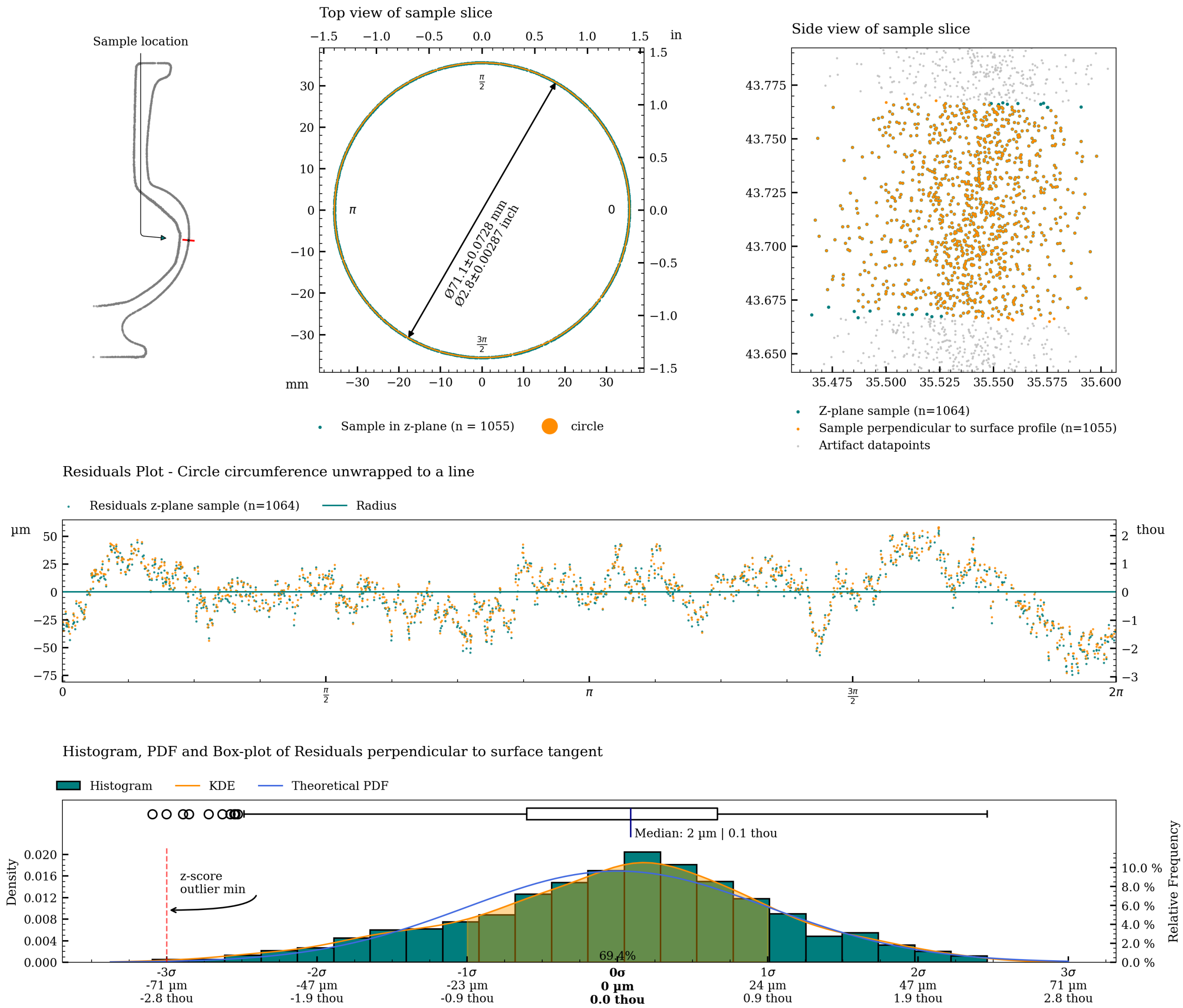


Figure 9: Charts with statistics for the measurement of c04.

Graphical overview of circularity measurement c05

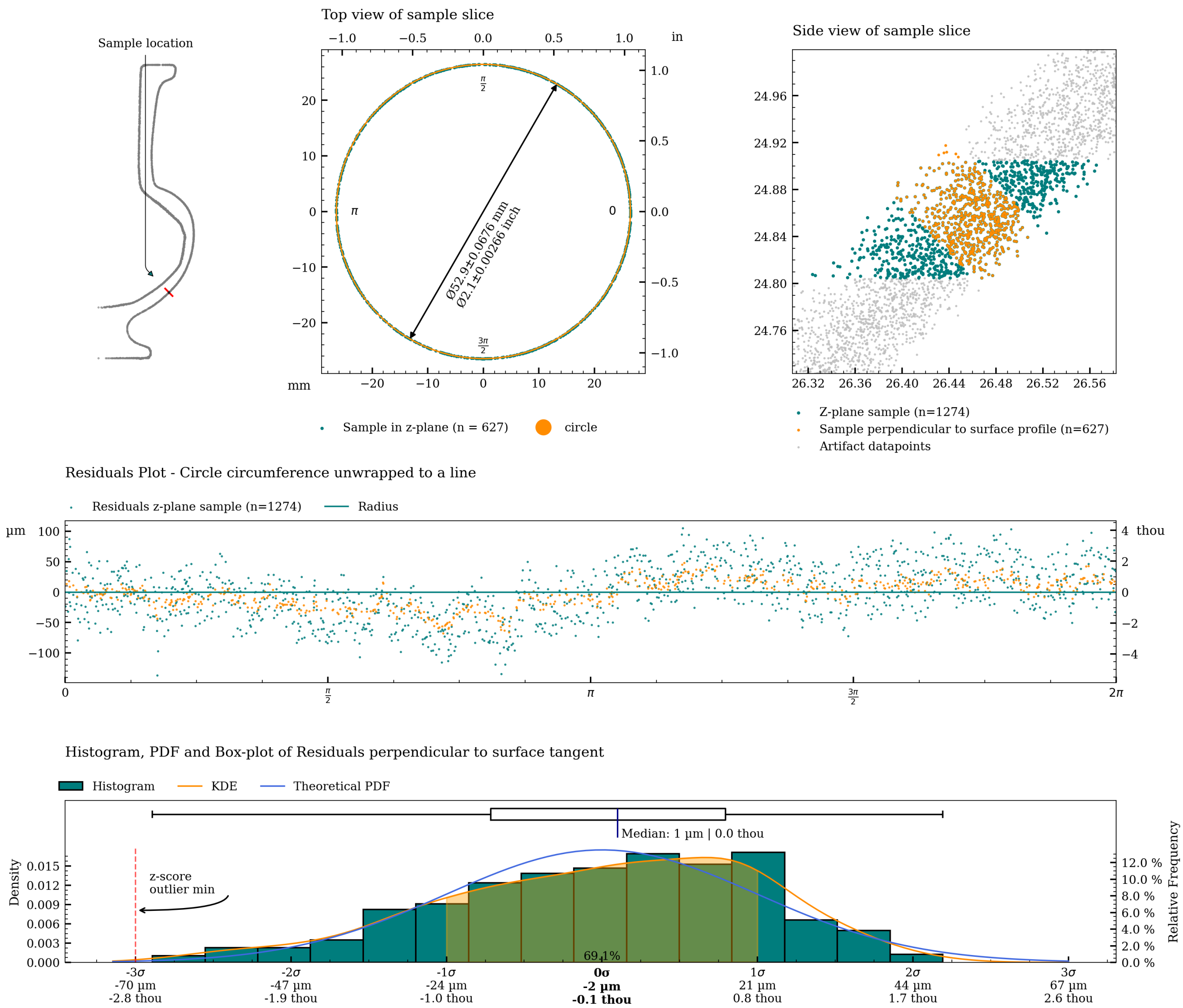


Figure 10: Charts with statistics for the measurement of c05.

Graphical overview of circularity measurement c06

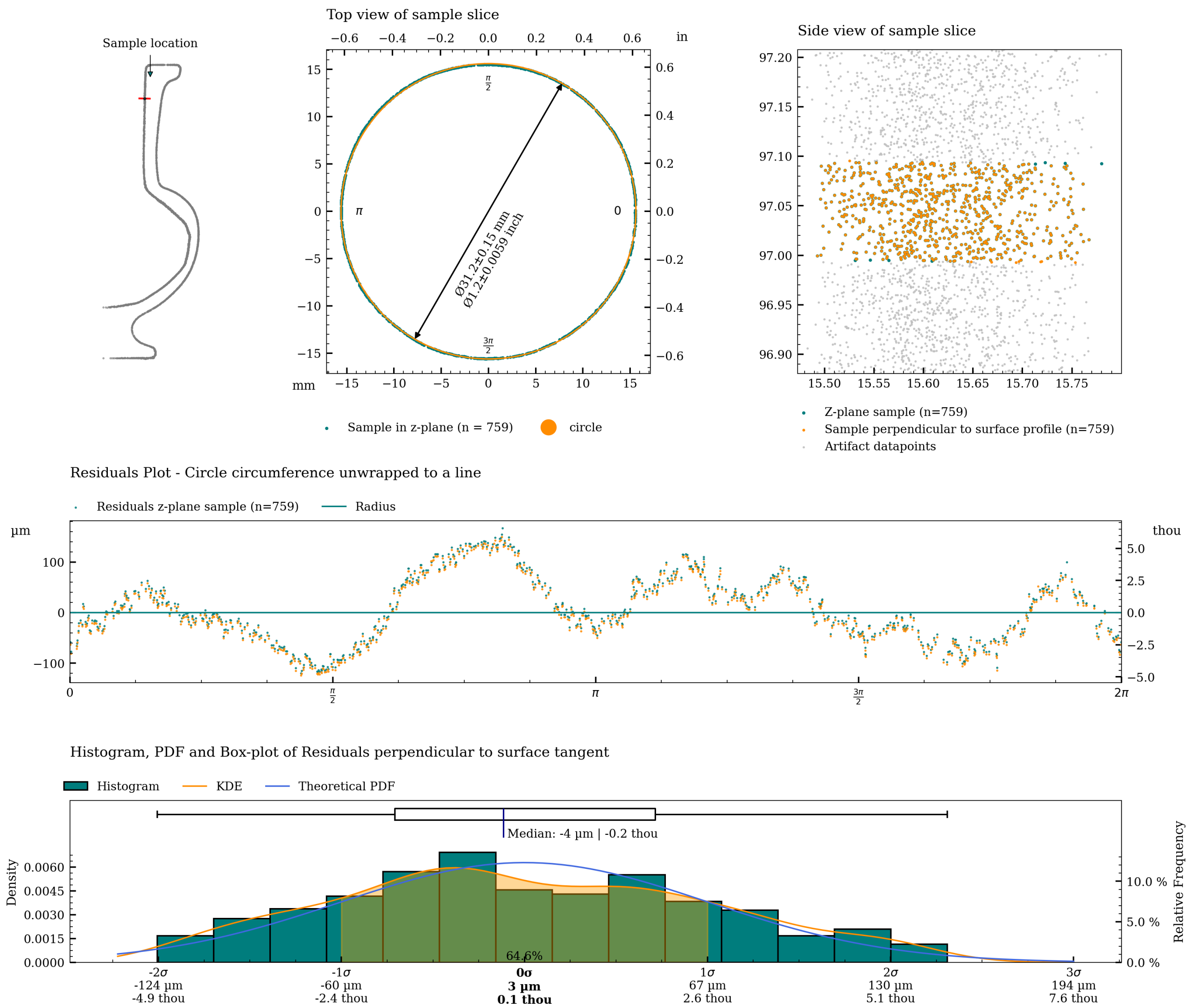


Figure 11: Charts with statistics for the measurement of c06.

Graphical overview of circularity measurement c06_s

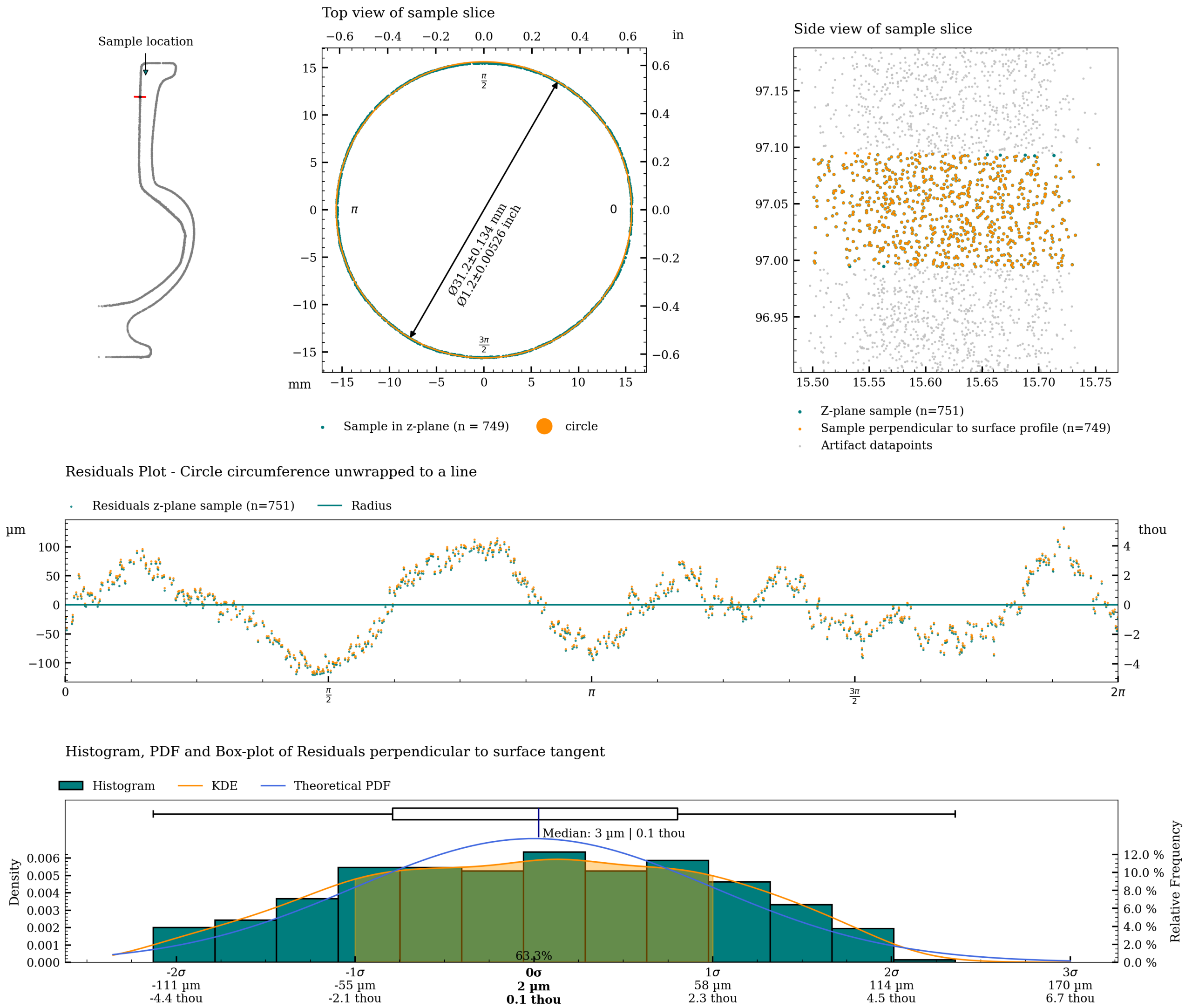


Figure 12: Charts with statistics for the measurement of c06_s.

Graphical overview of circularity measurement c07

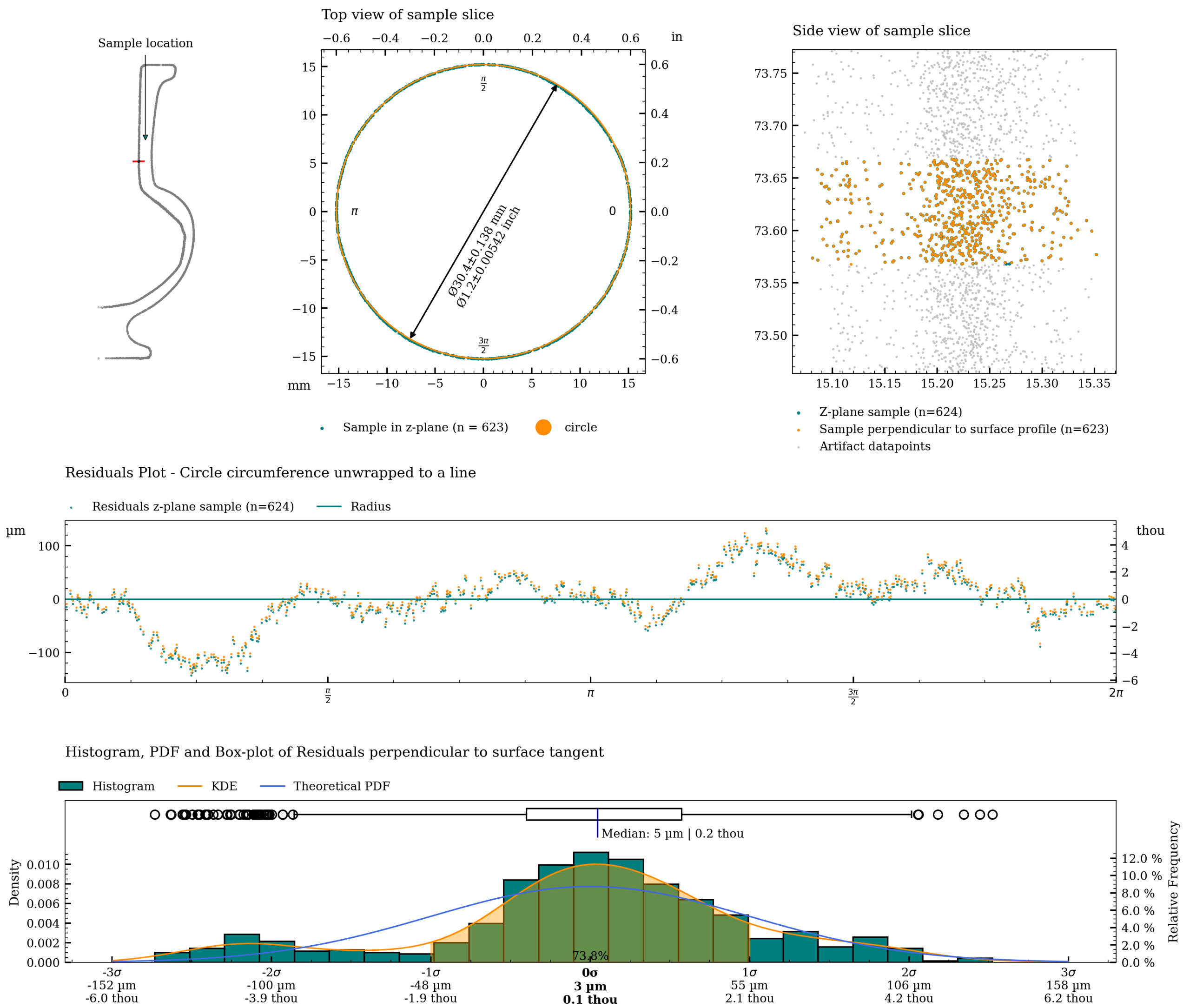


Figure 13: Charts with statistics for the measurement of c07.

Graphical overview of circularity measurement c07_s

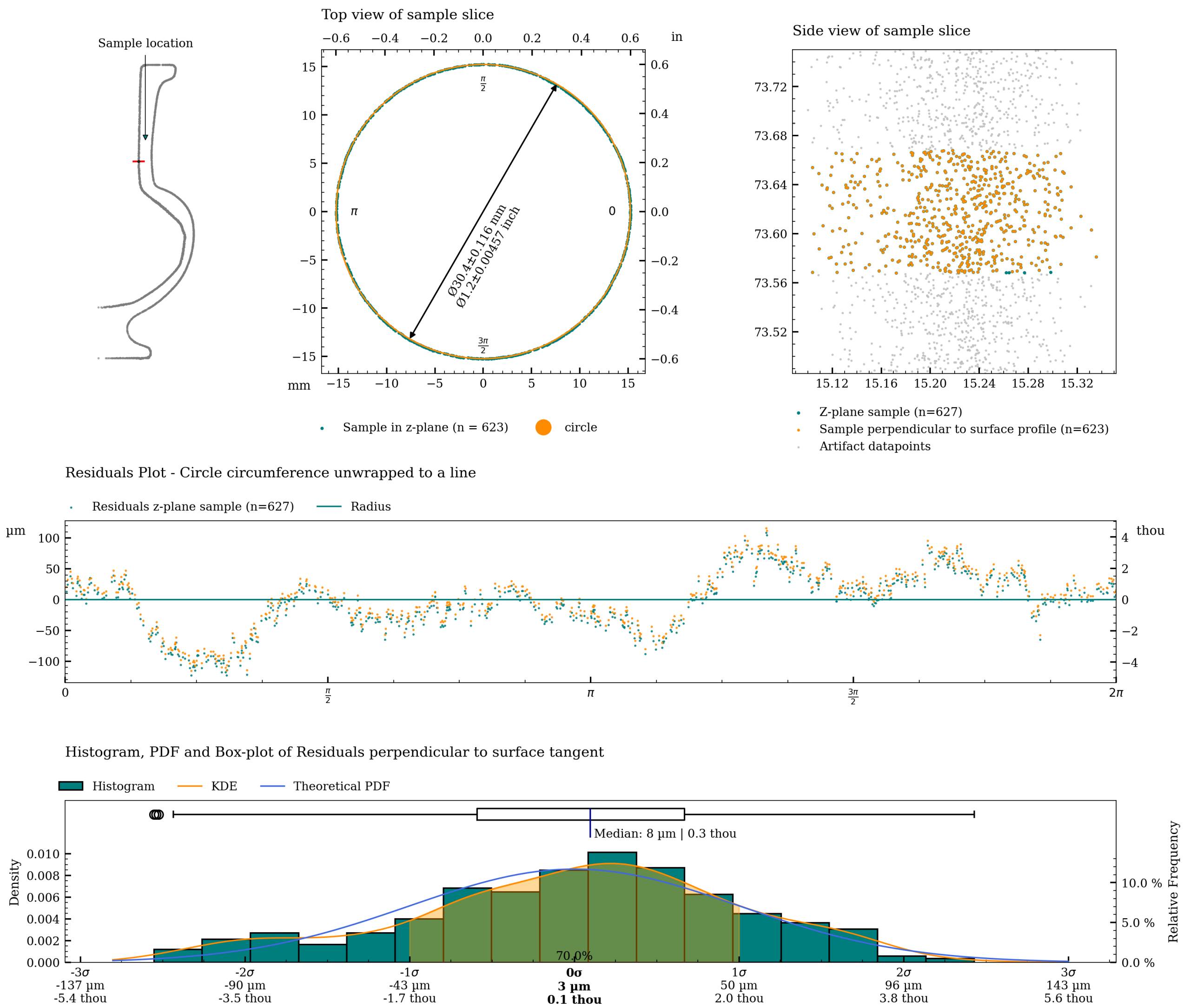


Figure 14: Charts with statistics for the measurement of c07_s.

Graphical overview of circularity measurement c08

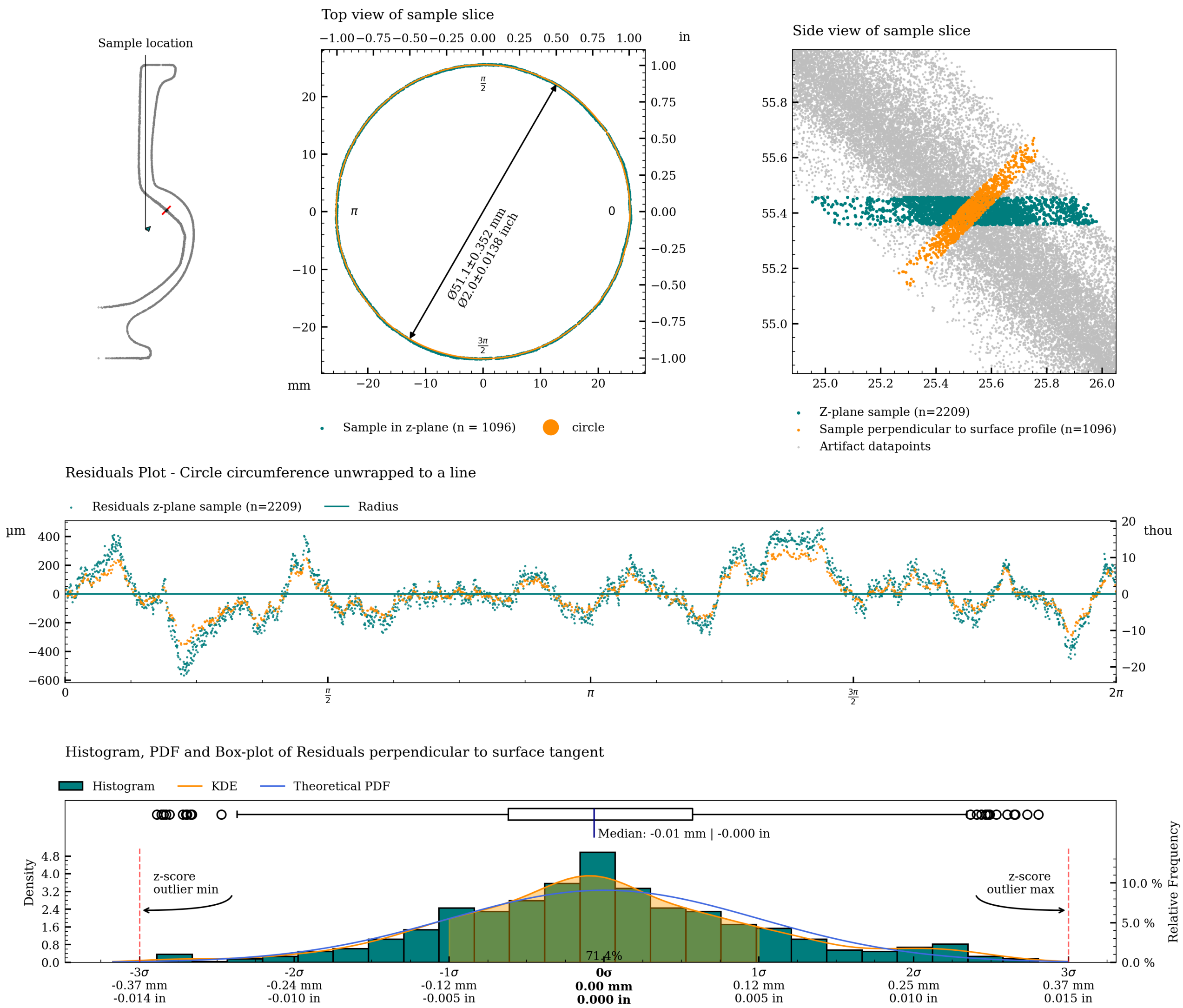


Figure 15: Charts with statistics for the measurement of c08.

Graphical overview of circularity measurement c08_s

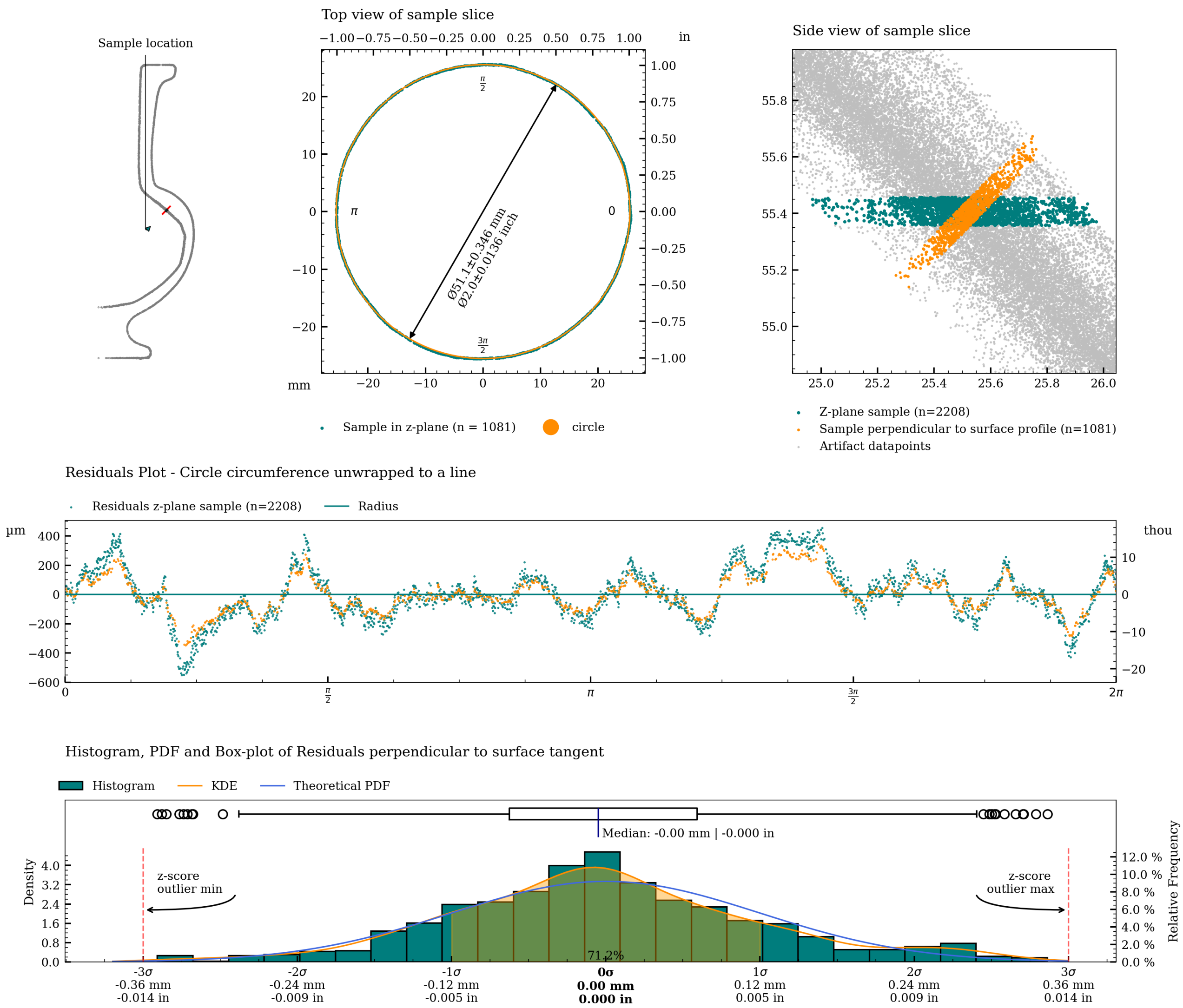


Figure 16: Charts with statistics for the measurement of c08_s.

Graphical overview of circularity measurement c09

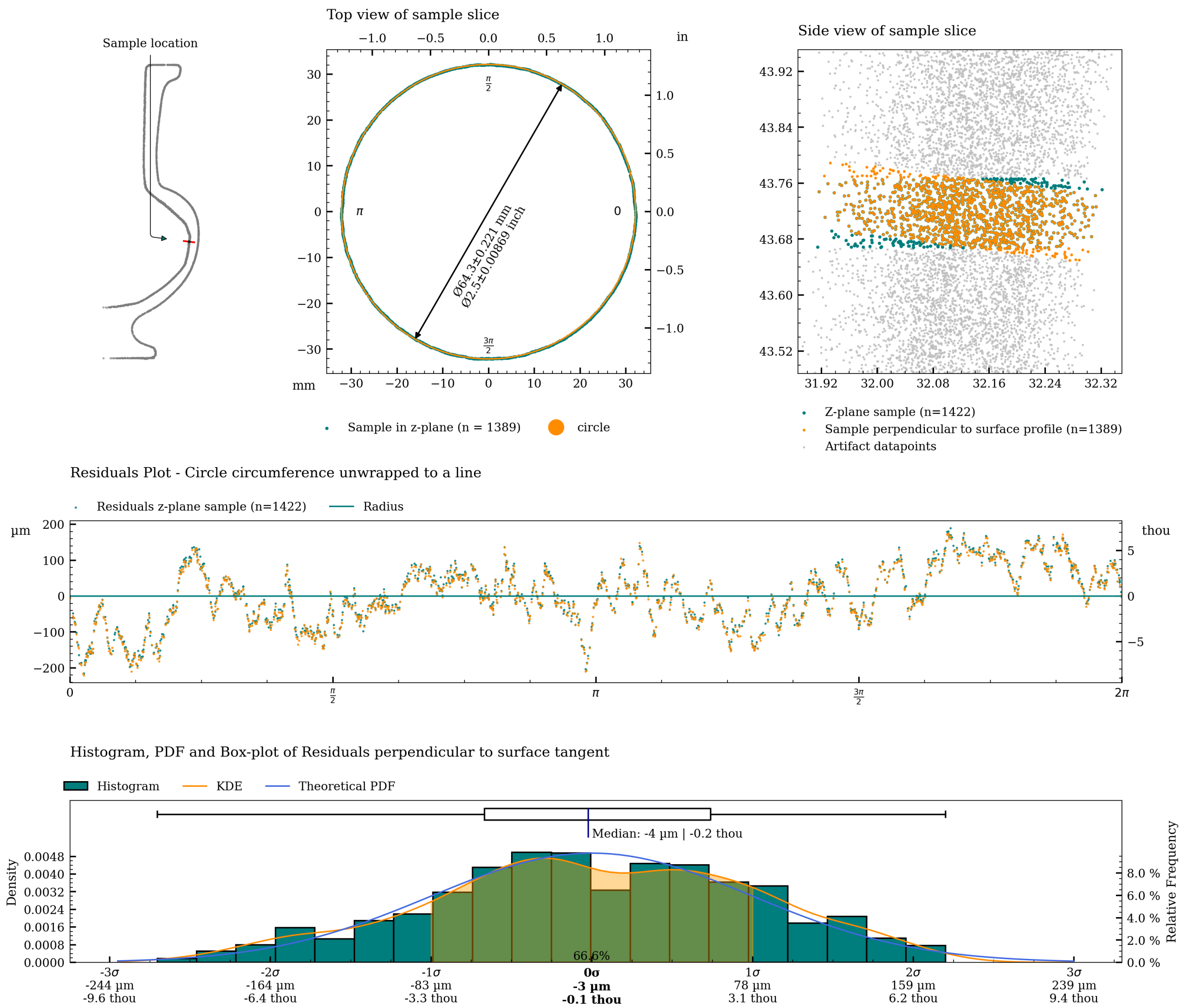


Figure 17: Charts with statistics for the measurement of c09.

Graphical overview of circularity measurement c09_s

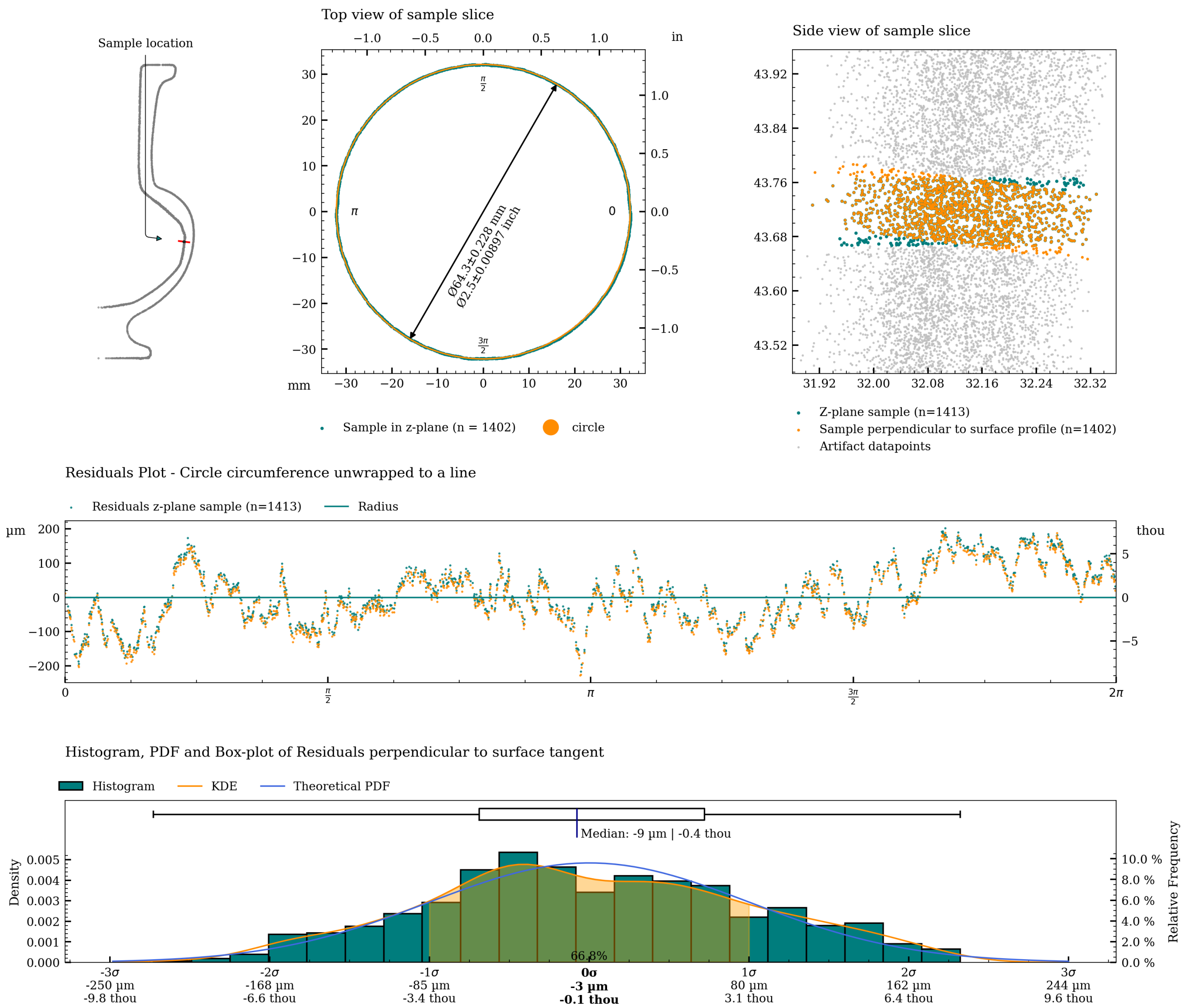


Figure 18: Charts with statistics for the measurement of c09_s.

Graphical overview of circularity measurement c10

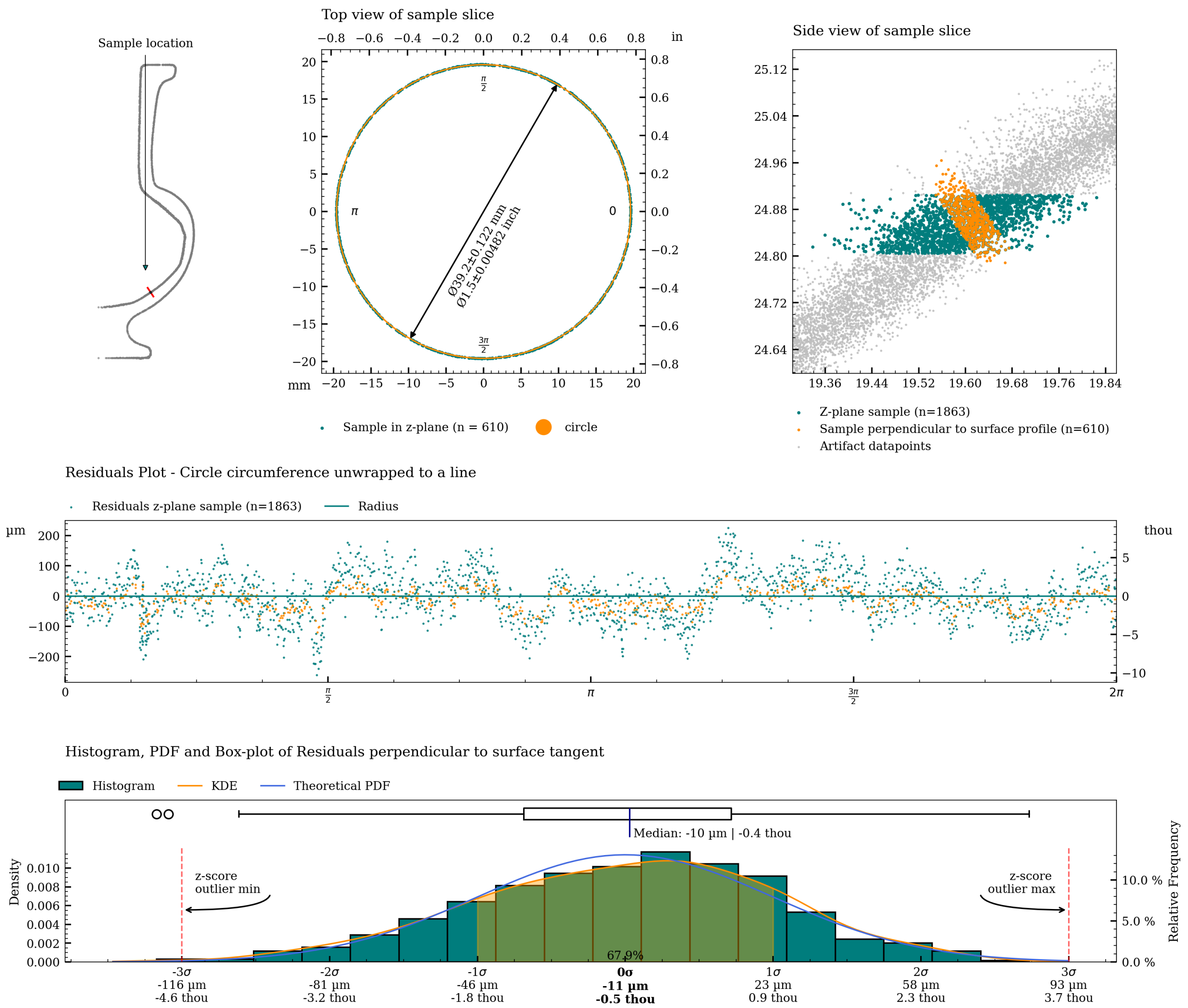


Figure 19: Charts with statistics for the measurement of c10.

Graphical overview of circularity measurement c10_s

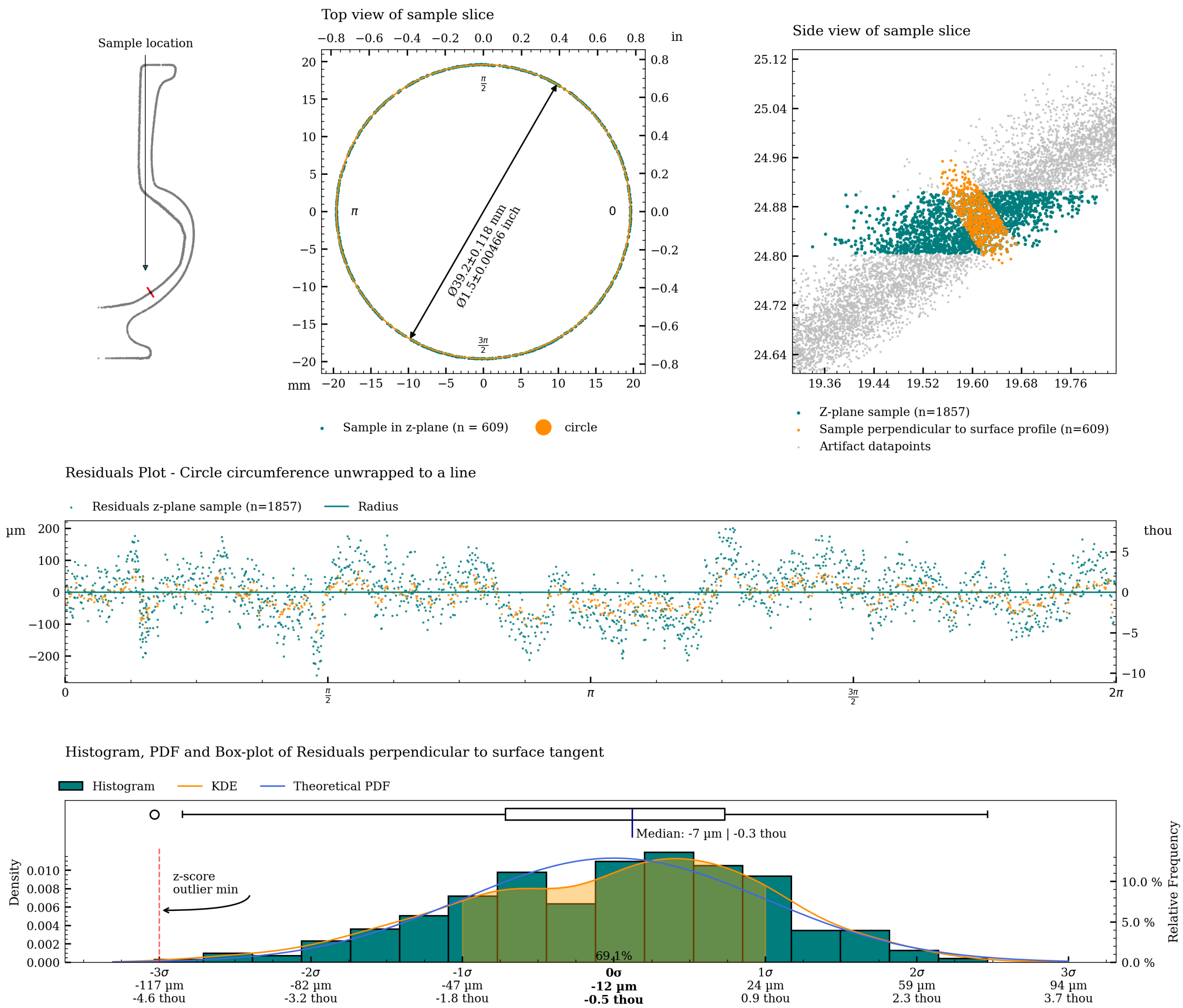


Figure 20: Charts with statistics for the measurement of c10_s.

Table 2 shows statistical measures of the circularity of the vessel, measured along the full height (areas on the artifact scan containing damaged parts have been removed to the best extent possible to reduce the influence of the measurement).

Metric

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.142	0.079	0.260	0.017	0.010	0.028	0.029	0.016	0.056	1941	0.050
Interior	0.324	0.116	0.727	0.041	0.013	0.084	0.068	0.021	0.138	1742	0.050
Interior separate	0.303	0.112	0.719	0.036	0.013	0.086	0.064	0.021	0.139	1741	0.050

Imperial

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.142	0.079	0.260	0.017	0.010	0.028	0.029	0.016	0.056	1941	0.050
Interior	0.324	0.116	0.727	0.041	0.013	0.084	0.068	0.021	0.138	1742	0.050
Interior separate	0.303	0.112	0.719	0.036	0.013	0.086	0.064	0.021	0.139	1741	0.050

Table 2: Perpendicular Circularity analysis of PV003.

Circularity analysis of exterior surface

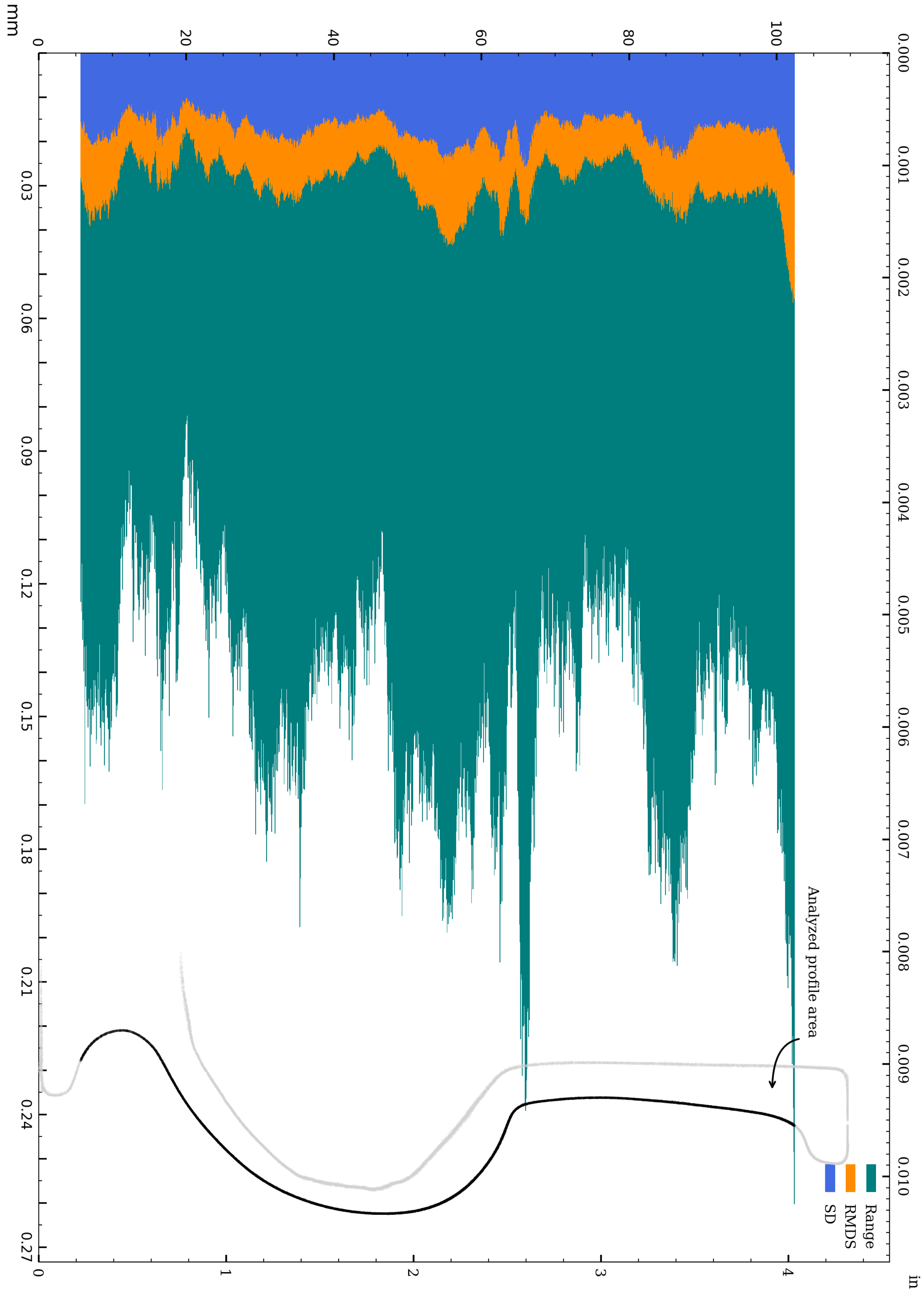


Figure 21: Circularity of exterior surface.

Circularity analysis of exterior surface, Standard Deviation and Root Mean Squared Deviation

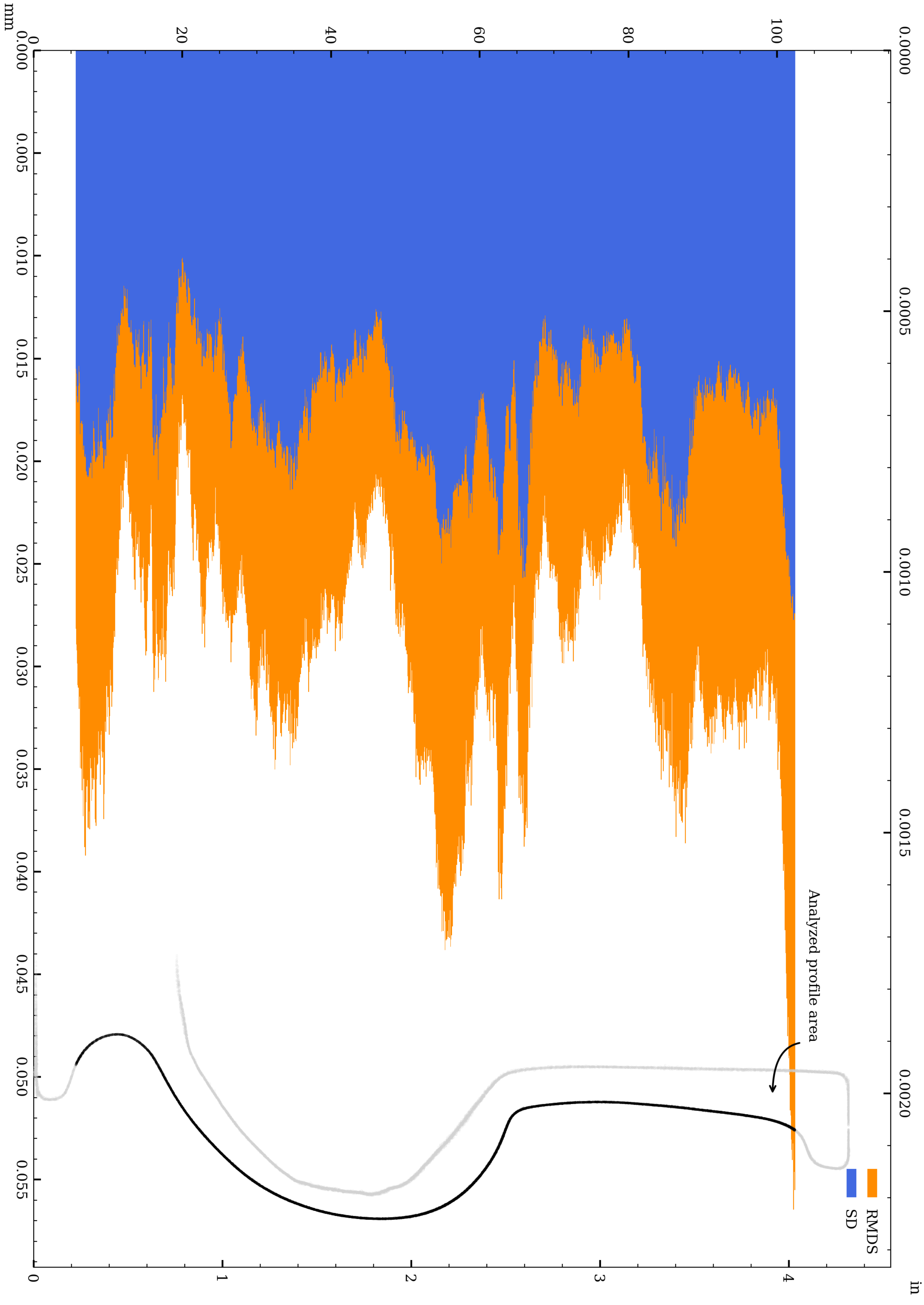


Figure 22: Vessel circularity of exterior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 1941 slices of the exterior surface are shown below.

Range measurement distribution across 1941 slices of exterior surface

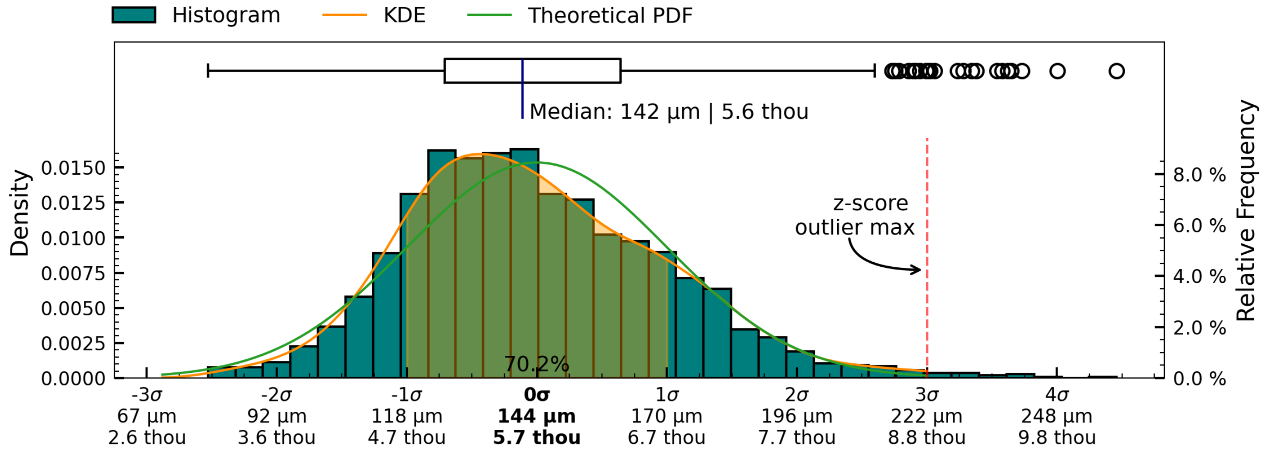


Figure 23: Range measurement distribution across measured slices of exterior surface

Standard Deviation measurement distribution across 1941 slices of exterior surface

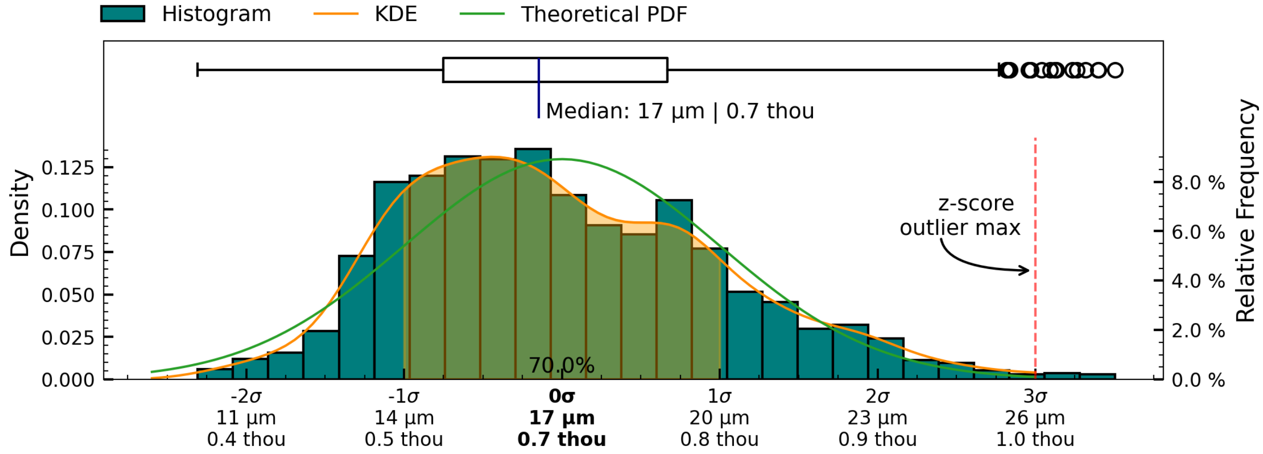


Figure 24: Standard Deviation measurement distribution across measured slices of “ + exterior + “ surface

Root Mean Squared Deviation measurement distribution across 1941 slices of exterior surface

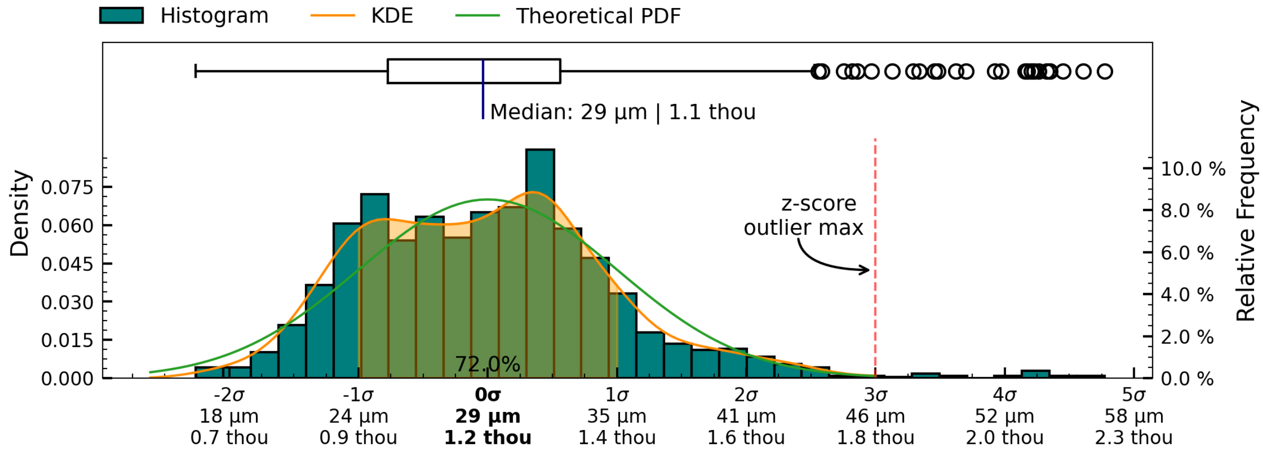


Figure 25: Root Mean Squared Deviation measurement distribution across measured slices of exterior surface

Circularity analysis of interior surface

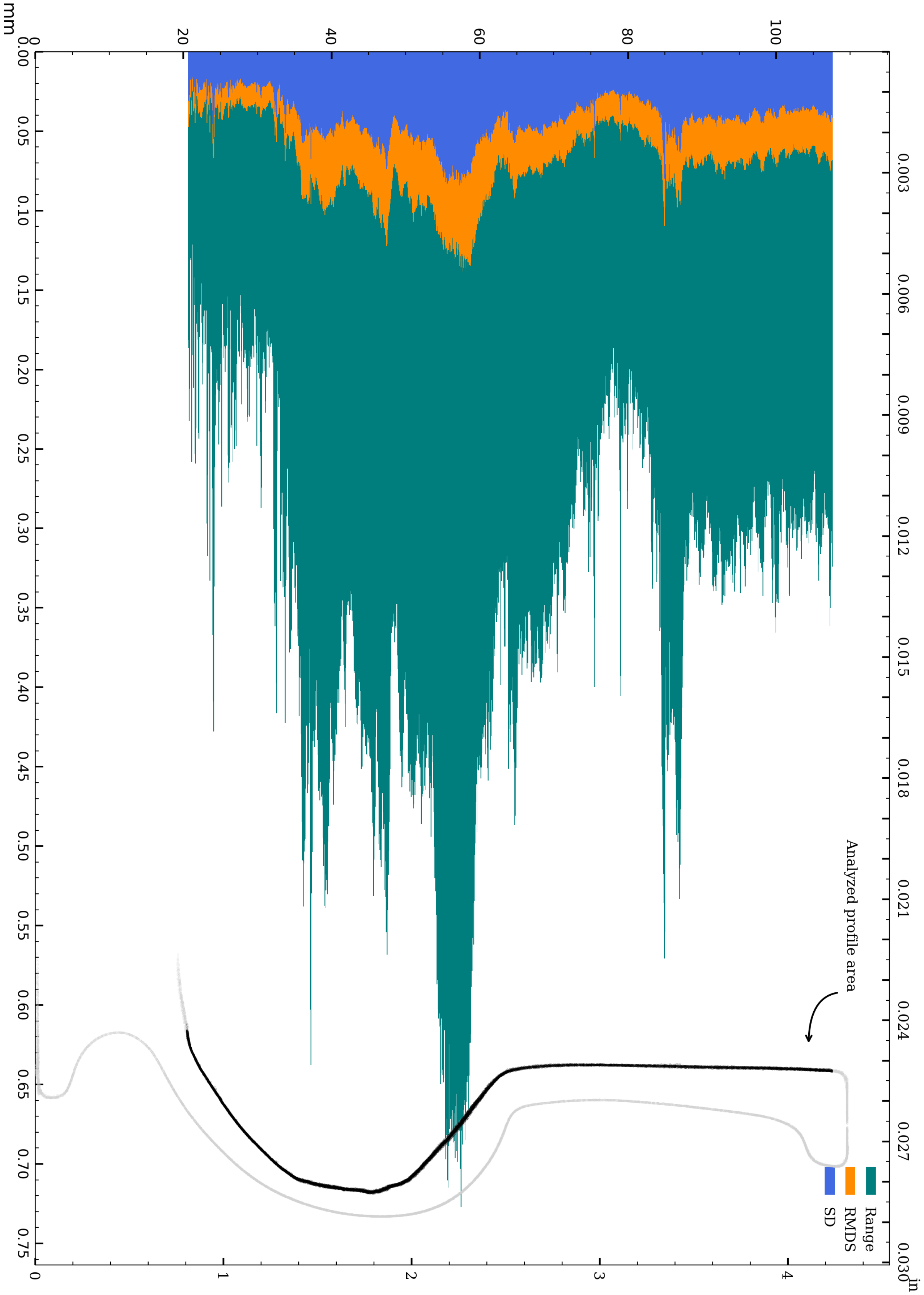


Figure 26: Circularity of interior surface.

Circularity analysis of interior surface, Standard Deviation and Root Mean Squared Deviation

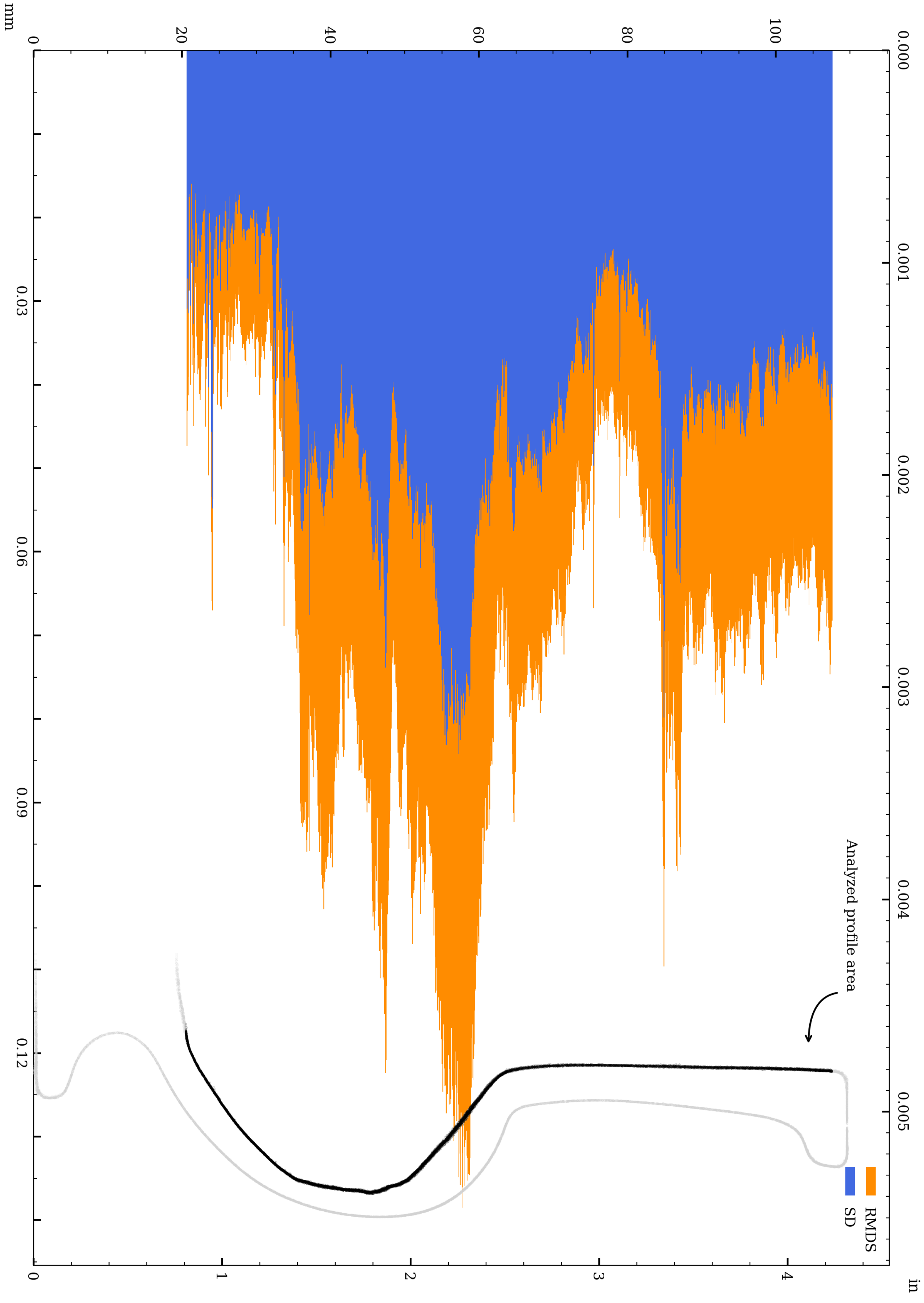


Figure 27: Vessel circularity of interior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 1742 slices of the interior surface are shown below.

Range measurement distribution across 1742 slices of interior surface

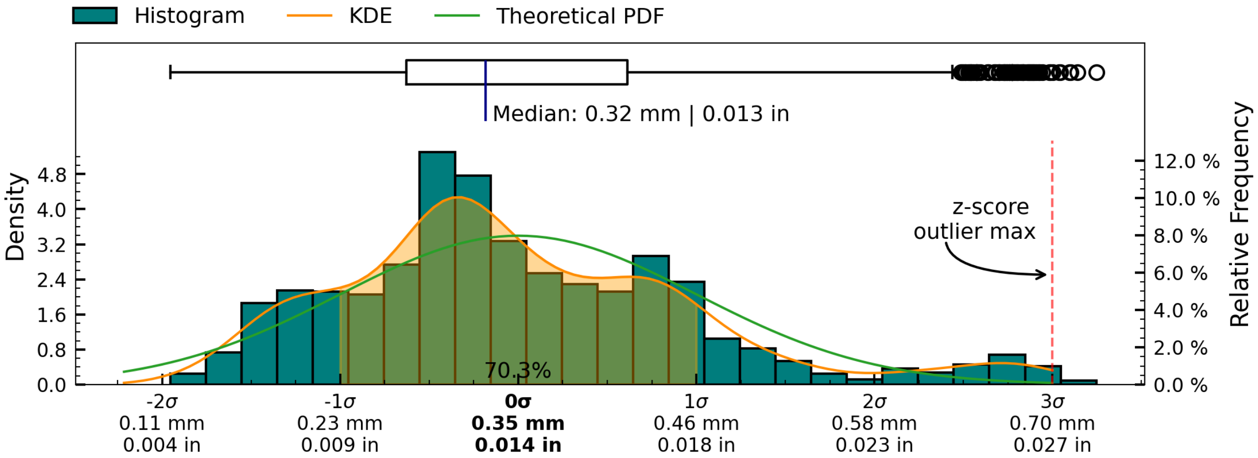


Figure 28: Range measurement distribution across measured slices of interior surface

Standard Deviation measurement distribution across 1742 slices of interior surface

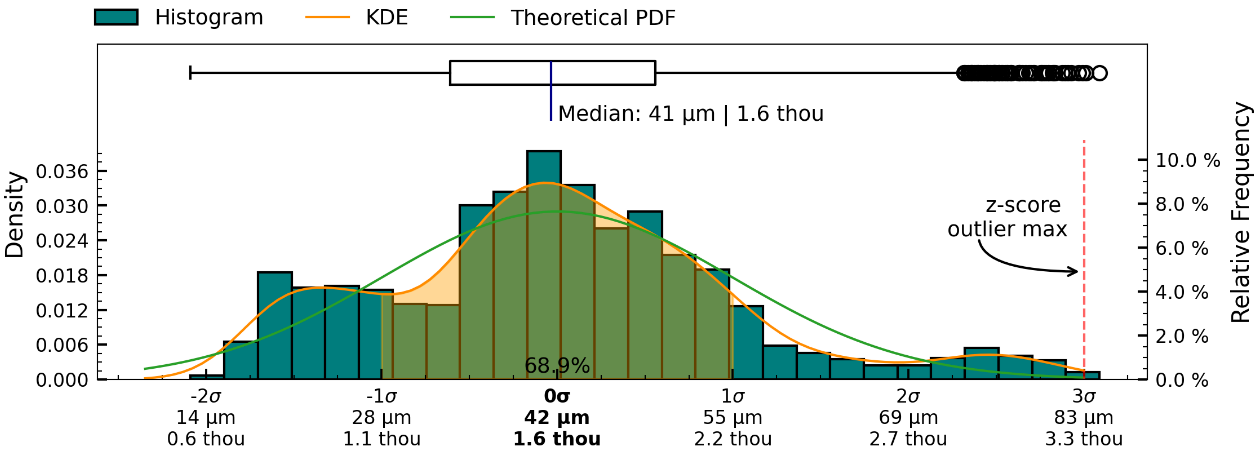


Figure 29: Standard Deviation measurement distribution across measured slices of " + interior + " surface

Root Mean Squared Deviation measurement distribution across 1742 slices of interior surface

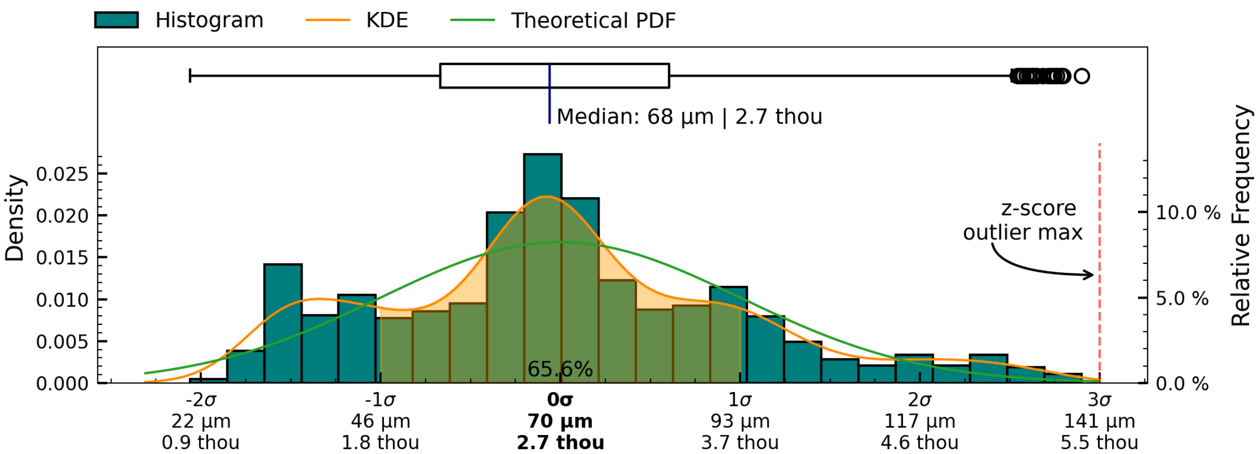


Figure 30: Root Mean Squared Deviation measurement distribution across measured slices of interior surface

Circularity analysis of interior separately aligned surface

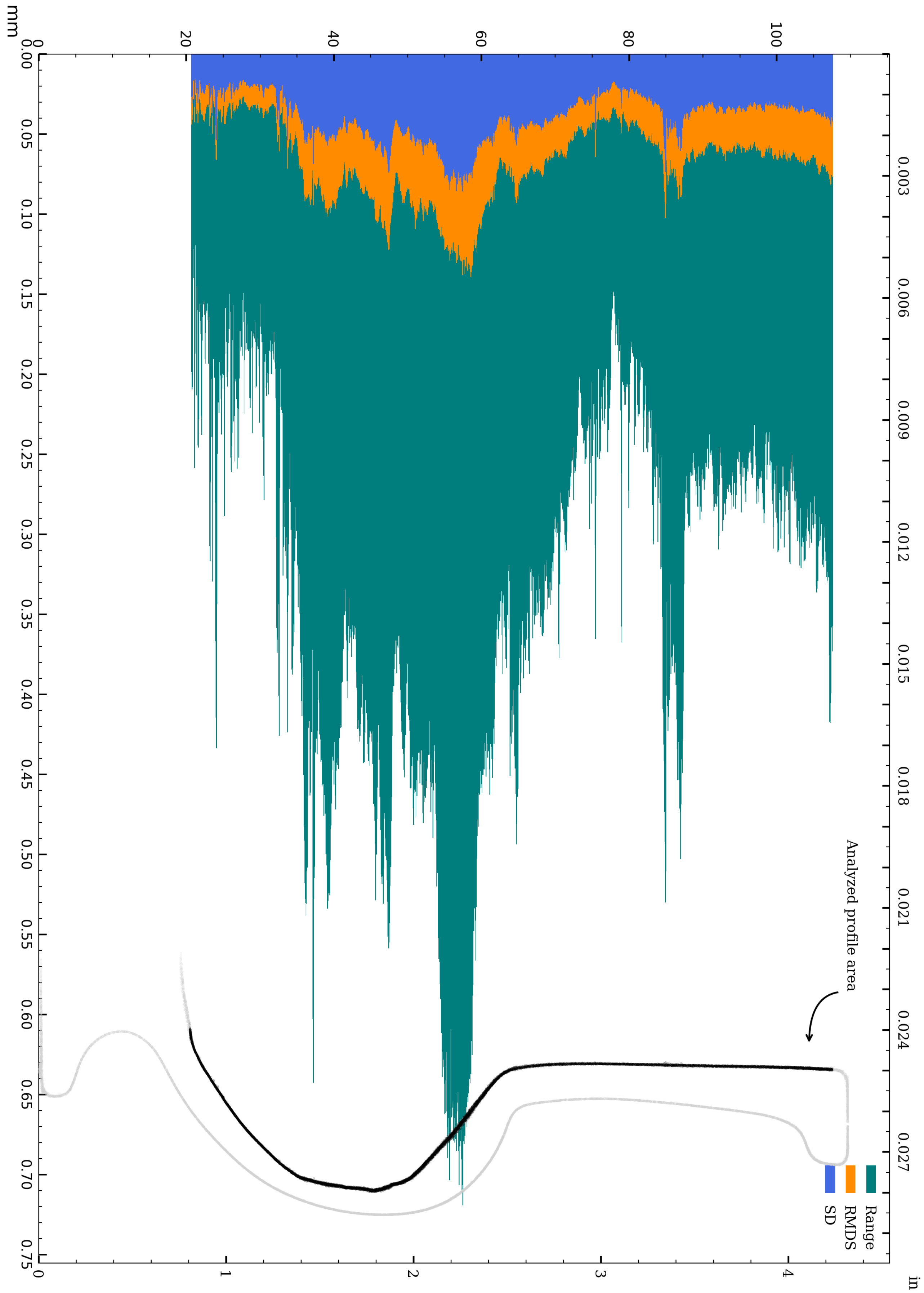


Figure 31: Circularity of interior_separate surface.

Circularity analysis of interior separately aligned surface, Standard Deviation and Root Mean Squared Deviation

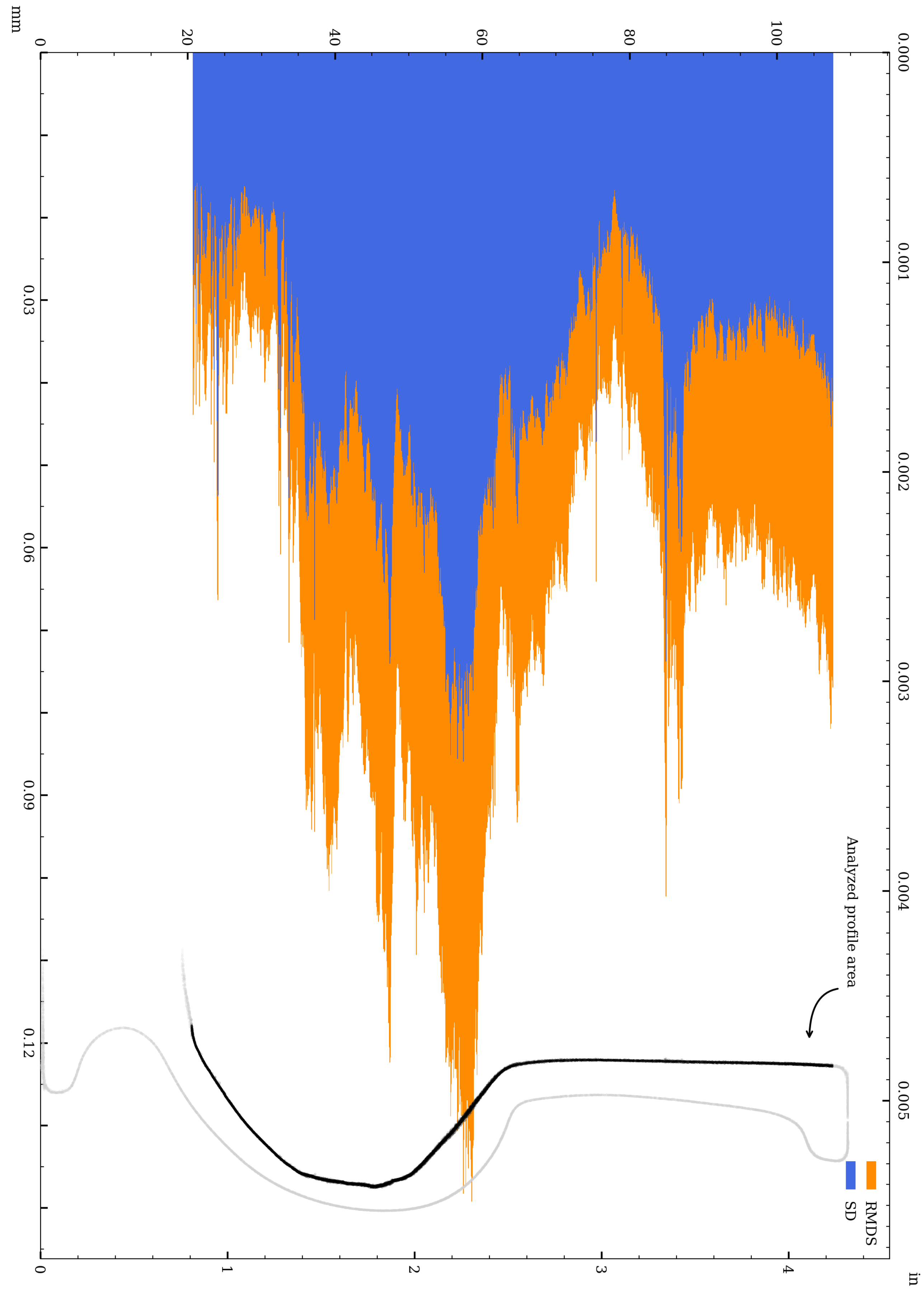


Figure 32: Vessel circularity of interior_separate surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 1741 slices of the interior_separate surface are shown below.

Range measurement distribution across 1741 slices of interior separately aligned surface

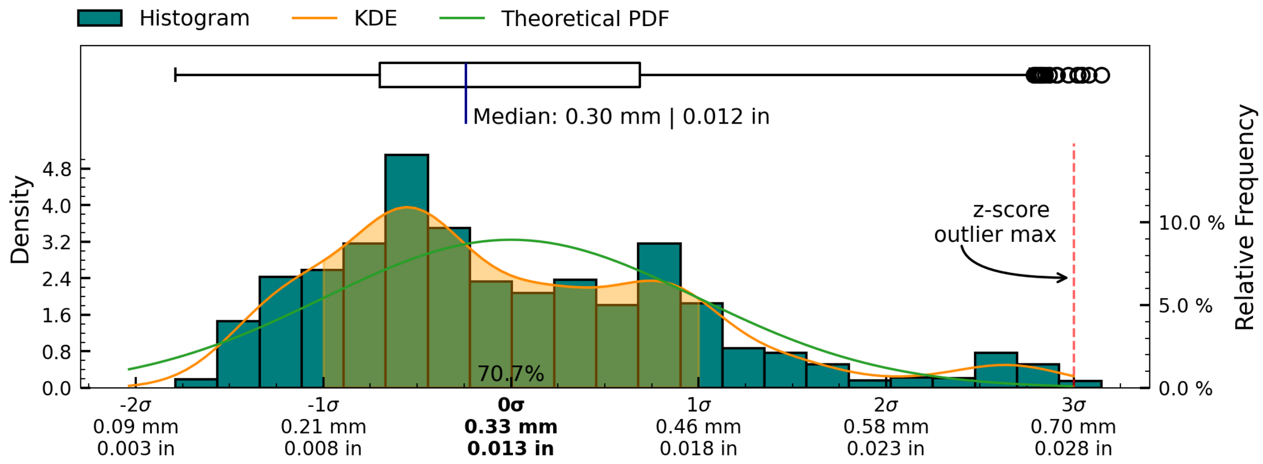


Figure 33: Range measurement distribution across measured slices of interior_separate surface

Standard Deviation measurement distribution across 1741 slices of interior separately aligned surface

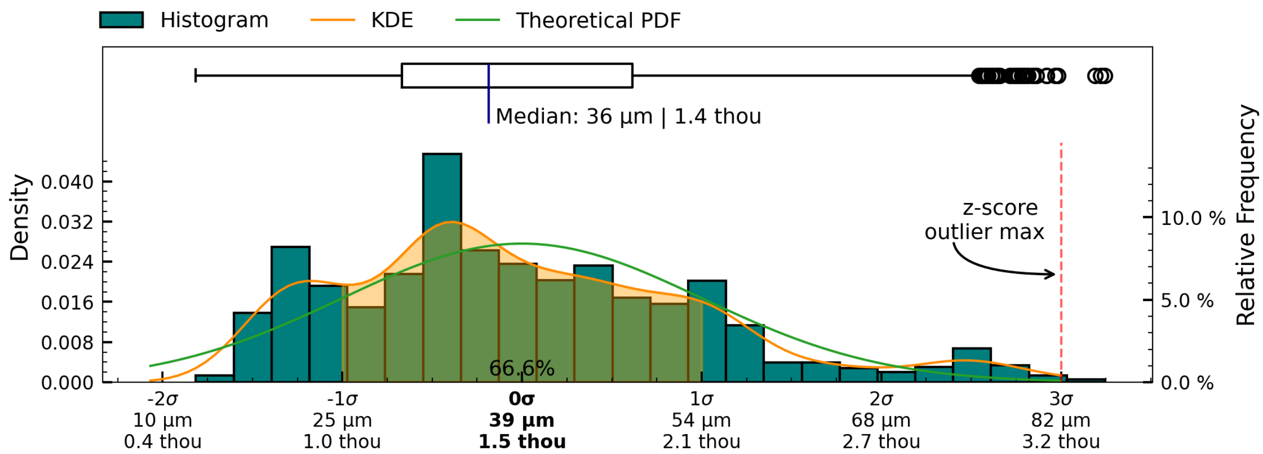


Figure 34: Standard Deviation measurement distribution across measured slices of “ + interior_separate + “ surface

Root Mean Squared Deviation measurement distribution across 1741 slices of interior separately aligned surface

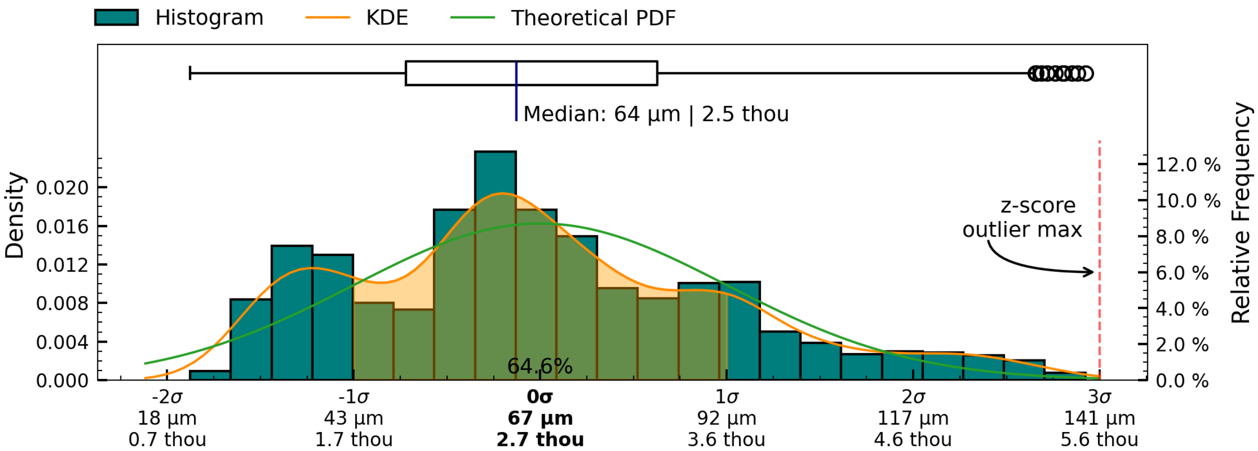


Figure 35: Root Mean Squared Deviation measurement distribution across measured slices of interior separately aligned surface

Concentricity

The concentricity metric describes the deviation in the center-point of the referenced features. As such, it is a measure to determine if several features of the object share the same center point/axis, and how closely. See Figure 36 for a visual representation of this metric.

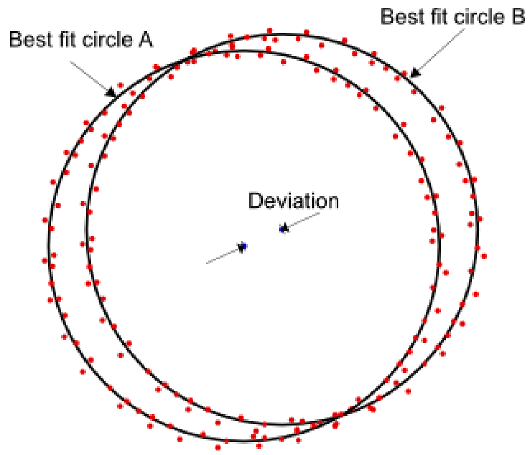


Figure 36: Concentricity measures the deviation (distance) between the center of two circles.

Determination of concentricity has been carried out by establishing the best fit circles of sample slices, using RANSAC (Random sample consensus) algorithm for outlier detection of a least squares circle regression on the scanned data-points at each cross-section, to estimate centers of each cross-section.

The concentricity between both the interior and exterior circular cross-sections is explored for cross-section measurements with the same Z-coordinates.

Additionally, the concentricity between each cross-section measurement defined in Figure 4 and the datum axis $(x, y) = (0, 0)$ has been calculated to establish the deviation of the feature center from the datum axis.

Figure 37: Circularity measurement sample locations, full mesh aligned to exterior surface

Figure 38: Circularity measurement sample location, separately aligned interior mesh

Metric

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	**Range inliers**	**RMSD full**	**RMDS inliers**	**SD full**	**SD inliers**	**Center (x,y)**
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.033	759	0.167	0.164	0.034	0.033	0.018	0.018	−32, −6
c02	z-axis	0.018	598	0.124	0.124	0.025	0.024	0.014	0.014	−15, 9
c03	z-axis	0.038	1042	0.186	0.179	0.039	0.039	0.021	0.021	−36, 9
c04	z-axis	0.008	1055	0.139	0.123	0.026	0.025	0.015	0.014	1, −8
c05	z-axis	0.016	627	0.133	0.133	0.029	0.029	0.015	0.015	4, −16
c06	z-axis	0.043	759	0.275	0.275	0.064	0.064	0.035	0.035	−43, −0
c06_s	z-axis	0.002	749	0.251	0.251	0.056	0.056	0.030	0.030	−2, 1
c07	z-axis	0.051	623	0.388	0.388	0.087	0.087	0.049	0.049	−29, −42
c07_s	z-axis	0.041	623	0.347	0.347	0.075	0.075	0.045	0.045	1, −41
c08	z-axis	0.049	1096	0.780	0.757	0.138	0.131	0.089	0.084	−19, −46
c08_s	z-axis	0.041	1081	0.762	0.699	0.131	0.125	0.084	0.079	−9, −40
c09	z-axis	0.027	1389	0.431	0.431	0.087	0.087	0.052	0.052	9, −25
c09_s	z-axis	0.035	1402	0.424	0.424	0.085	0.085	0.050	0.050	26, −23
c10	z-axis	0.003	610	0.208	0.208	0.036	0.036	0.022	0.022	−2, −1
c10_s	z-axis	0.004	609	0.195	0.195	0.036	0.036	0.022	0.022	4, −1
c01	c06_s	0.031								−30, −7
c02	c07_s	0.053								−16, 50
c03	c08_s	0.056								−27, 49
c04	c09_s	0.030								−26, 15
c05	c10_s	0.015								−0, −15

Imperial

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	**Range inliers**	**RMSD full**	**RMDS inliers**	**SD full**	**SD inliers**	**Center (x,y)**
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0013	759	0.0066	0.0065	0.0013	0.0013	0.0007	0.0007	−1.3, −0.2
c02	z-axis	0.0007	598	0.0049	0.0049	0.0010	0.0009	0.0006	0.0005	−0.6, 0.4
c03	z-axis	0.0015	1042	0.0073	0.0070	0.0015	0.0015	0.0008	0.0008	−1.4, 0.4
c04	z-axis	0.0003	1055	0.0055	0.0049	0.0010	0.0010	0.0006	0.0006	0.0, −0.3
c05	z-axis	0.0006	627	0.0052	0.0052	0.0011	0.0011	0.0006	0.0006	0.1, −0.6
c06	z-axis	0.0017	759	0.0108	0.0108	0.0025	0.0025	0.0014	0.0014	−1.7, −0.0
c06_s	z-axis	0.0001	749	0.0099	0.0099	0.0022	0.0022	0.0012	0.0012	−0.1, 0.0
c07	z-axis	0.0020	623	0.0153	0.0153	0.0034	0.0034	0.0019	0.0019	−1.1, −1.7
c07_s	z-axis	0.0016	623	0.0137	0.0137	0.0030	0.0030	0.0018	0.0018	0.0, −1.6
c08	z-axis	0.0019	1096	0.0307	0.0298	0.0054	0.0051	0.0035	0.0033	−0.7, −1.8
c08_s	z-axis	0.0016	1081	0.0300	0.0275	0.0052	0.0049	0.0033	0.0031	−0.4, −1.6
c09	z-axis	0.0011	1389	0.0170	0.0170	0.0034	0.0034	0.0020	0.0020	0.4, −1.0
c09_s	z-axis	0.0014	1402	0.0167	0.0167	0.0033	0.0033	0.0020	0.0020	1.0, −0.9
c10	z-axis	0.0001	610	0.0082	0.0082	0.0014	0.0014	0.0009	0.0009	−0.1, −0.1
c10_s	z-axis	0.0002	609	0.0077	0.0077	0.0014	0.0014	0.0009	0.0009	0.2, −0.0
c01	c06_s	0.0012								−1.2, −0.3
c02	c07_s	0.0021								−0.6, 2.0
c03	c08_s	0.0022								−1.1, 1.9
c04	c09_s	0.0012								−1.0, 0.6
c05	c10_s	0.0006								−0.0, −0.6

Table 3: Concentricity analysis of PV003.

Concentricity analysis of c01

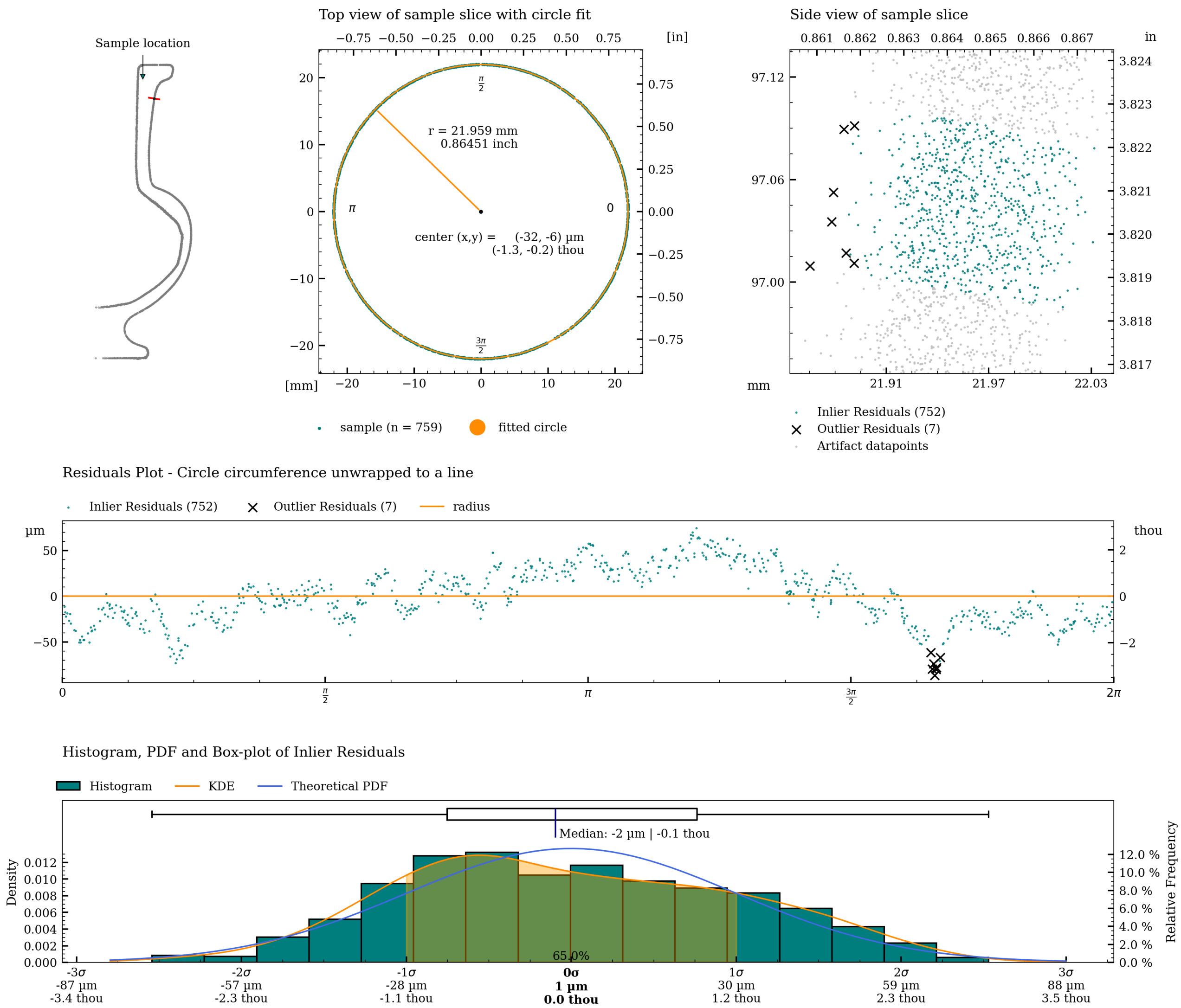


Figure 39: Detailed plot of concentricity measurement for c01.

Concentricity analysis of c02

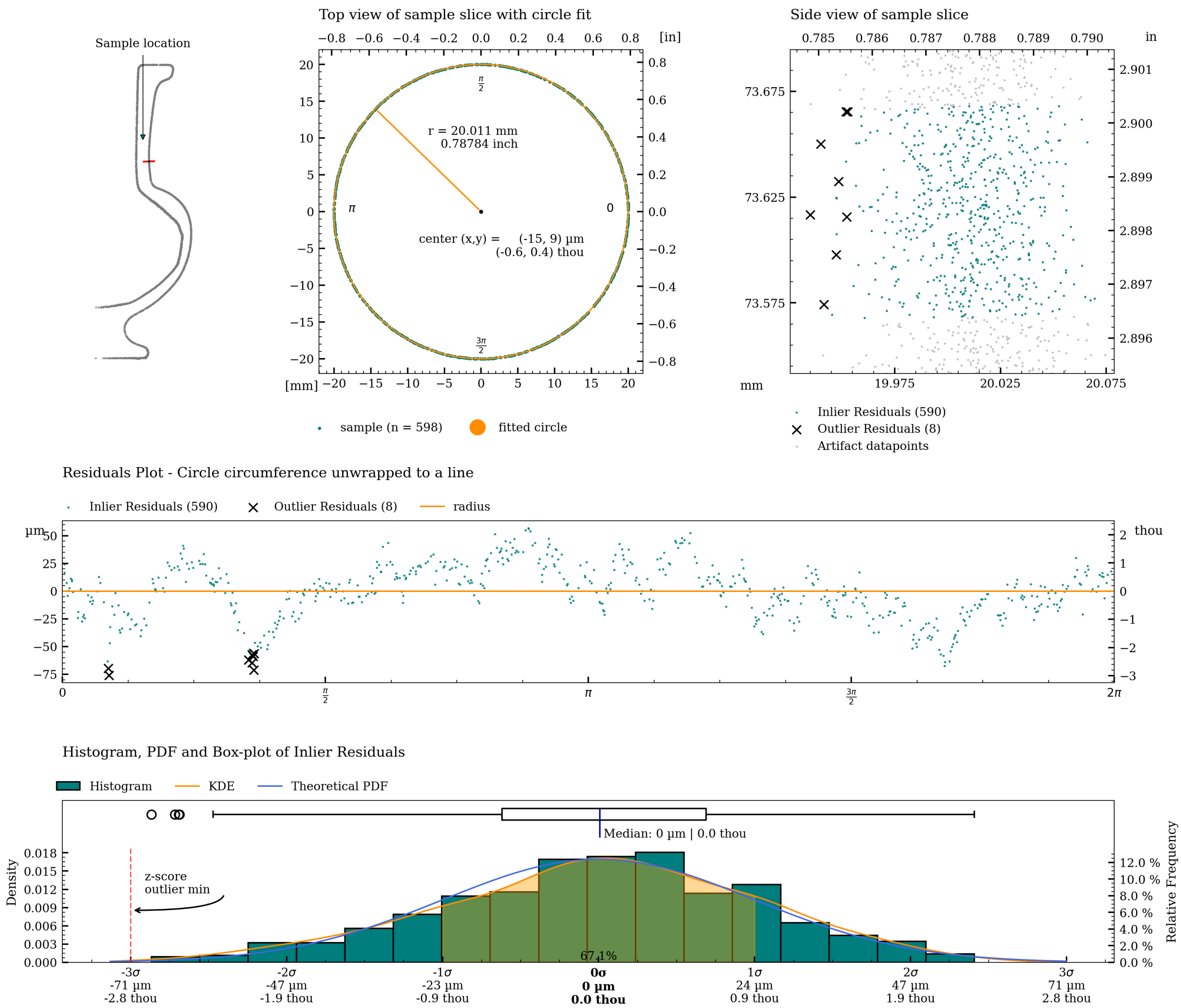


Figure 40: Detailed plot of concentricity measurement for c02.

Concentricity analysis of c03

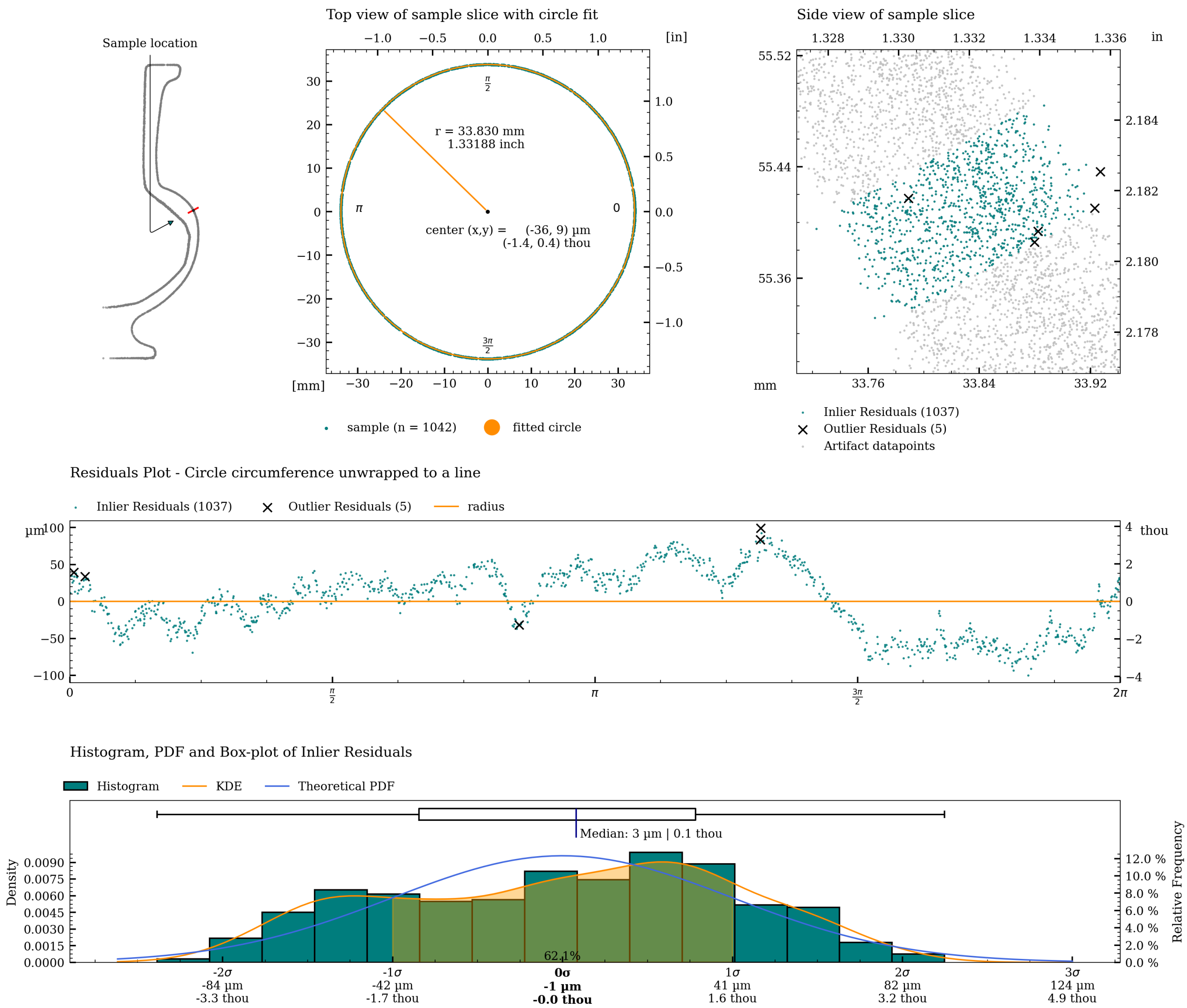


Figure 41: Detailed plot of concentricity measurement for c03.

Concentricity analysis of c04

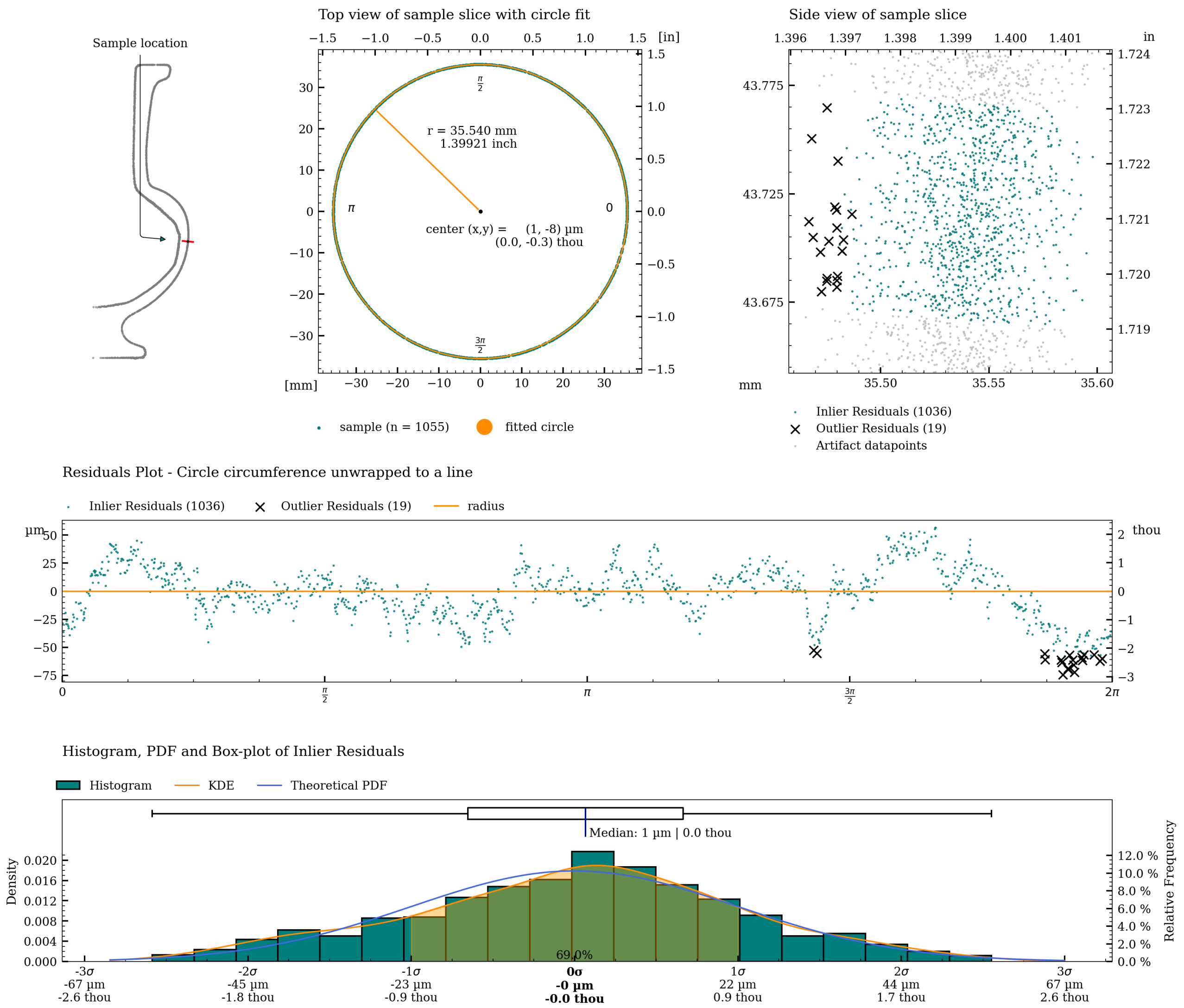


Figure 42: Detailed plot of concentricity measurement for c04.

Concentricity analysis of c05

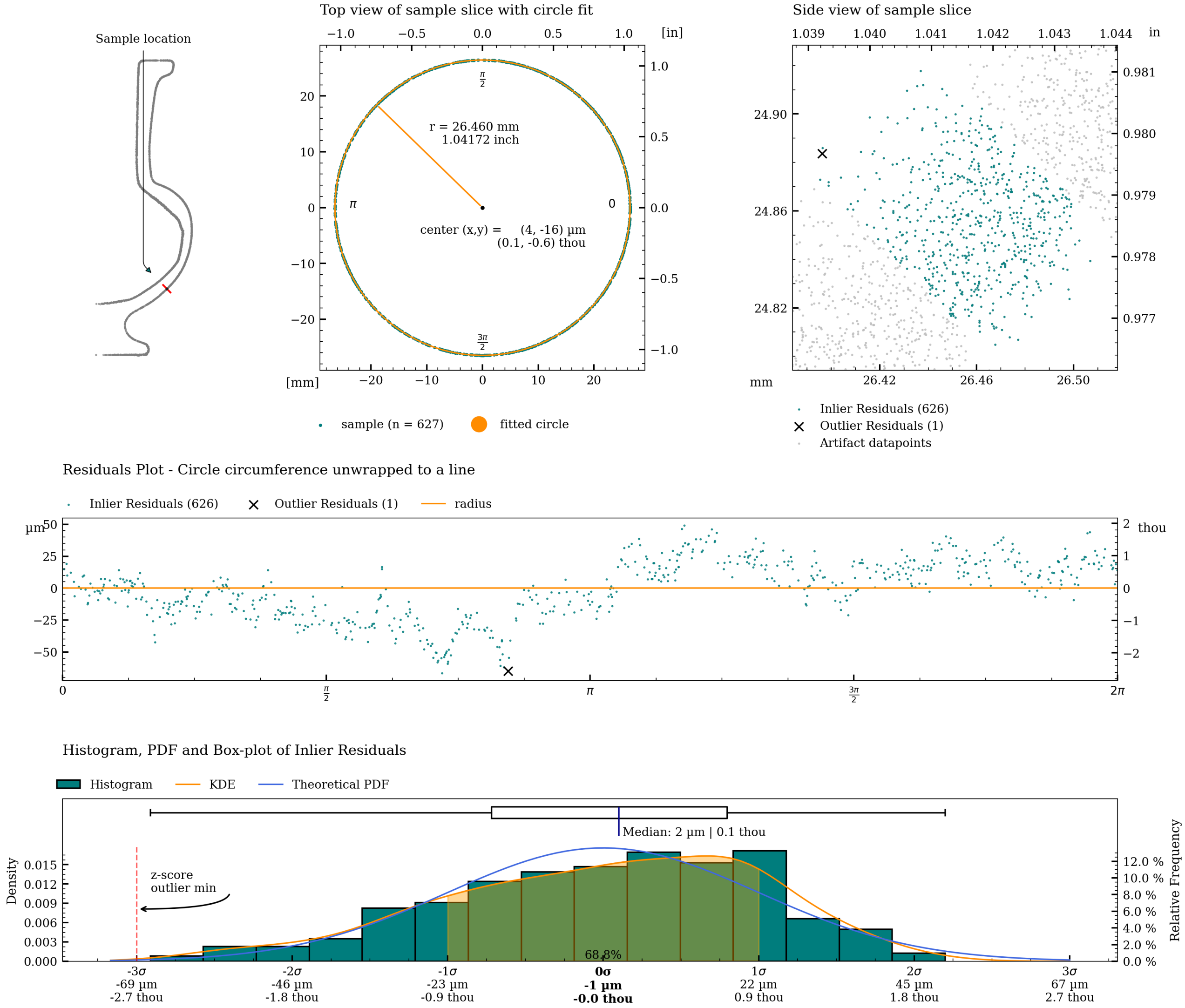


Figure 43: Detailed plot of concentricity measurement for c05.

Concentricity analysis of c06

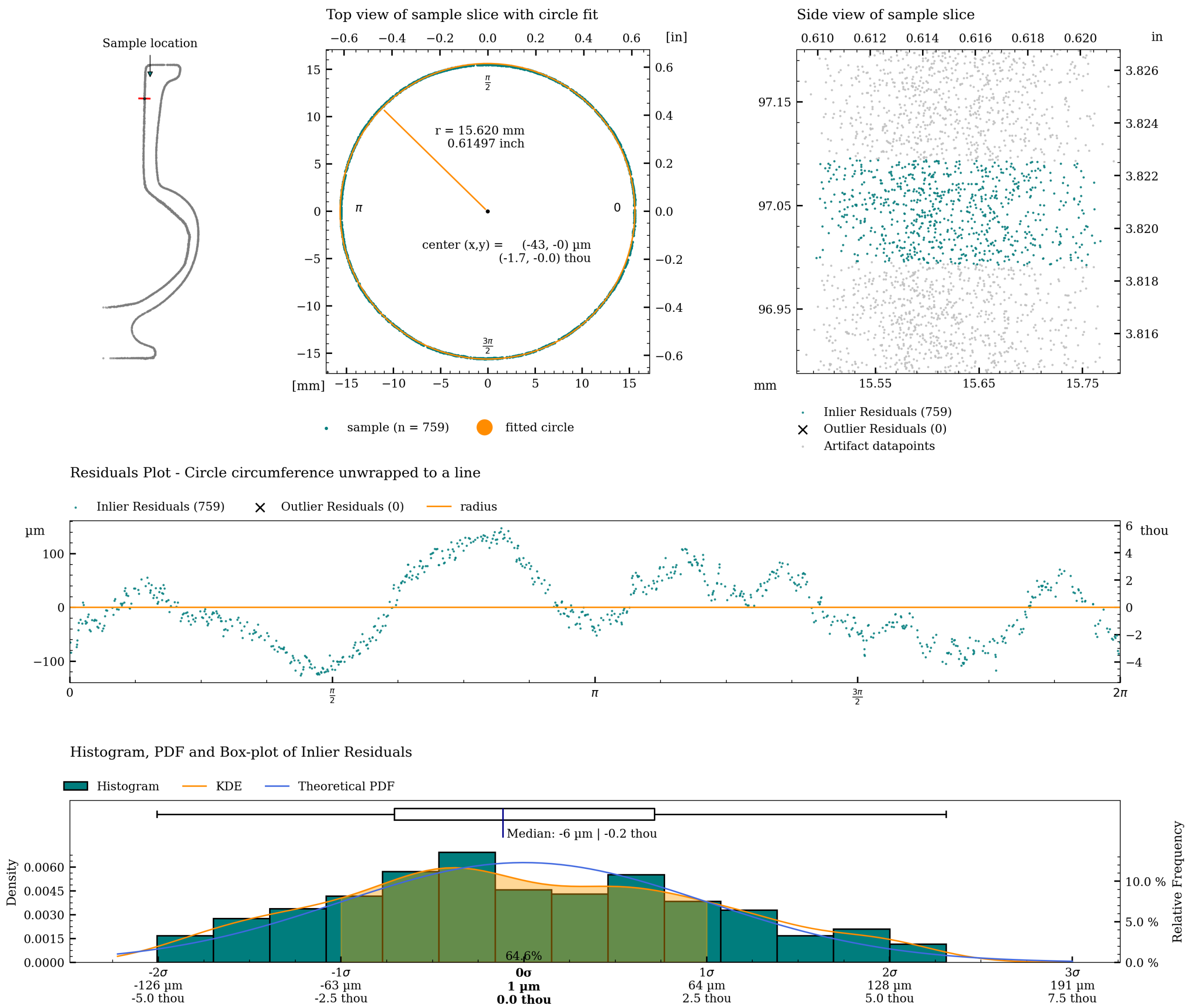


Figure 44: Detailed plot of concentricity measurement for c06.

Concentricity analysis of c06_s

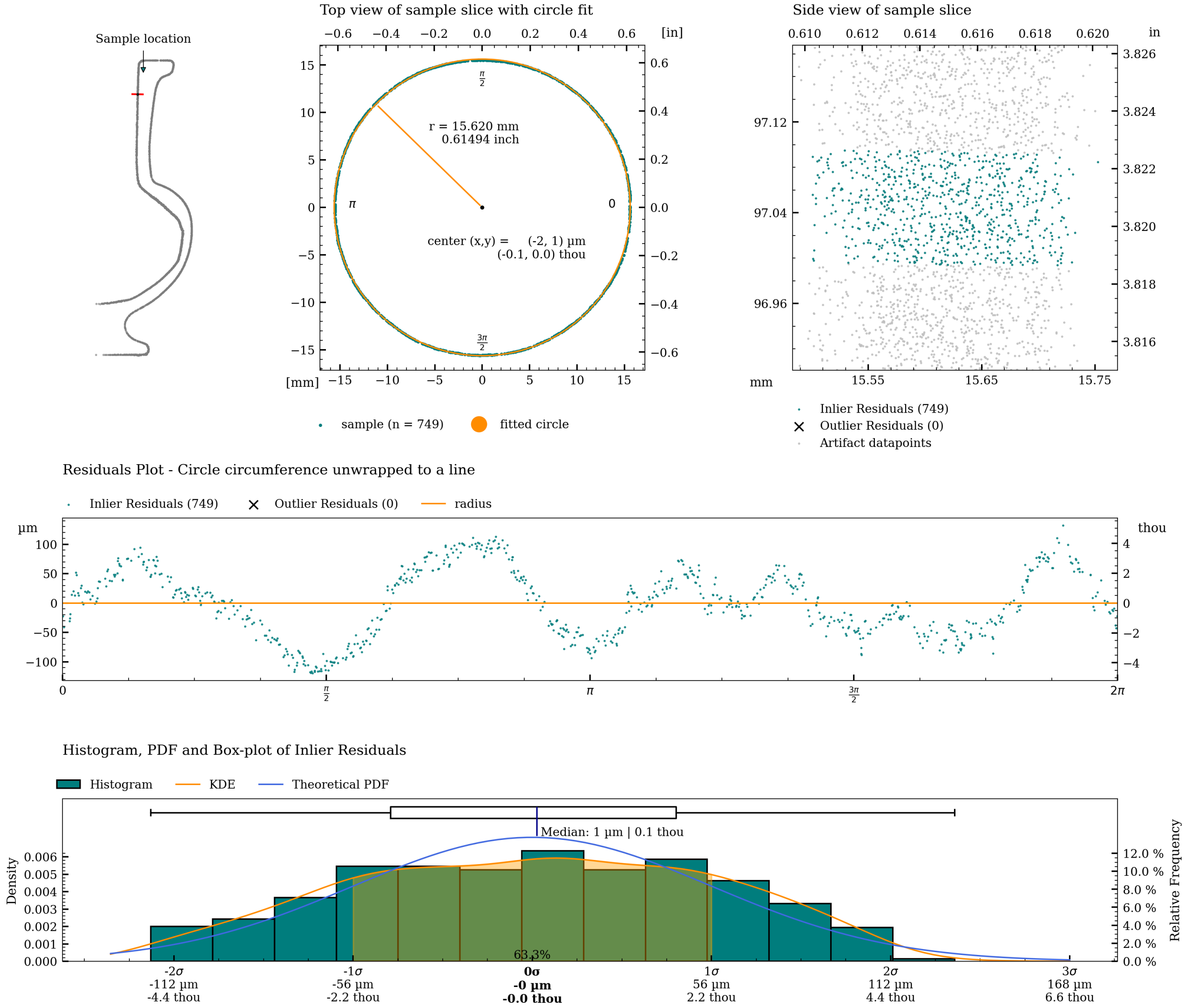


Figure 45: Detailed plot of concentricity measurement for c06_s.

Concentricity analysis of c07

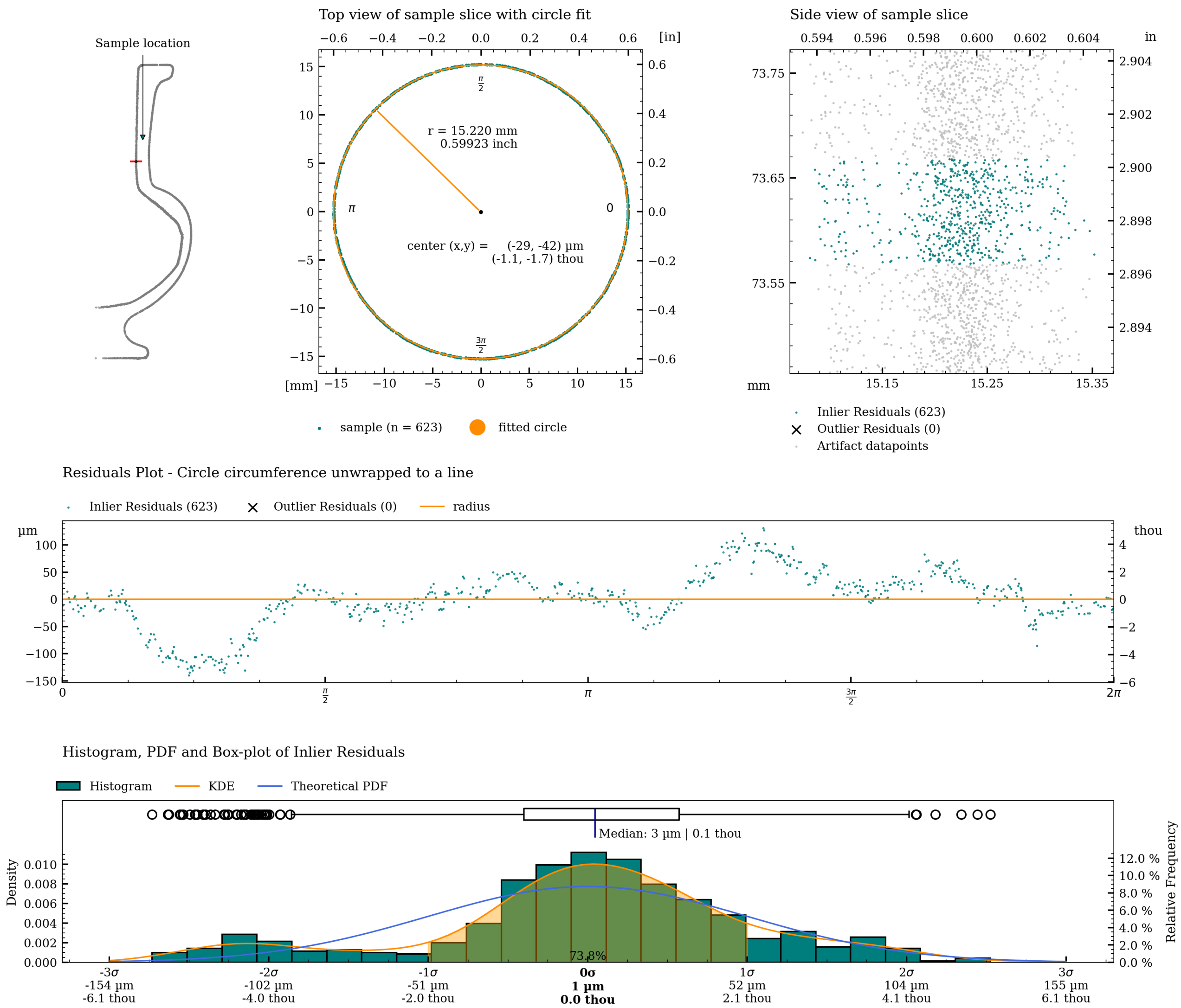


Figure 46: Detailed plot of concentricity measurement for c07.

Concentricity analysis of c07_s

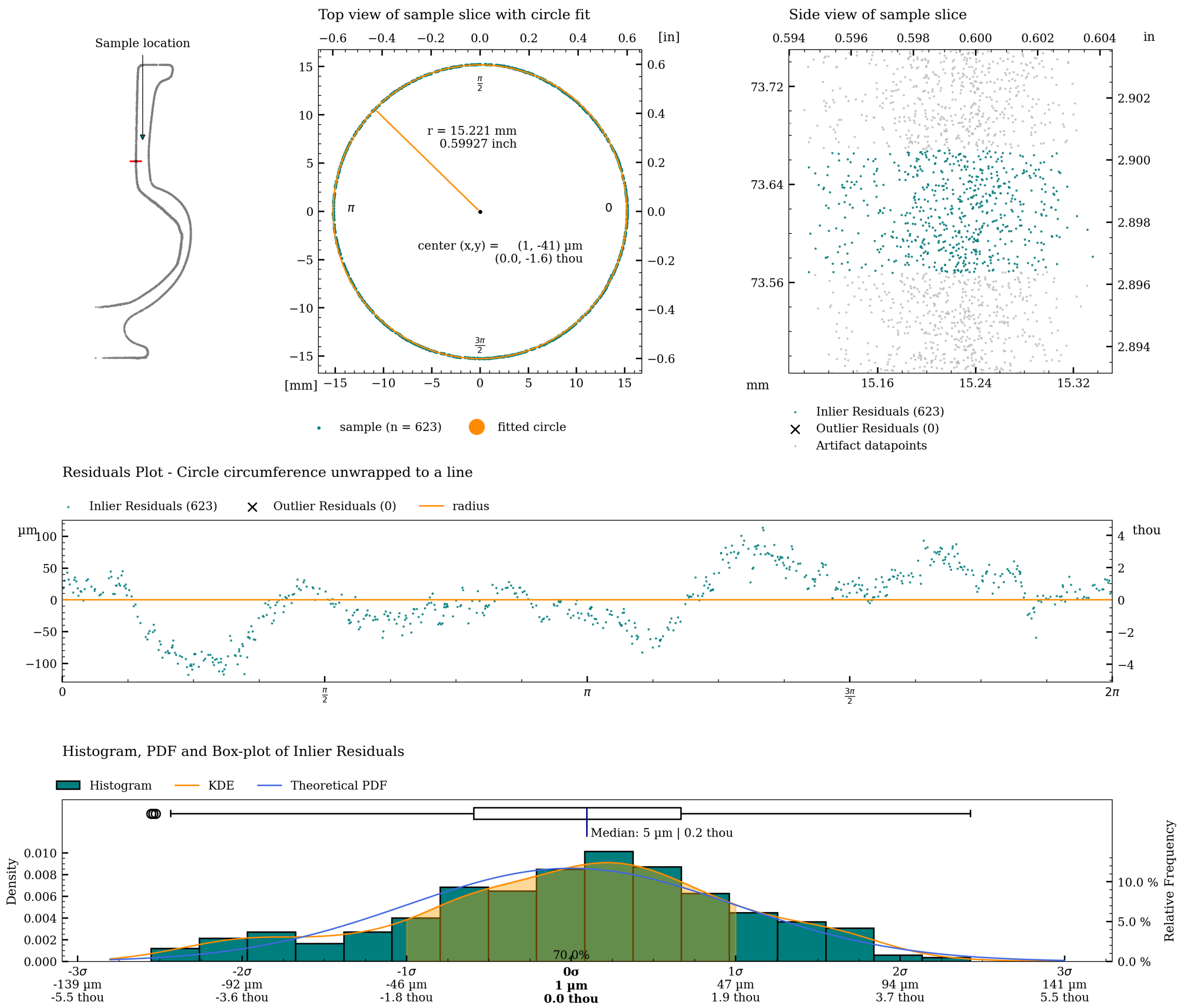


Figure 47: Detailed plot of concentricity measurement for c07_s.

Concentricity analysis of c08

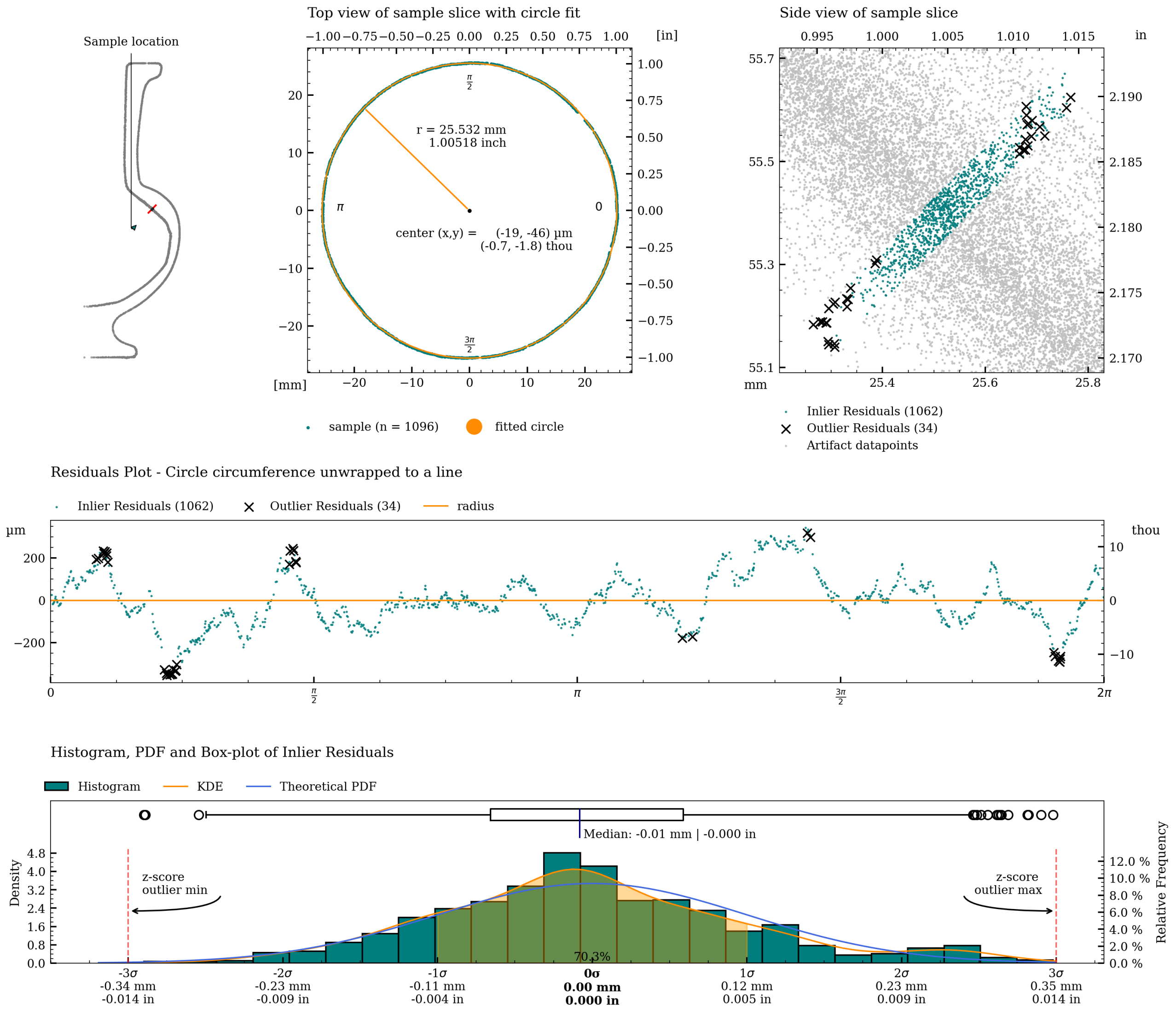


Figure 48: Detailed plot of concentricity measurement for c08.

Concentricity analysis of c08_s

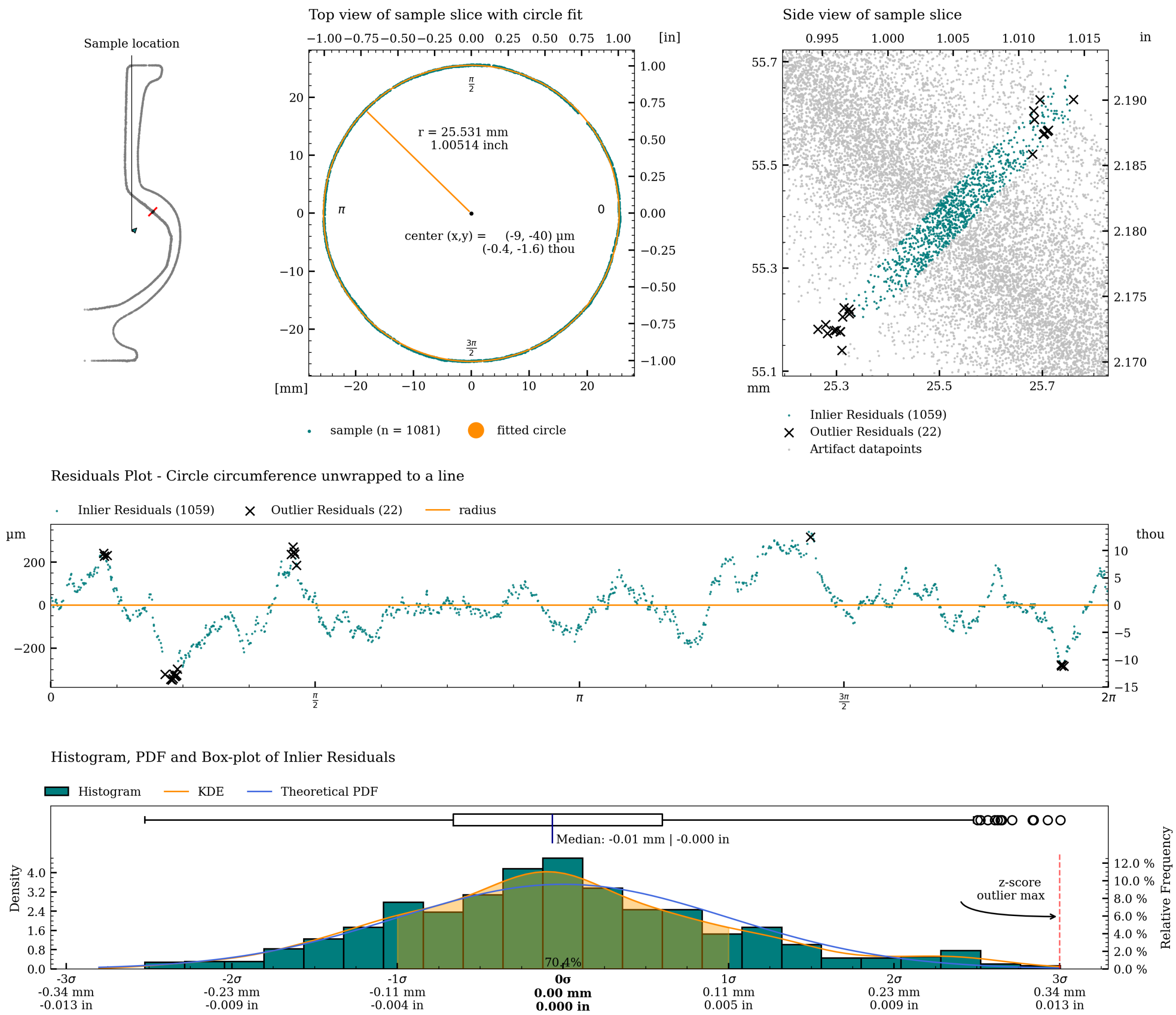


Figure 49: Detailed plot of concentricity measurement for c08_s.

Concentricity analysis of c09

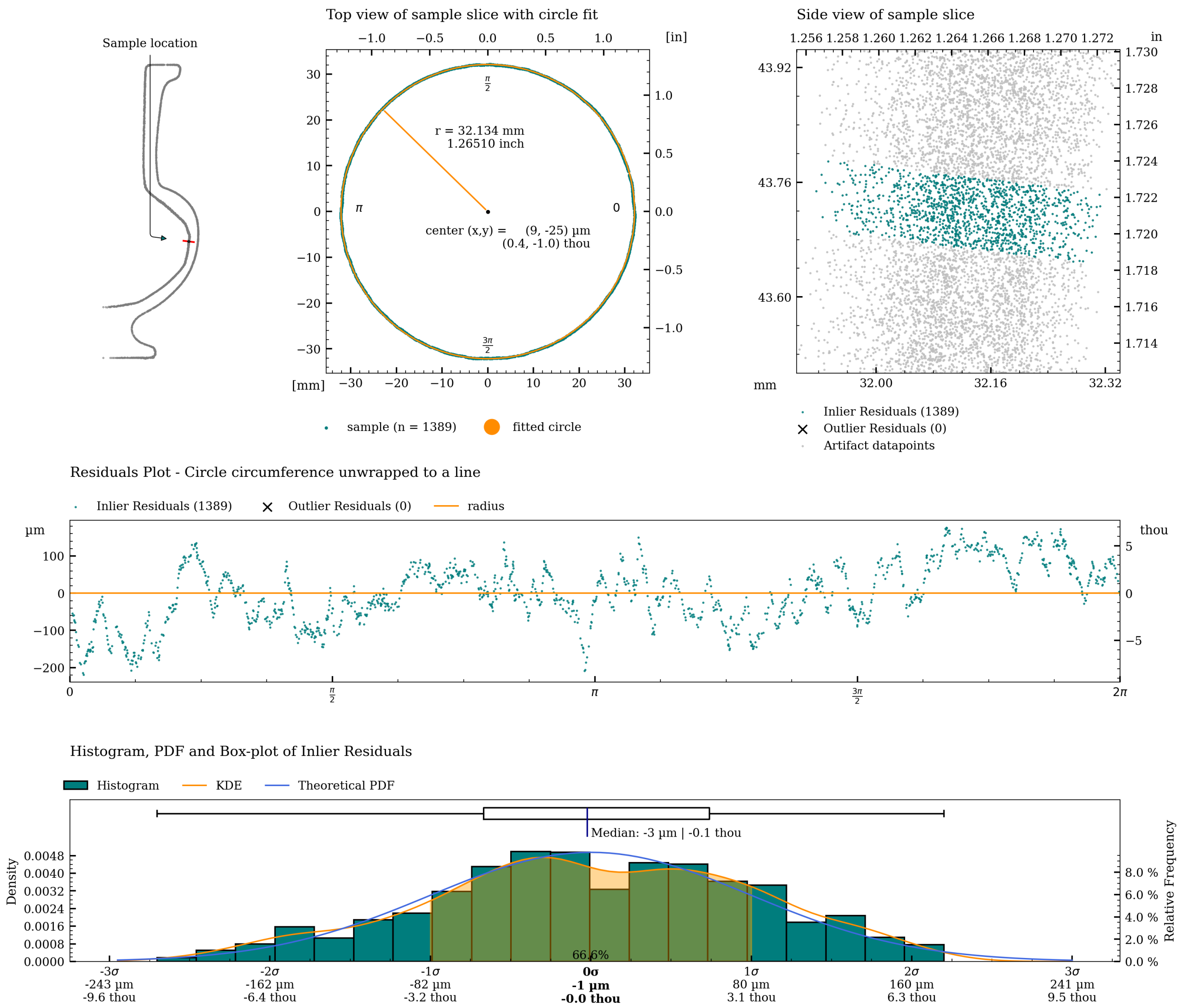


Figure 50: Detailed plot of concentricity measurement for c09.

Concentricity analysis of c09_s

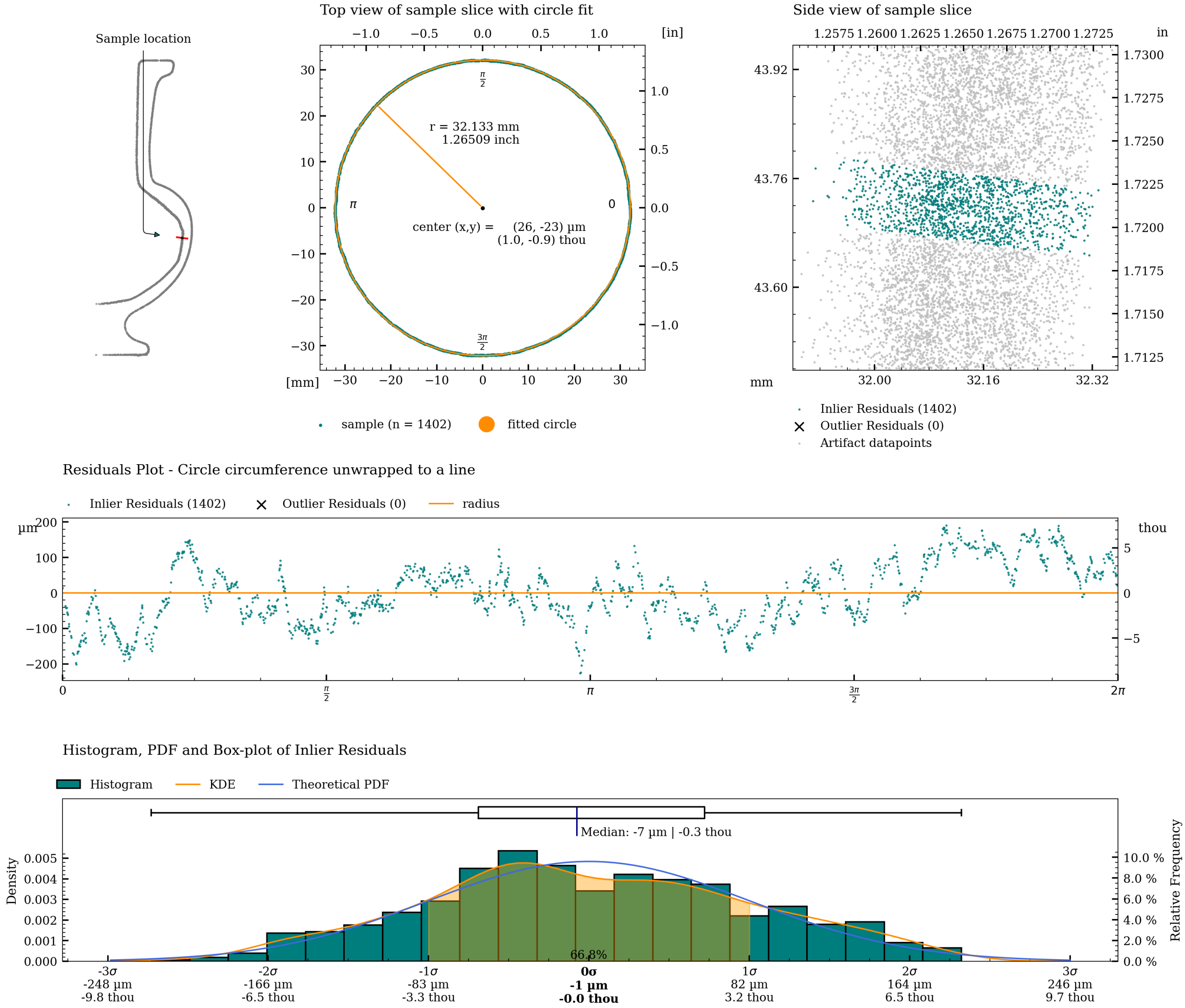


Figure 51: Detailed plot of concentricity measurement for c09_s.

Concentricity analysis of c10

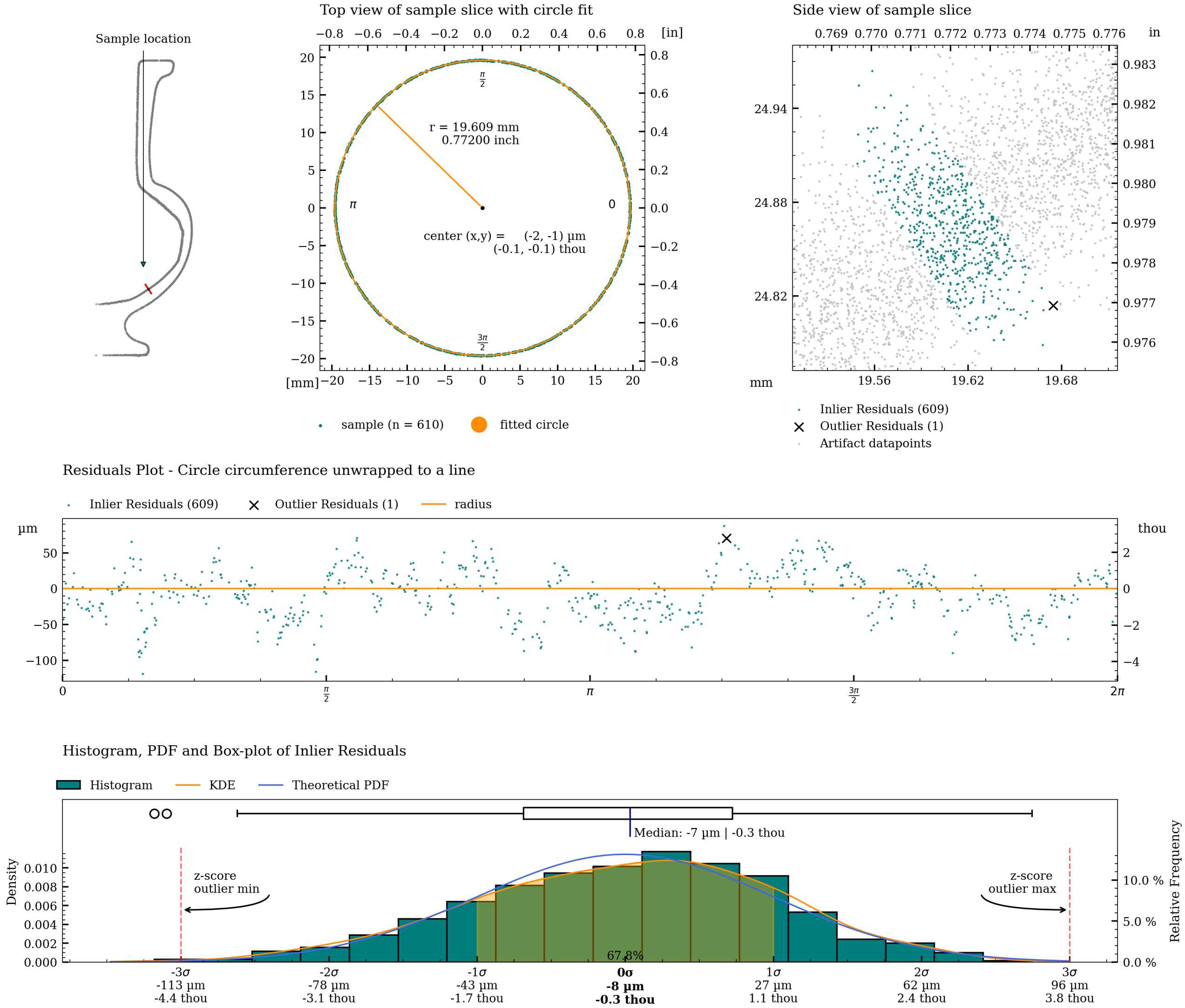


Figure 52: Detailed plot of concentricity measurement for c10.

Concentricity analysis of c10_s

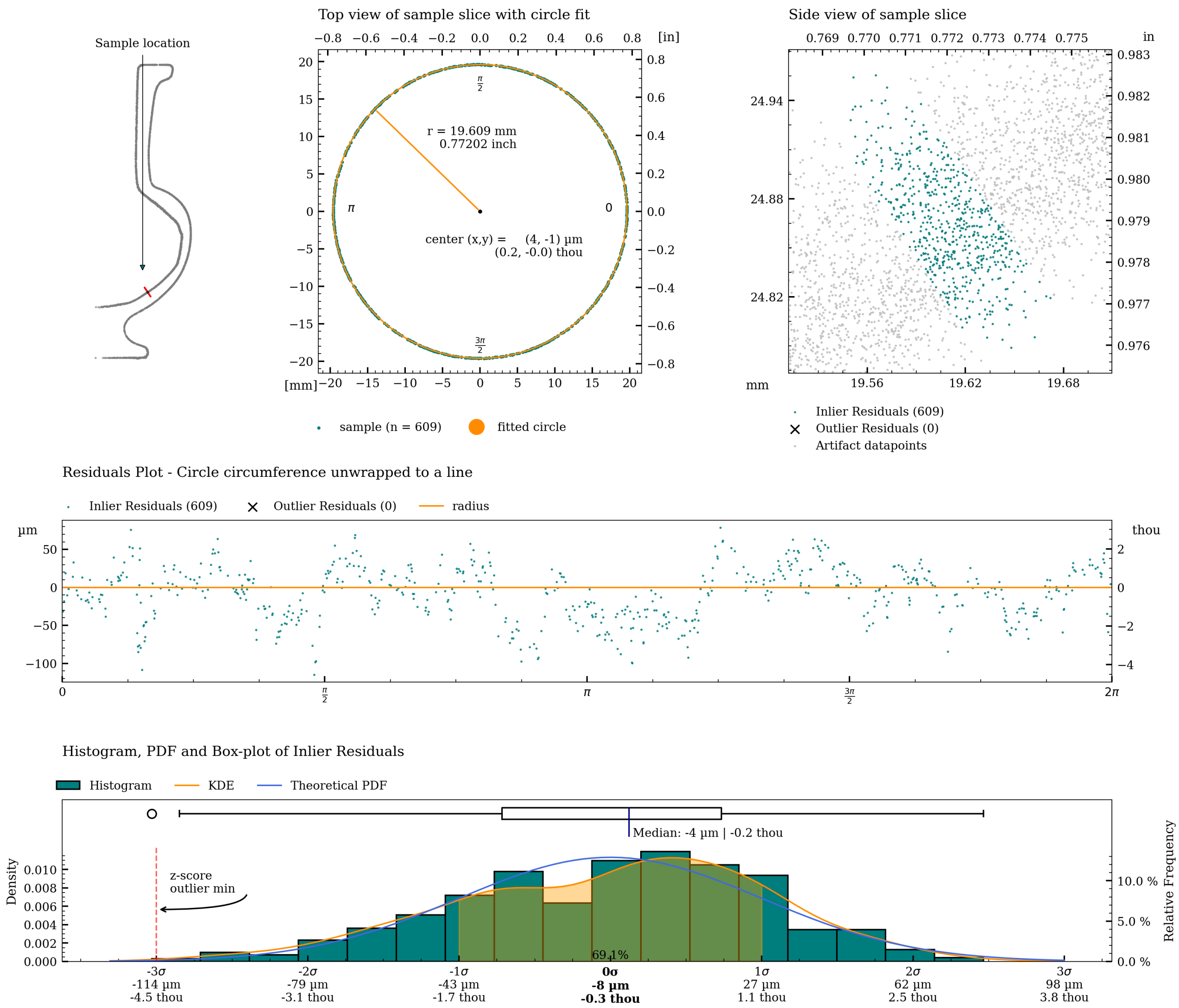


Figure 53: Detailed plot of concentricity measurement for c10_s.

Coaxiality

Coaxiality refers to the straightness and consistency of a central line running through the center of the vase. It measures how aligned the core of the vase remains along its vertical axis.

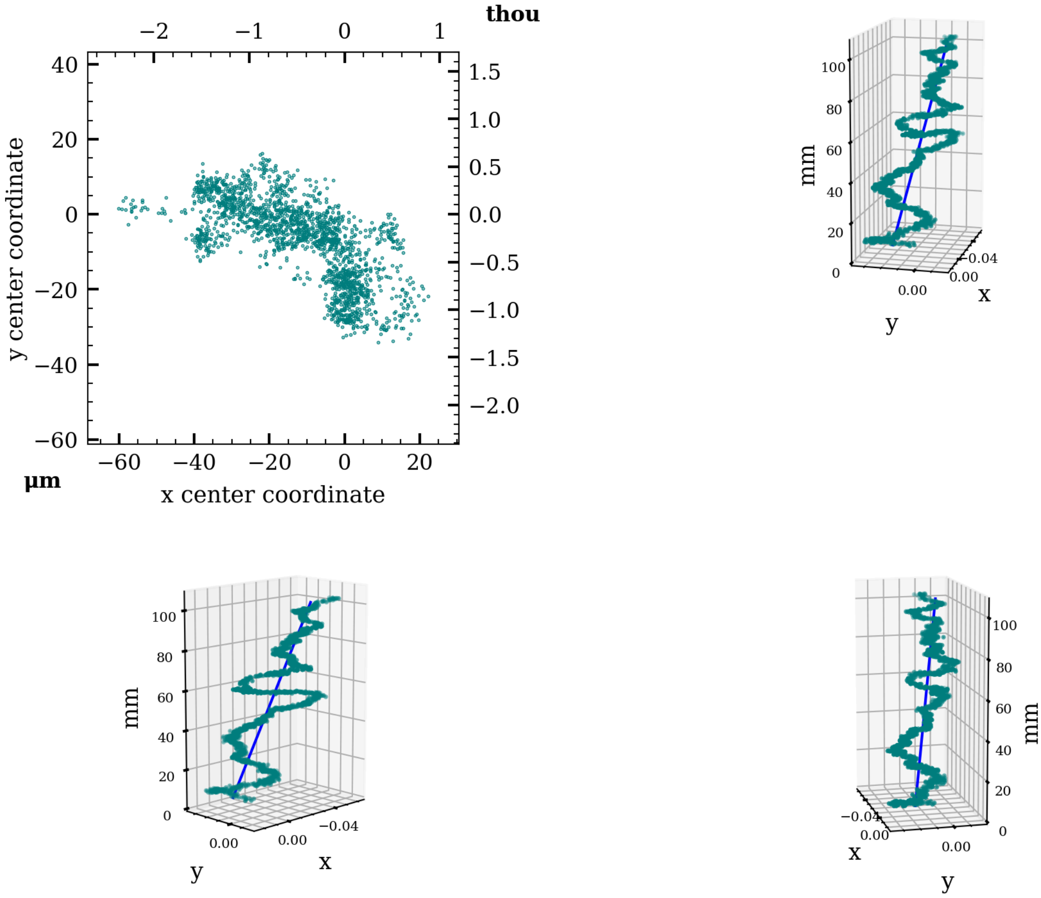
The coaxiality measurements are calculated using RANSAC (Random sample consensus) algorithm for outlier detection on least squares circle regression on cross-sections of the vessel (excluding potential handles), to estimate the best fit circle centers for each slice of the vessel. A best-fit line connects these centers, showing whether the vessels's shape twists or remains straight. This concept helps describe the symmetry and structural uniformity in a visual and analytical way.

Coaxiality is measured for:
- The exterior surface (excluding handles)
- The interior surface

	Exterior		Interior		Interior separate	
Analyzed Slices		1941		1742		1741
Median sample size		332		397		399
Slice Height	50 µm	2.0 thou	50 µm	2.0 thou	50 µm	2.0 thou
Statistics with Z-axis as Reference						
Median Absolute Deviation (MAD)	20 µm	0.8 thou	42 µm	1.6 thou	28 µm	1.1 thou
Standard Deviation (SD)	10 µm	0.4 thou	18 µm	0.7 thou	20 µm	0.8 thou
Root Mean Square Deviation (RMSD)	24 µm	0.9 thou	43 µm	1.7 thou	36 µm	1.4 thou
Statistics with Best Fit Central Axis as Reference						
Best fit Central Axis Equation (in metric coordinate system with unit [mm])	x = 0.011 + t-0.00044 y = -0.020 + t0.00024 z = 0.000 + t1.00000		x = 0.011 + t-0.00046 y = -0.025 + t0.00026 z = 0.000 + t1.00000		x = 0.001 + t-0.00005 y = -0.024 + t-0.00027 z = 0.000 + t-1.00000	
Axis tilt		−0.025°		−0.026°		−0.003°
Median Absolute Deviation (MAD)	10 µm	0.4 thou	28 µm	1.1 thou	27 µm	1.1 thou
Standard Deviation (SD)	7 µm	0.3 thou	18 µm	0.7 thou	17 µm	0.7 thou
Root Mean Square Deviation (RMSD)	13 µm	0.5 thou	35 µm	1.4 thou	35 µm	1.4 thou

Table 4: Coaxiality analysis of vessel PV003.

Coaxiality plots, exterior surface



Coaxiality residuals from fitted axis, exterior surface

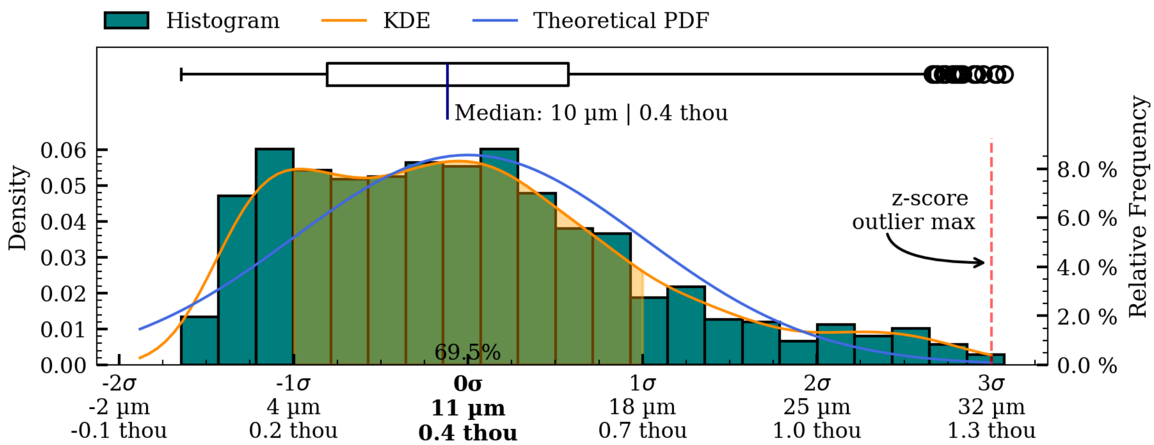
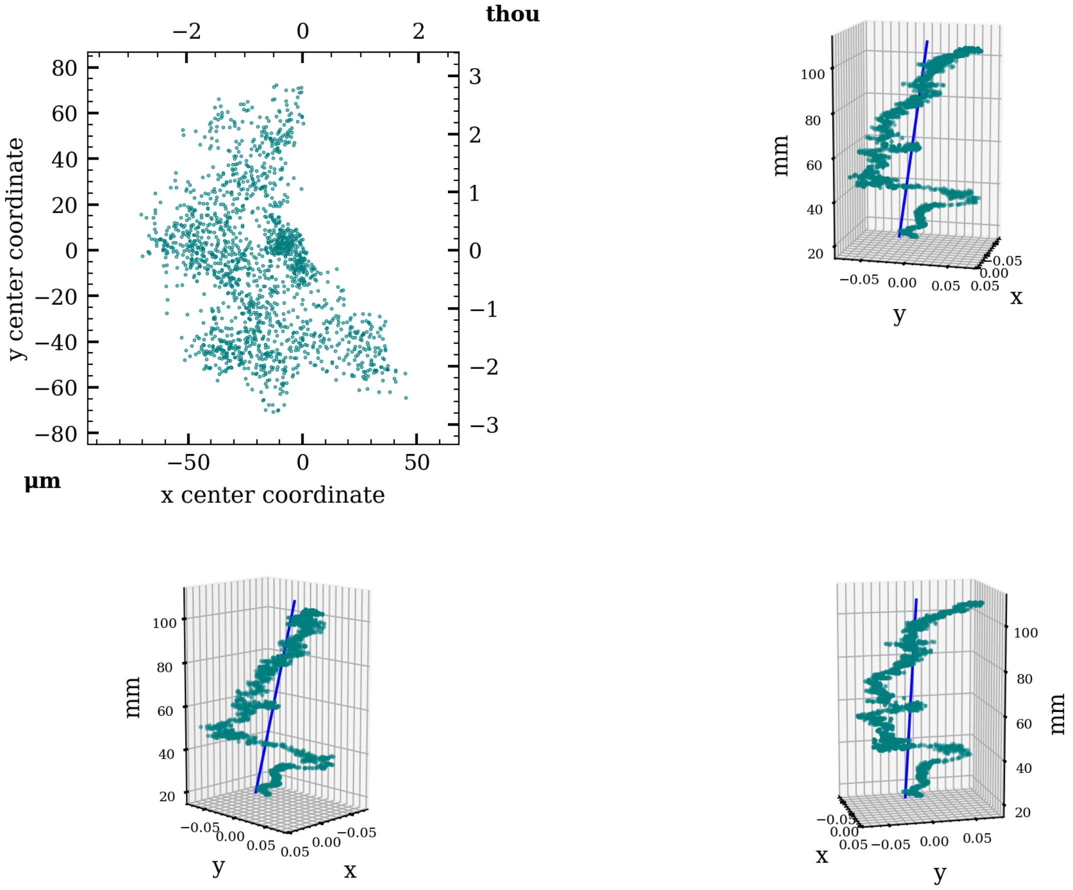


Figure 54: Coaxiality residual plots of exterior surface, PV003.

Coaxiality plots, interior surface



Coaxiality residuals from fitted axis, interior surface

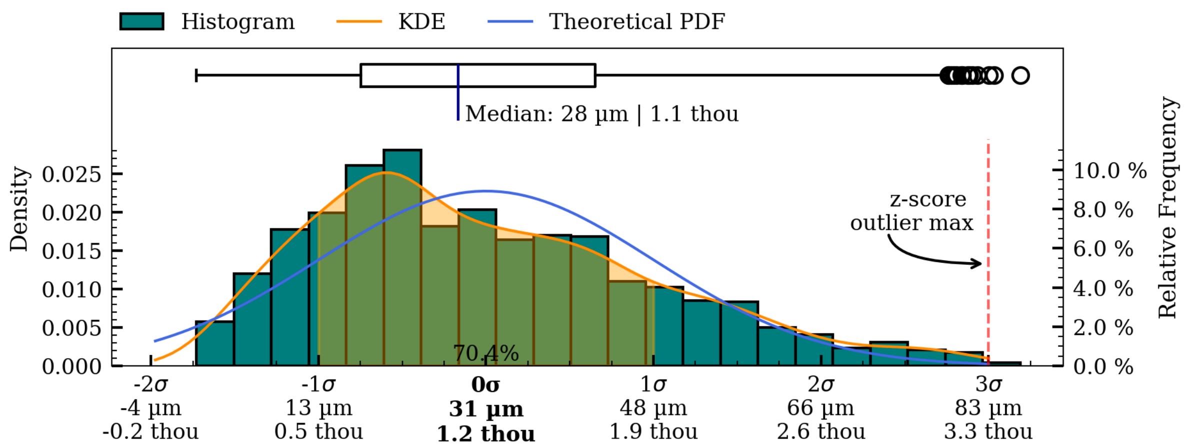
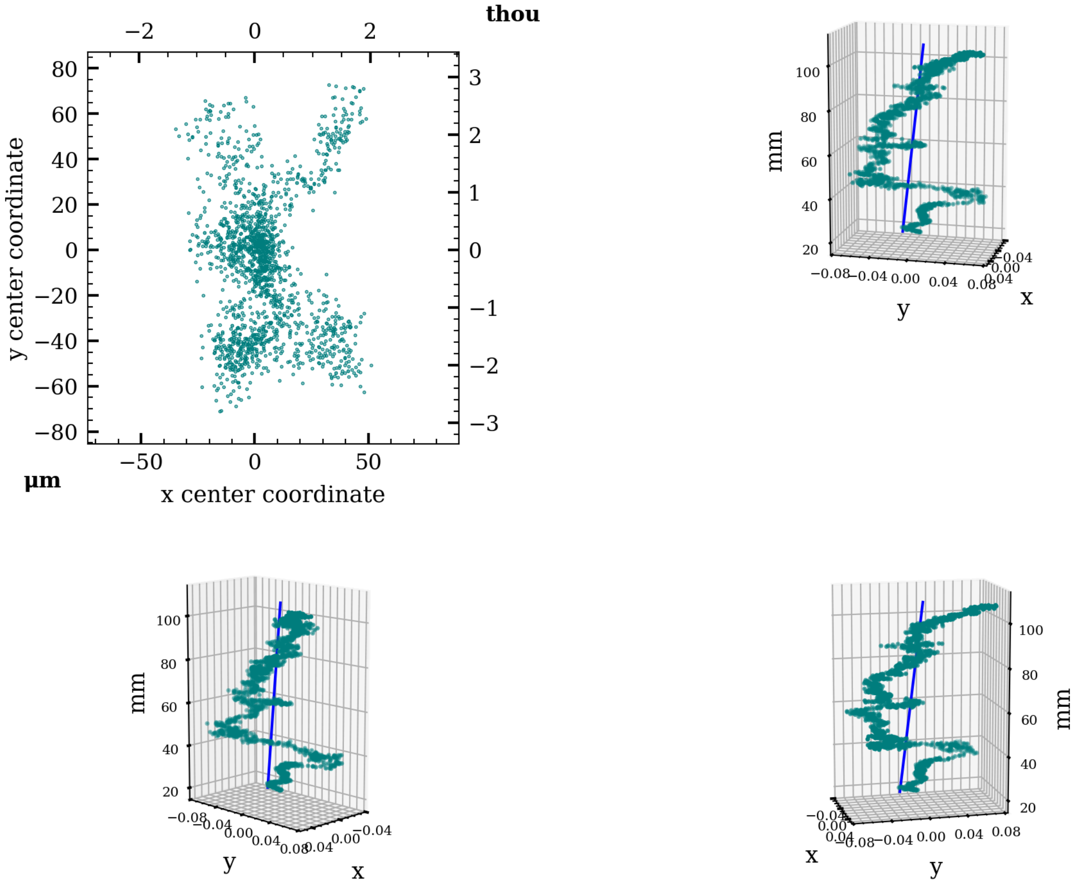


Figure 55: Coaxiality residual plots of interior surface, PV003.

Coaxiality plots, interior separately aligned surface



Coaxiality residuals from fitted axis, interior separately aligned surface

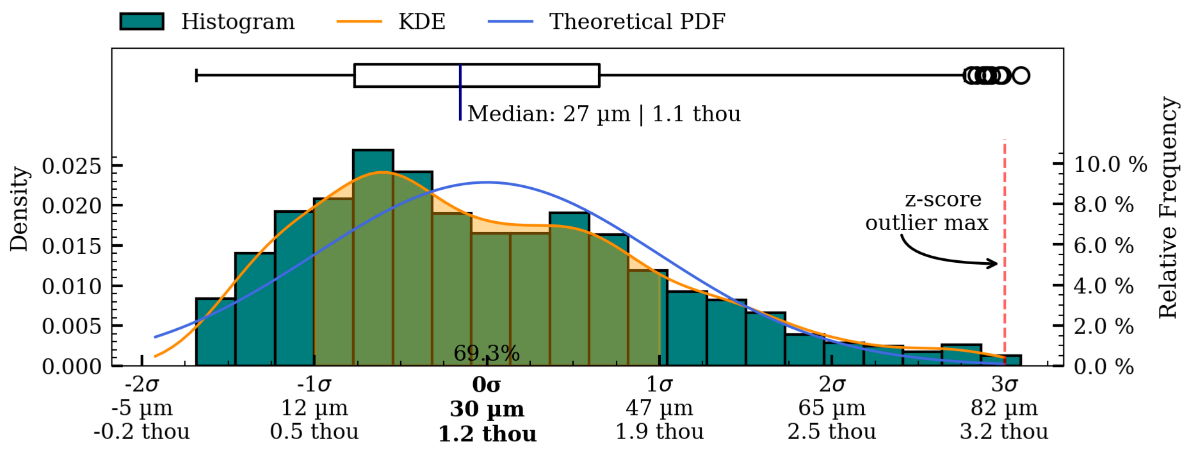


Figure 56: Coaxiality residual plots of interior_separate surface, PV003.

Surface Variability

To illustrate the overall surface deviations of the object, a surface variability heatmap has been created. This heatmap provides an accessible overview of the topography of the manufacturing precision and surface structure of the object.

When CT scanning hard-stone objects, their internal crystalline structure will be captured by the scanner. This can result in strange-looking structures inside the walls of the scan. These structures have not all been filtered from the dataset, and can therefore be seen on the interior heatmap and will result in an increased total range of the surface deviation statistics.

The surface variability measurements are created by fitting a number of higher-order polynomials to the two-dimensional folded profile of the scan data. This process creates an idealized mathematical representation of actual surface curvature of object, and as such provides a continuous model representation of the actual object. It is important to note that only such a non-discretized representation is sufficient to avoid introducing inconsistently varying errors in the mapping of the final surface deviation results, that the rendered heatmaps are based on.

To produce the final surface variability map, the distance from each scanned vertex to the fitted polynomial is calculated and used as the mapping function input, for applying colours to the surface of the object.

It is important to note that this variability map does not describe deviations from the original *intended* shape of the artifact (if any), as this shape (the *intended design*, so to speak) will have been lost to time. It does however provide a very informative visualization of the texture and structure of the surface and very importantly, *does* hightlight potential manufacturing-relevant patterns in the surface texture (if present). Such patterns are, as an example, clearly evident on the interior surface of artifact PV001.

Exterior surface

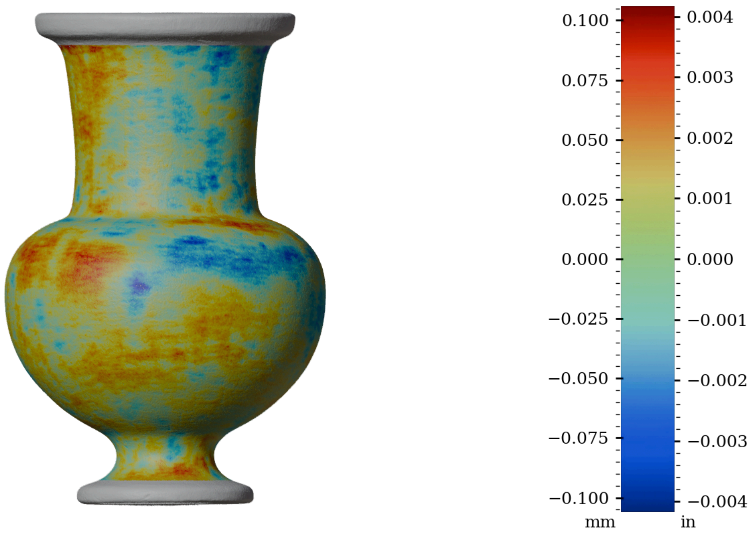


Figure 57: Surface variability heatmap of PV003, front view

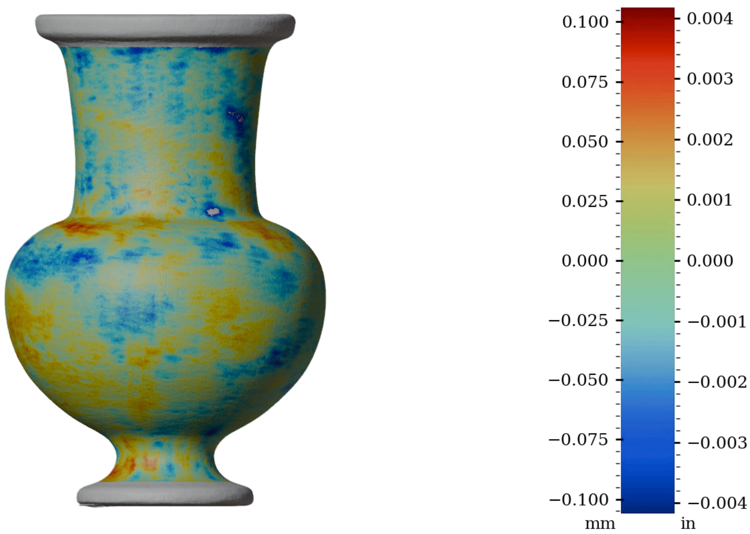


Figure 58: Surface variability heatmap of PV003, rotated 90°

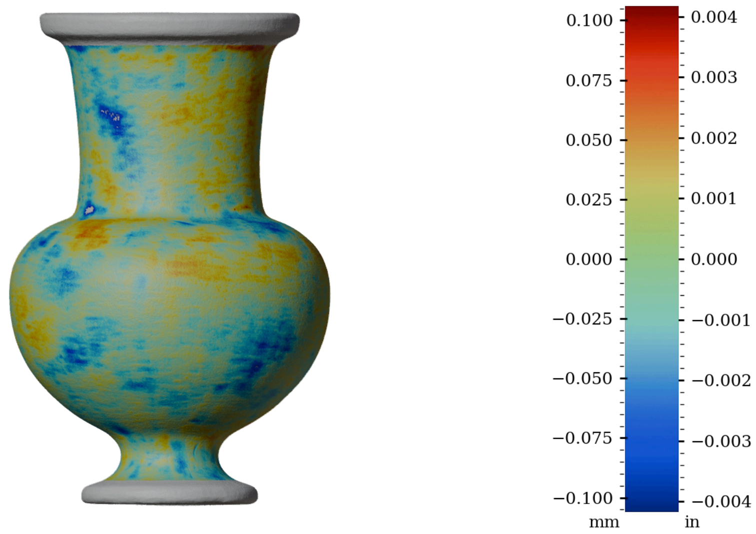


Figure 59: Surface variability heatmap of PV003, rotated 180°

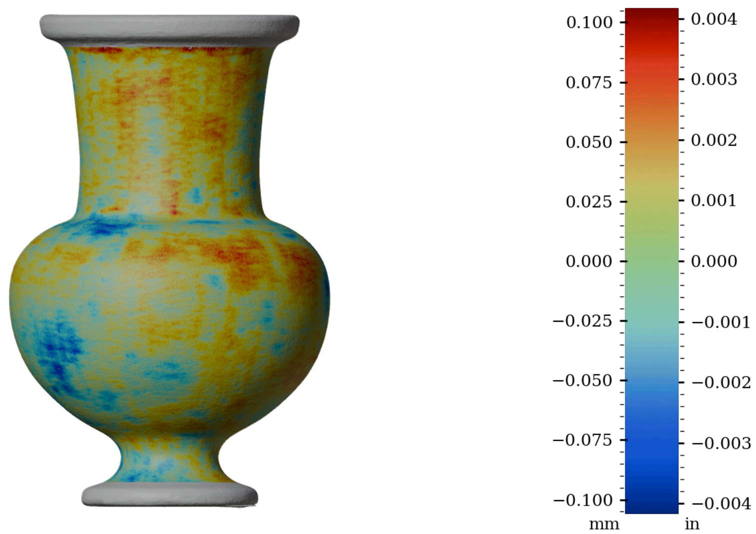


Figure 60: Surface variability heatmap of PV003, rotated 270°

Interior surface

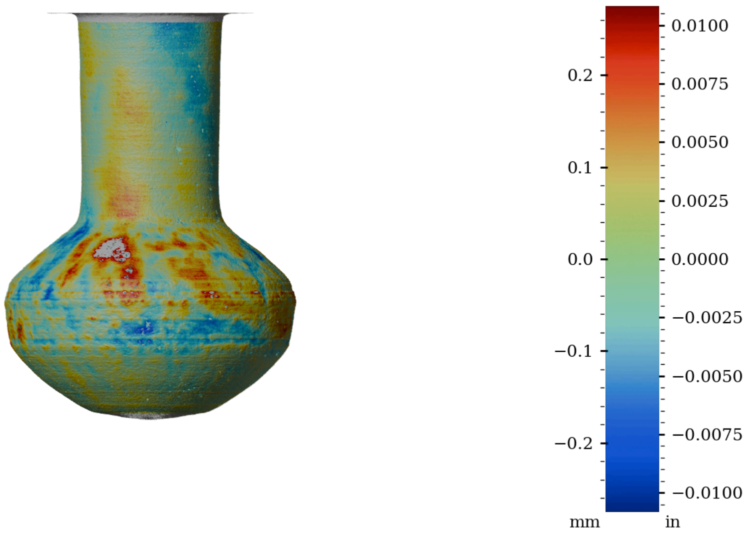


Figure 61: Surface variability heatmap of PV003, front view

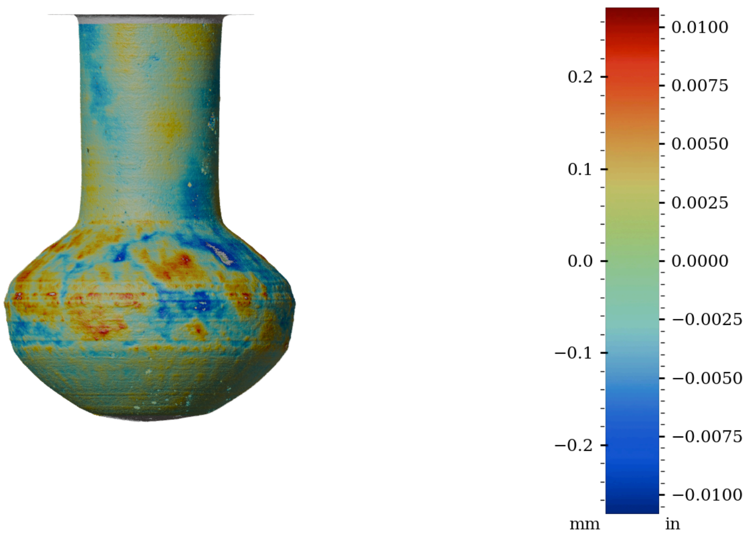


Figure 62: Surface variability heatmap of PV003, rotated 90°

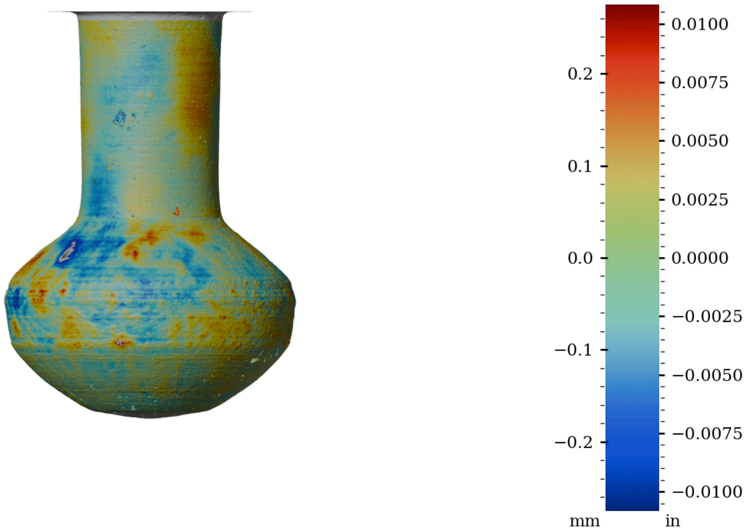


Figure 63: Surface variability heatmap of PV003, rotated 180°

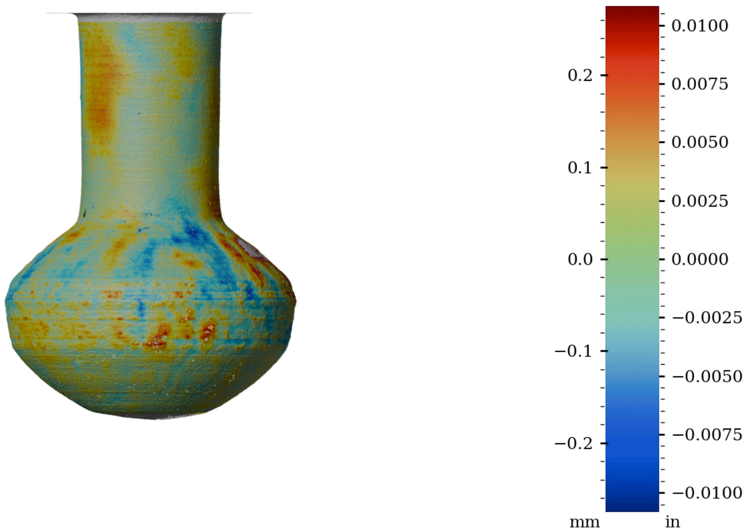


Figure 64: Surface variability heatmap of PV003, rotated 270°

Interior surface aligned separately

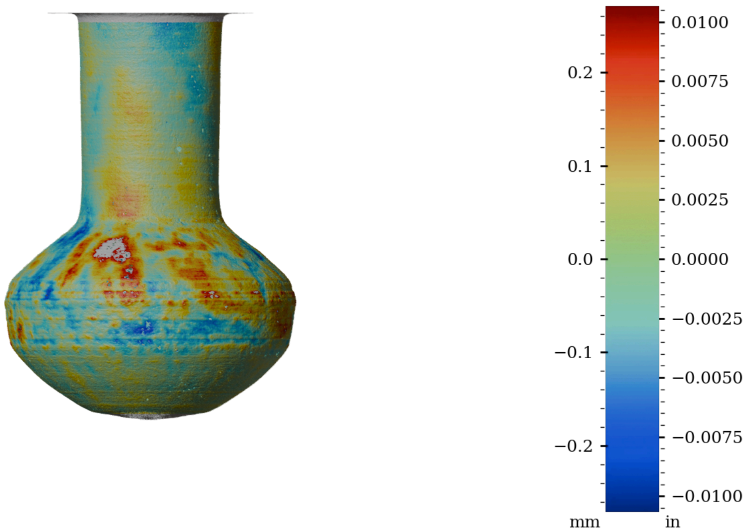


Figure 65: Surface variability heatmap of PV003, front view

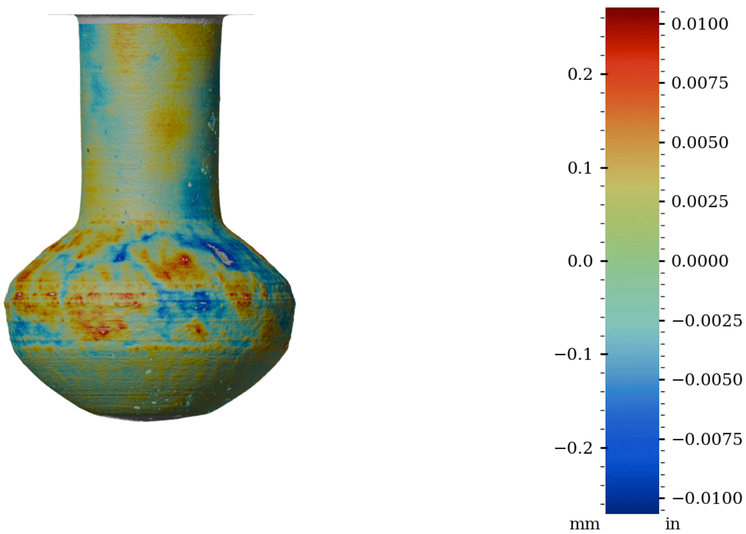


Figure 66: Surface variability heatmap of PV003, rotated 90°

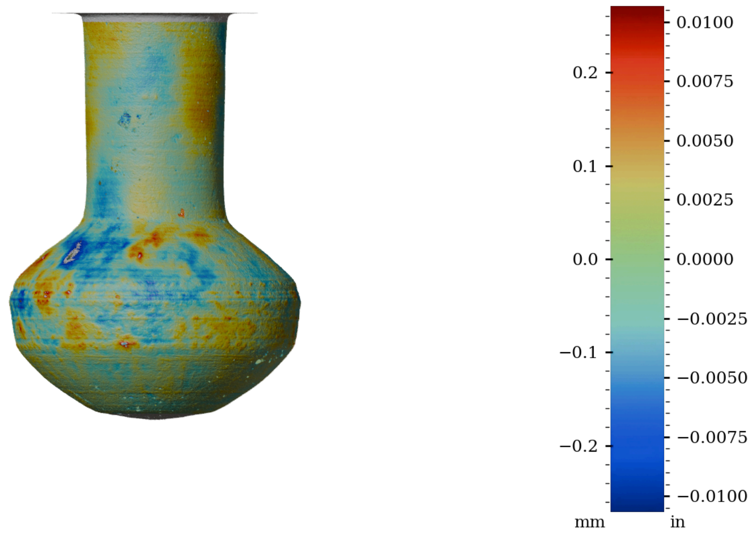


Figure 67: Surface variability heatmap of PV003, rotated 180°

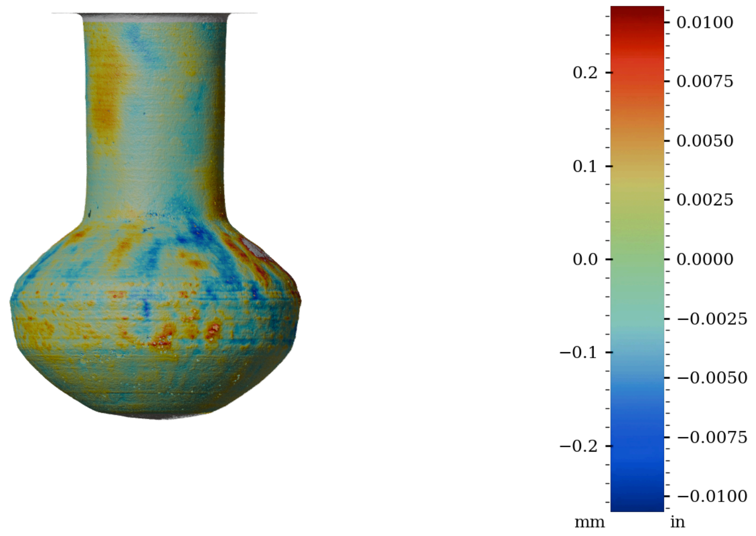


Figure 68: Surface variability heatmap of PV003, rotated 270°

Surface variability statistics

Area	MSD	RMSD	SD	Median AD	Range	Min	Max	Sample size
	mm^2	mm	mm	mm	mm	mm	mm	
Exterior	0.0009	0.030	0.018	0.012	0.273	−0.148	0.124	926863
Interior	0.0061	0.078	0.052	0.029	0.869	−0.456	0.412	1106960
Interior separate	0.0060	0.077	0.051	0.028	0.881	−0.472	0.410	1106973
	in^2	in	in	in	in	in	in	
Exterior	0.000001	0.0012	0.0007	0.0005	0.0107	−0.0058	0.0049	926863
Interior	0.000010	0.0031	0.0020	0.0011	0.0342	−0.0180	0.0162	1106960
Interior separate	0.000009	0.0030	0.0020	0.0011	0.0347	−0.0186	0.0161	1106973

Table 5: Surface variability statistics, PV003

Table 5 shows the statistics of the distance from the scan vertices to the best fit object model. These statistics are briefly explained below.

Histogram, KDE and Box-plot of measured surface variability - exterior surface

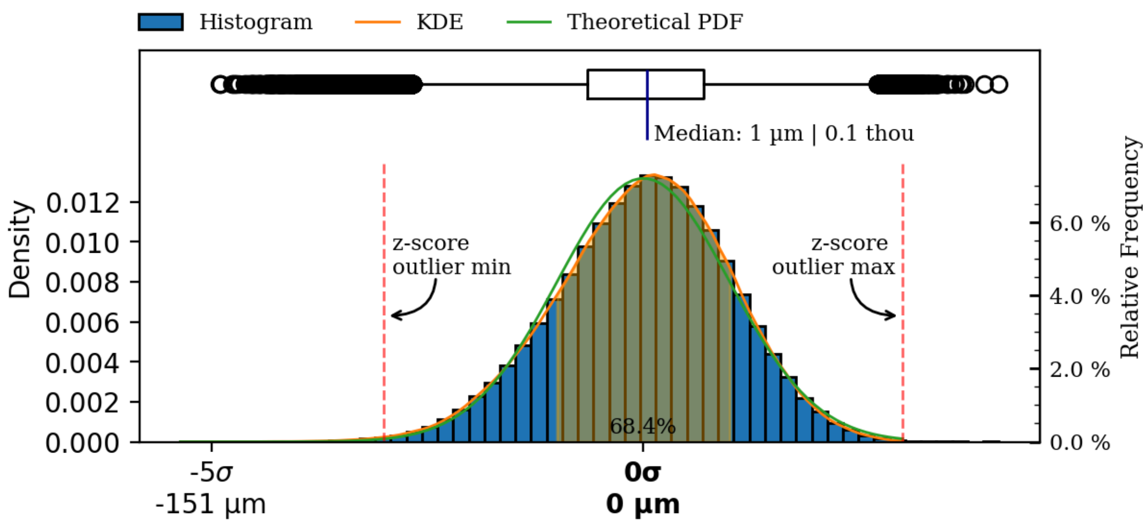


Figure 69: Exterior surface variability boxplot, kds and histogram.

Histogram, KDE and Box-plot of measured surface variability - interior surface

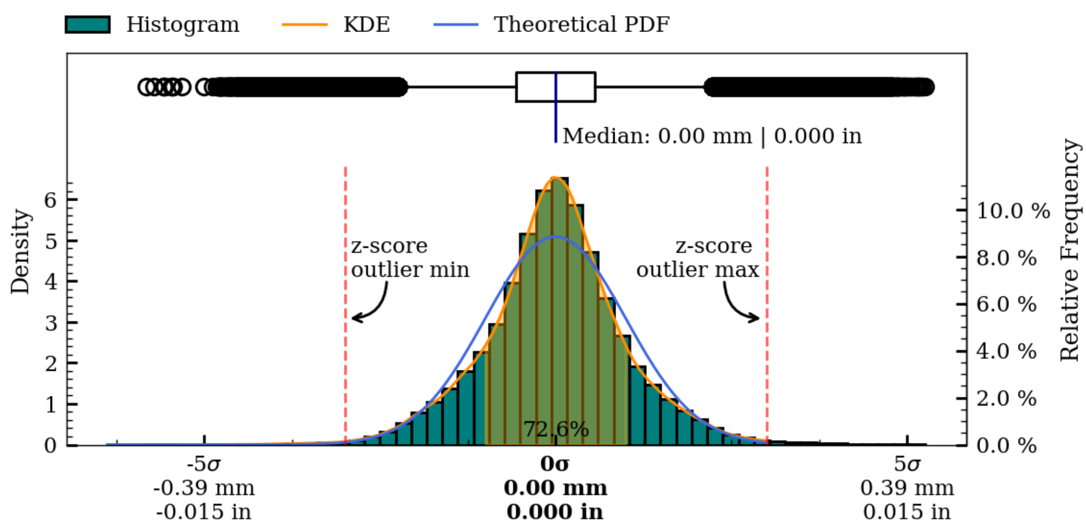


Figure 70: Interior surface variability boxplot, kds and histogram.

Histogram, KDE and Box-plot of measured surface variability - interior separately aligned surface

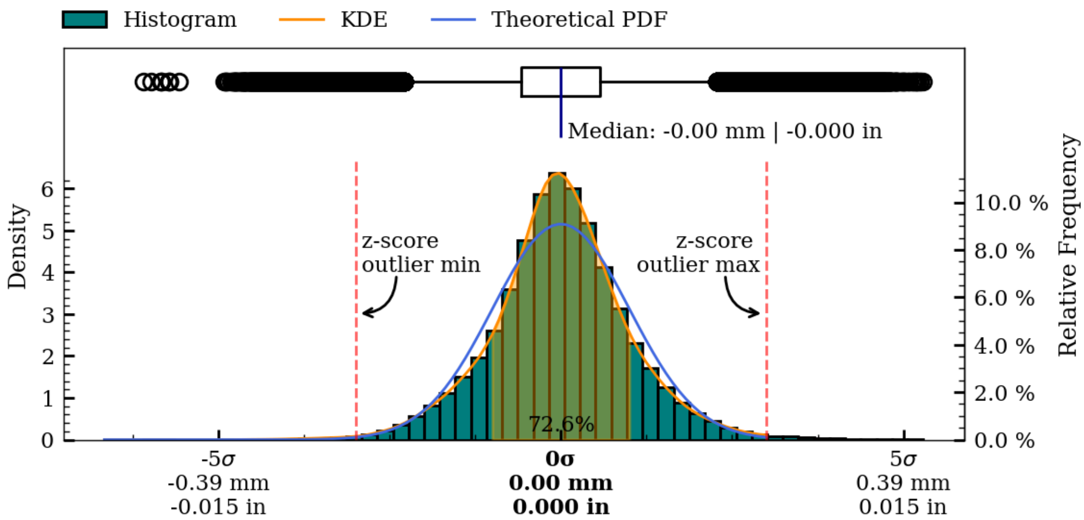


Figure 71: Interior separately aligned surface variability boxplot, kds and histogram.

Precision Score Of The Artifact

To enable valid comparison of the manufacturing precision of different artifacts, a metric that robustly quantifies the overall precision of the object is required. The considerations for such a metric will be explored in this section.

Based on these considerations, a *Precision Score* metric will be defined.

For an object to be described as having been manufactured with high precision, several qualities must be present *concurrently*, and throughout the *entire* geometry of the final object. A given object may exhibit high levels of one or more *components* of precision, but be lacking in others. For example:

- An object may present high levels of coaxialility, but lack circularity.
- An object may exhibit good circularity, but show imperfections in the surface structure.
- An object may be smoothed to perfection *without* any circularity or coaxiality.
- An object may exhibit high levels of all of the above metrics in *some* areas, but not in others.

Therefore, a precision score metric **must** account for *all* aspects of the individual, underlying precision metrics (circularity, concentricity, coaxiality and surface variability) throughout the *entire* surface area of the object.

The composite high order polynomial model, used to generate the surface variability map (described in Surface Variability, p. 59) is the best continuous mathematical representation of the object available to us (lacking any original design plans, as would normally be available in metrological analysis). This idealized model encompasses all of the above component metrics.

In the creation of the model, all scan data-points are taken into account (excluding areas with extensive damage), making it the best possible idealized representation we can achieve. When this model has been accurately created, the deviation between the model and the scanned data-points can be calculated over the non-discretized polynomials, *without* the need for an "original" CAD model (and importantly, unless such a CAD model *actually* corresponded to the original design intent, it would be an insufficient comparison basis).

Within the context of defining a valid, overall precision metric, this approach satisfies the incorporation of all of the necessary metrics:

- **Circularity**: Because the reconstructed polynomial model is revolved around the Z-plane, the idealized representation is perfectly circular, and thus incorporates the circularity component.
- **Concentricity and coaxiality**: Because the Z-axis (datum axis) is the center axis of the model, it incorporates the concentricity and coaxiality components.
- **Surface variability**: Because the model is continuous and non-discretized, it can be used accurately for all points of the scan data, and incorporates the surface variability component.

The level of precision ultimately achieved in a physical object does not share a linear relationship with its manufacturing requirements. Since continuously higher levels of final precision becomes progressively harder to achieve, an overall precision metric must take this relationship into account.

A robust statistical metric that satisfies this requirement is the *Mean Squared Deviation* (MSD or MSE). Here specifically, we can utilize the mean square of the deviations between the model ($\hat{y}$) and the data-points (y_i).

Combining all of the above considerations, we can express a well-defined *Precision Score* metric, that provides an immediately accessible way to understand the overall precision of an object, while being statistically valid. Since the Mean Squared Deviation tends towards zero as the overall precision increases, the inverse of the Mean Squared Deviation is taken to obtain a precision score metric that increases as precision increases[12]:

$$\text{Precision Score} = \frac{n}{\sum_{i=1}^{n} \left(y_i - \hat{y}\right)^2}$$

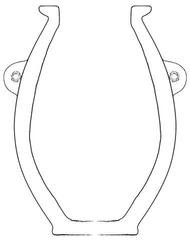
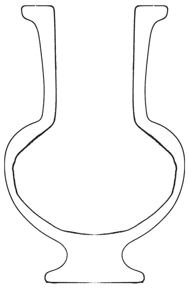
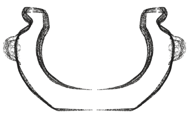
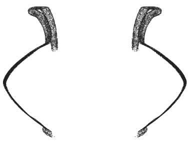
[12]The precision score unit is $\frac{1}{\text{mm}^2}$

The precision score of PV003 have been calculated separately for:

- Precision score, exterior surface: 1092
- Precision score, separately aligned interior surface: 167
- Precision score, interior surface: 163
- Precision score, full surface: 272

The precision score of a Zeiss 1.00000 inch reference sphere have been calculated to 43,943 (RMSE = 0.00477 mm / 0.00010 in). The scan was obtained by Max Fomitchev-Zamilov using a Keyence VL −500 scanner with a rated accuracy of 10 microns. The precision analysis of the reference sphere scan indicates at the maximum possible precision score obtainable.

Table 6 shows the precision score of this artifact (PV003), compared to the two most precise, and the two least precise vessels currently analyzed.

Artifact			**Material**	**Precision Score**	**Link to Report**
		PV001	Red Granite	**1980** Full: 1177 Exterior: 1980 Interior separate: 798 Interior: 722	Report Publication
		PV003		**1092** Full: 272 Exterior: 1092 Interior separate: 167 Interior: 163	Report Publication
		RV003	Marble breccia	**1.46** Full: 1.49 Exterior: 1.46 Interior separate: 1.53 Interior: 0.54	Report Publication
		MV010	Calcite (Egyptian Alabaster)	**1.17** Full: 1.32 Exterior: 1.17 Interior separate: 11 Interior: 0.17	Report Publication

Analysis Roadmap

While the current iteration of this work already provides valuable results, continued future additions and improvements will enhance their utility further. This section details planned iterative updates and improvements, to both the reports themselves, and to the underlying methodology and software they are created with.

Alignment Section

- Detailed exploration of different circle regression algorithms
- If handles are present on the vessel, exploring alignment of the vessels so the handle positions match each other
- Add optimization of the perpendicular surface deviation, with the best results of the coaxial alignment
- Align by minimizing circularity results (of rotated sample slice, to compensate for sample height distortions)

Measurements of Precision

- Section detailing how measurements perpendicular to the surface curvature are obtained
- Detailed surface area analysis, exploring the residual patterns throughout subsequent sample slices of the artifact surface
- Wall thickness deviation color map
- Robust outlier identification on circularity, to better handle analysis of damaged areas of the artifacts in addition to removal of interior crystalline structure points present in CT scans
- Layout updates to the charts and tables

Visibility of Outliers and Damaged Sections

- Identification and marking of damaged parts
- Visualization of outliers on the artifact surface

Exploration of Mathematical Primitives

- Analysis of selected curvatures and flat surfaces on the vessel in both the horizontal and vertical planes
  - Circles
  - Parabolas
  - Ellipsoids
  - Hyperbolas
  - Cones
- Implementation of robust regressions models suitable for this domain, based on RANSAC.

Metrics on Primary Features

- Measurements of features in the horizontal plane
- Measurements of features in the vertical plane
- Measurements of angles
- Measurements of volume

Exploration of Potential Design Ratios

- π, φ, e, 1, 2, 3, 4 etc.

Raw Dataset Attachments

- Including all measurement and sample coordinates as CSV-files embedded in the report
- Including an STL file of the aligned object alongside the report, for easier external replication and validation of the research results

Appendix A - Comparison Of Circularity Measurements (Z-plane vs. surface-perpendicular)

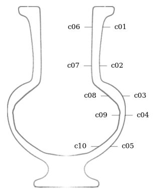


Figure 72: Circularity measurement sample locations, full mesh aligned to exterior surface

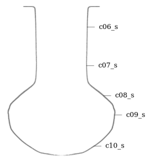


Figure 73: Circularity measurement sample location, separately aligned interior mesh

Samples perpendicular to the surface curvature

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø43.913±0.085	0.161	0.030	0.012	0.017	759	0.100	97.044	21.957
c02	exterior	Ø40.017±0.073	0.133	0.025	0.010	0.015	598	0.100	73.618	20.008
c03	exterior	Ø67.658±0.100	0.199	0.042	0.018	0.022	1042	0.100	55.408	33.829
c04	exterior	Ø71.077±0.073	0.131	0.024	0.010	0.015	1055	0.100	43.717	35.538
c05	exterior	Ø52.922±0.068	0.116	0.023	0.009	0.013	627	0.100	24.855	26.461
c06	interior	Ø31.236±0.150	0.274	0.064	0.026	0.036	759	0.100	97.044	15.618
c06_s	interior sep.	Ø31.236±0.134	0.251	0.056	0.024	0.030	749	0.100	97.044	15.618
c07	interior	Ø30.436±0.138	0.271	0.052	0.018	0.035	623	0.100	73.618	15.218
c07_s	interior sep.	Ø30.438±0.116	0.232	0.047	0.018	0.028	623	0.100	73.618	15.219
c08	interior	Ø51.056±0.352	0.697	0.122	0.049	0.079	1096	0.100	55.408	25.528
c08_s	interior sep.	Ø51.056±0.346	0.690	0.120	0.049	0.077	1081	0.100	55.408	25.528
c09	interior	Ø64.270±0.221	0.396	0.081	0.031	0.046	1389	0.100	43.717	32.135
c09_s	interior sep.	Ø64.271±0.228	0.416	0.082	0.033	0.047	1402	0.100	43.717	32.135
c10	interior	Ø39.230±0.122	0.206	0.037	0.014	0.022	610	0.100	24.855	19.615
c10_s	interior sep.	Ø39.231±0.118	0.194	0.037	0.015	0.023	609	0.100	24.855	19.616

Table 7: Detailed circularity measurements at selected samples in z-plane, vessel PV003.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø43.914±0.092	0.168	0.031	0.012	0.018	781	0.100	97.044	21.957
c02	exterior	Ø40.022±0.076	0.134	0.025	0.010	0.015	600	0.100	73.618	20.011
c03	exterior	Ø67.670±0.136	0.267	0.051	0.021	0.029	1349	0.100	55.408	33.835
c04	exterior	Ø71.080±0.074	0.132	0.024	0.009	0.015	1064	0.100	43.717	35.540
c05	exterior	Ø52.921±0.137	0.242	0.043	0.019	0.025	1274	0.100	24.855	26.460
c06	interior	Ø31.226±0.167	0.287	0.064	0.028	0.037	759	0.100	97.044	15.613
c06_s	interior sep.	Ø31.243±0.131	0.251	0.056	0.024	0.030	751	0.100	97.044	15.621
c07	interior	Ø30.448±0.143	0.271	0.051	0.017	0.035	624	0.100	73.618	15.224
c07_s	interior sep.	Ø30.454±0.123	0.232	0.047	0.019	0.029	627	0.100	73.618	15.227
c08	interior	Ø51.041±0.567	1.026	0.178	0.068	0.114	2209	0.100	55.408	25.520
c08_s	interior sep.	Ø51.038±0.550	1.005	0.175	0.068	0.112	2208	0.100	55.408	25.519
c09	interior	Ø64.264±0.217	0.406	0.081	0.032	0.046	1422	0.100	43.717	32.132
c09_s	interior sep.	Ø64.253±0.217	0.419	0.083	0.033	0.048	1413	0.100	43.717	32.126
c10	interior	Ø39.198±0.261	0.486	0.075	0.031	0.045	1863	0.100	24.855	19.599
c10_s	interior sep.	Ø39.200±0.260	0.462	0.076	0.032	0.046	1857	0.100	24.855	19.600

Table 8: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel PV003.

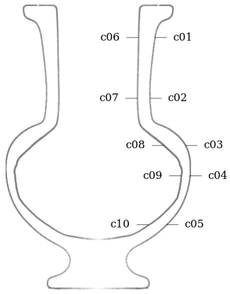


Figure 74: Circularity measurement sample locations, full mesh aligned to exterior surface

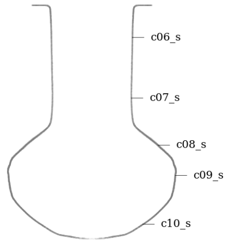


Figure 75: Circularity measurement sample location, separately aligned interior mesh

Samples perpendicular to the surface curvature

Tag	Area	Measured deviation[8]	Residuals				Sam-ple size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø1.7289±0.0033	0.0064	0.0012	0.0005	0.0007	759	0.0039	3.8206	0.8644
c02	exterior	Ø1.5755±0.0029	0.0052	0.0010	0.0004	0.0006	598	0.0039	2.8983	0.7877
c03	exterior	Ø2.6637±0.0039	0.0078	0.0016	0.0007	0.0009	1042	0.0039	2.1814	1.3319
c04	exterior	Ø2.7983±0.0029	0.0052	0.0009	0.0004	0.0006	1055	0.0039	1.7211	1.3991
c05	exterior	Ø2.0835±0.0027	0.0046	0.0009	0.0004	0.0005	627	0.0039	0.9785	1.0418
c06	interior	Ø1.2297±0.0059	0.0108	0.0025	0.0010	0.0014	759	0.0039	3.8206	0.6149
c06_s	interior sep.	Ø1.2298±0.0053	0.0099	0.0022	0.0010	0.0012	749	0.0039	3.8206	0.6149
c07	interior	Ø1.1983±0.0054	0.0107	0.0020	0.0007	0.0014	623	0.0039	2.8983	0.5991
c07_s	interior sep.	Ø1.1984±0.0046	0.0091	0.0018	0.0007	0.0011	623	0.0039	2.8983	0.5992
c08	interior	Ø2.0101±0.0138	0.0274	0.0048	0.0019	0.0031	1096	0.0039	2.1814	1.0050
c08_s	interior sep.	Ø2.0101±0.0136	0.0272	0.0047	0.0019	0.0030	1081	0.0039	2.1814	1.0050
c09	interior	Ø2.5303±0.0087	0.0156	0.0032	0.0012	0.0018	1389	0.0039	1.7211	1.2652
c09_s	interior sep.	Ø2.5303±0.0090	0.0164	0.0032	0.0013	0.0018	1402	0.0039	1.7211	1.2652
c10	interior	Ø1.5445±0.0048	0.0081	0.0014	0.0006	0.0009	610	0.0039	0.9785	0.7723
c10_s	interior sep.	Ø1.5445±0.0047	0.0076	0.0015	0.0006	0.0009	609	0.0039	0.9785	0.7723

Table 9: Detailed circularity measurements at selected samples in z-plane, vessel PV003.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sam-ple size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø1.7289±0.0036	0.0066	0.0012	0.0005	0.0007	781	0.0039	3.8206	0.8645
c02	exterior	Ø1.5757±0.0030	0.0053	0.0010	0.0004	0.0006	600	0.0039	2.8983	0.7878
c03	exterior	Ø2.6642±0.0053	0.0105	0.0020	0.0008	0.0011	1349	0.0039	2.1814	1.3321
c04	exterior	Ø2.7984±0.0029	0.0052	0.0009	0.0004	0.0006	1064	0.0039	1.7211	1.3992
c05	exterior	Ø2.0835±0.0054	0.0095	0.0017	0.0007	0.0010	1274	0.0039	0.9785	1.0417
c06	interior	Ø1.2294±0.0066	0.0113	0.0025	0.0011	0.0014	759	0.0039	3.8206	0.6147
c06_s	interior sep.	Ø1.2300±0.0052	0.0099	0.0022	0.0009	0.0012	751	0.0039	3.8206	0.6150
c07	interior	Ø1.1987±0.0056	0.0107	0.0020	0.0007	0.0014	624	0.0039	2.8983	0.5994
c07_s	interior sep.	Ø1.1990±0.0048	0.0091	0.0018	0.0007	0.0011	627	0.0039	2.8983	0.5995
c08	interior	Ø2.0095±0.0223	0.0404	0.0070	0.0027	0.0045	2209	0.0039	2.1814	1.0047
c08_s	interior sep.	Ø2.0094±0.0217	0.0396	0.0069	0.0027	0.0044	2208	0.0039	2.1814	1.0047
c09	interior	Ø2.5301±0.0085	0.0160	0.0032	0.0013	0.0018	1422	0.0039	1.7211	1.2650
c09_s	interior sep.	Ø2.5296±0.0085	0.0165	0.0033	0.0013	0.0019	1413	0.0039	1.7211	1.2648
c10	interior	Ø1.5432±0.0103	0.0192	0.0030	0.0012	0.0018	1863	0.0039	0.9785	0.7716
c10_s	interior sep.	Ø1.5433±0.0103	0.0182	0.0030	0.0013	0.0018	1857	0.0039	0.9785	0.7717

Table 10: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel PV003.

Comparison of circularity on the full vessel surface

Metric

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.142	0.079	0.260	0.017	0.010	0.028	0.029	0.016	0.056	1941	0.050
Interior	0.324	0.116	0.727	0.041	0.013	0.084	0.068	0.021	0.138	1742	0.050
Interior separate	0.303	0.112	0.719	0.036	0.013	0.086	0.064	0.021	0.139	1741	0.050

Table 11: Detailed circularity measurements at selected samples in z-plane, vessel PV003.

Samples in the z-plane

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.170	0.094	0.619	0.020	0.012	0.063	0.033	0.018	0.110	1935	0.050
Interior	0.362	0.183	2.687	0.043	0.022	0.241	0.070	0.037	0.372	1741	0.050
Interior separate	0.351	0.140	2.681	0.040	0.016	0.237	0.067	0.033	0.366	1741	0.050

Table 12: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel PV003.

Imperial

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.142	0.079	0.260	0.017	0.010	0.028	0.029	0.016	0.056	1941	0.050
Interior	0.324	0.116	0.727	0.041	0.013	0.084	0.068	0.021	0.138	1742	0.050
Interior separate	0.303	0.112	0.719	0.036	0.013	0.086	0.064	0.021	0.139	1741	0.050

Table 13: Detailed circularity measurements at selected samples in z-plane, vessel PV003.

Samples in the z-plane

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.170	0.094	0.619	0.020	0.012	0.063	0.033	0.018	0.110	1935	0.050
Interior	0.362	0.183	2.687	0.043	0.022	0.241	0.070	0.037	0.372	1741	0.050
Interior separate	0.351	0.140	2.681	0.040	0.016	0.237	0.067	0.033	0.366	1741	0.050

Table 14: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel PV003.

Circularity analysis of exterior surface - perpendicular to surface curvature

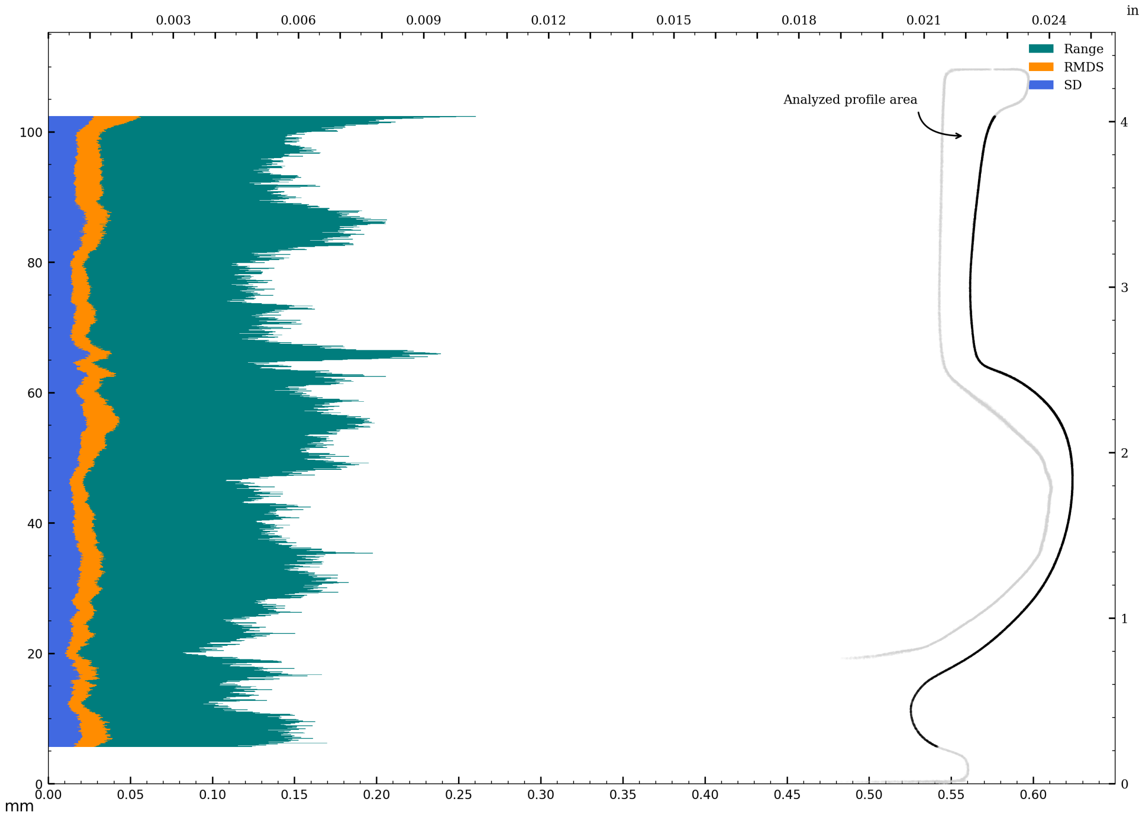


Figure 76: Circularity of exterior surface - perpendicular to surface curvature.

Circularity analysis of exterior surface - in z-plane

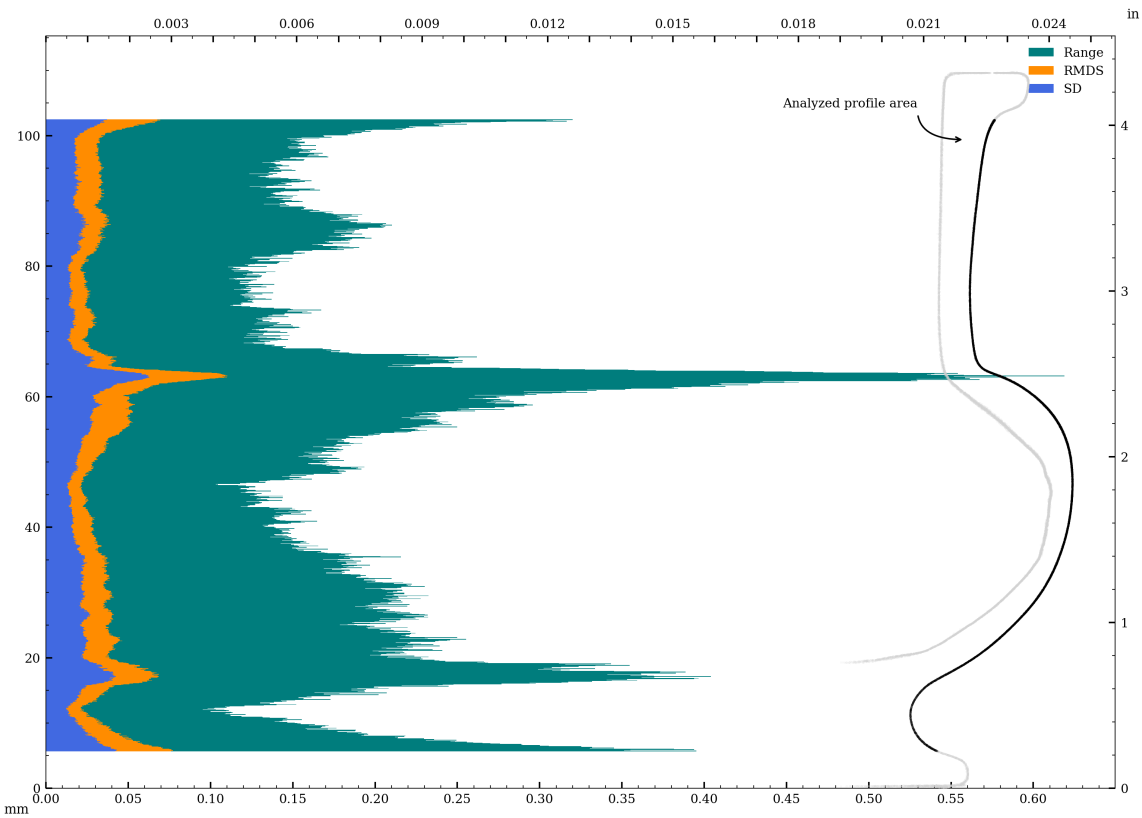


Figure 77: Circularity of exterior surface - in z-plane.

Circularity analysis of exterior surface, perpendicular to surface curvature, Standard Deviation and Root Mean Squared Deviation

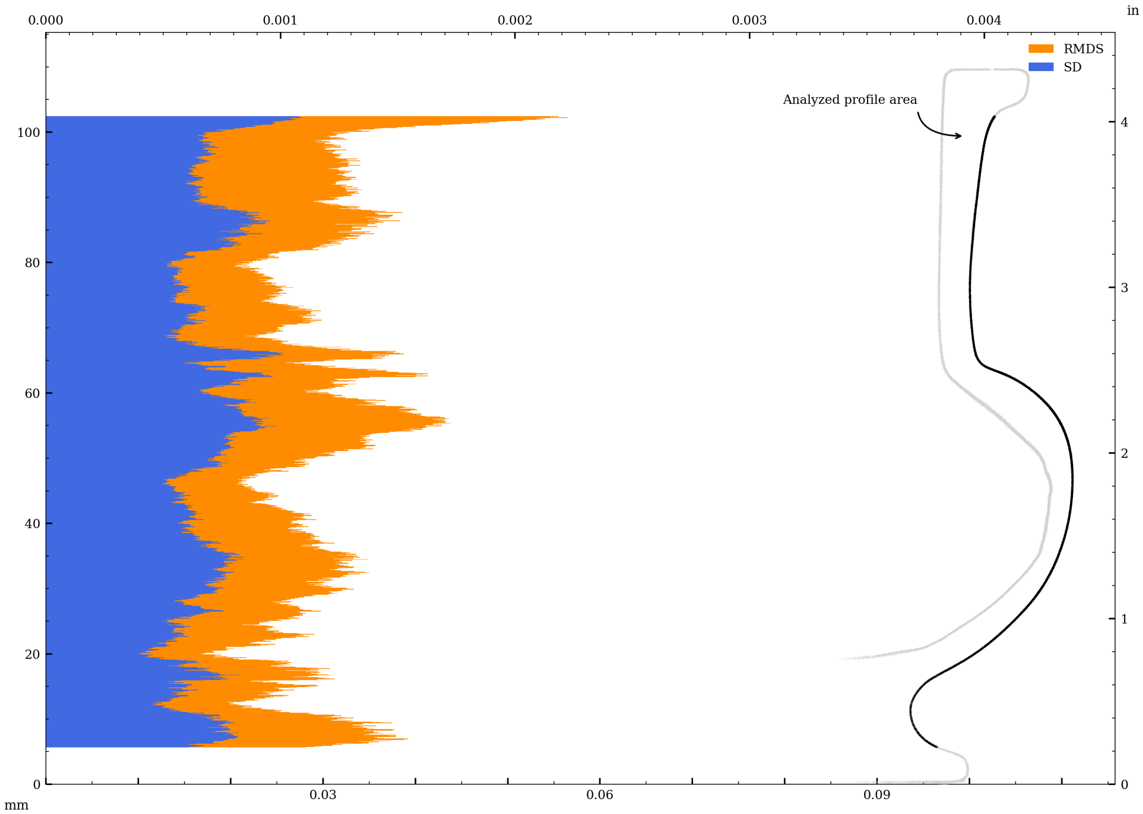


Figure 78: Vessel circularity of exterior surface, perpendicular to surface curvature, standard deviation and median absolute deviation.

Circularity analysis of exterior surface, in z-plane, Standard Deviation and Root Mean Squared Deviation

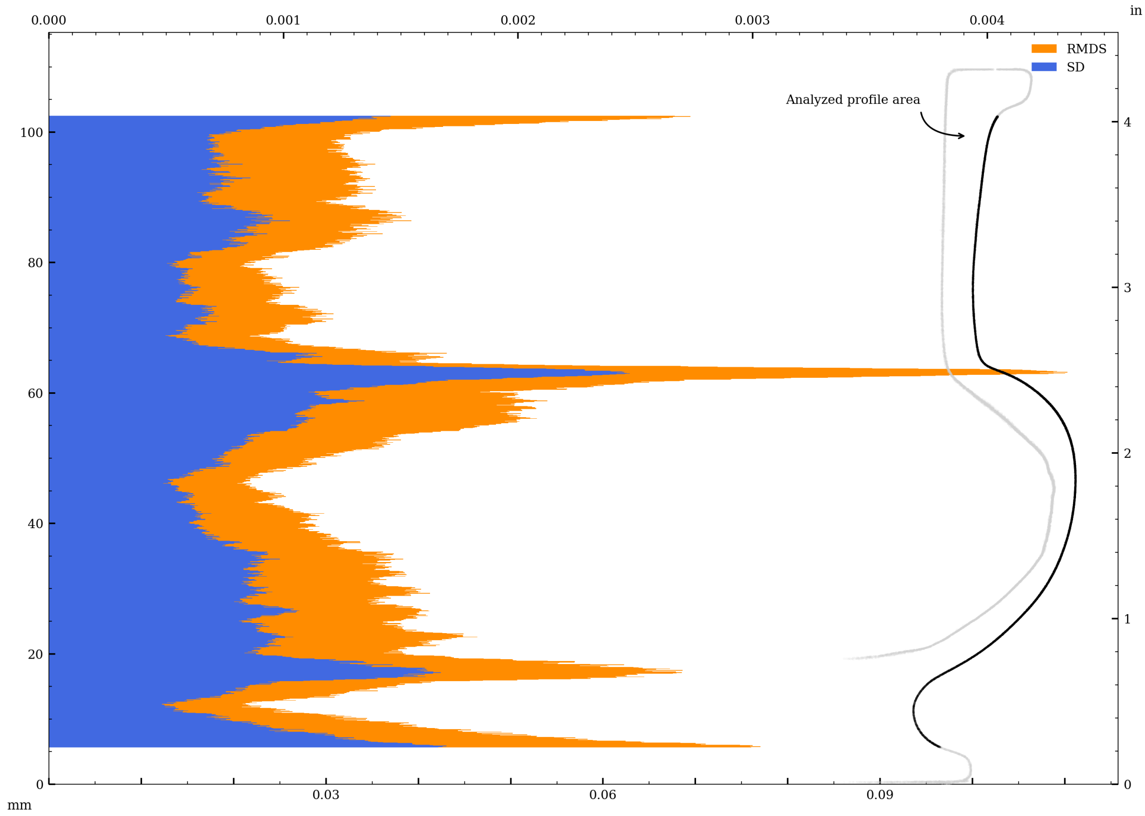


Figure 79: Vessel circularity of exterior surface, in z-plane, standard deviation and median absolute deviation.

Circularity analysis of interior surface - perpendicular to surface curvature

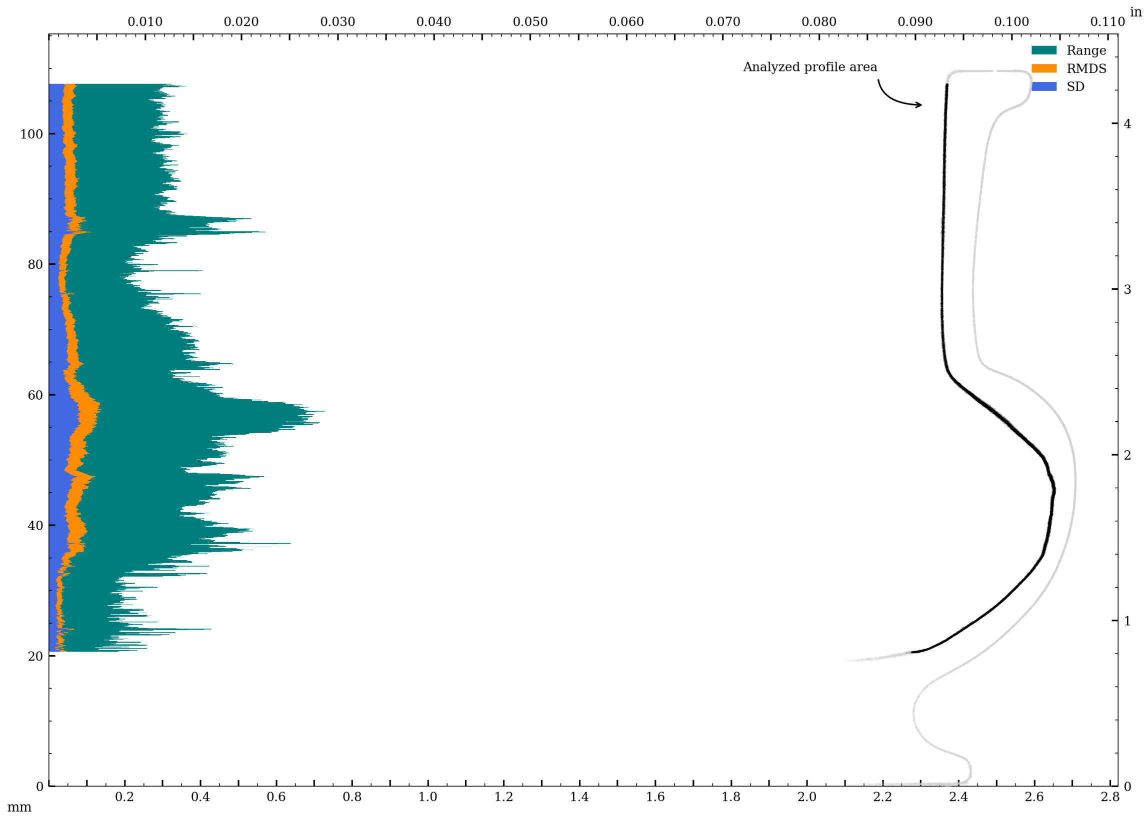


Figure 80: Circularity of interior surface - perpendicular to surface curvature.

Circularity analysis of interior surface - in z-plane

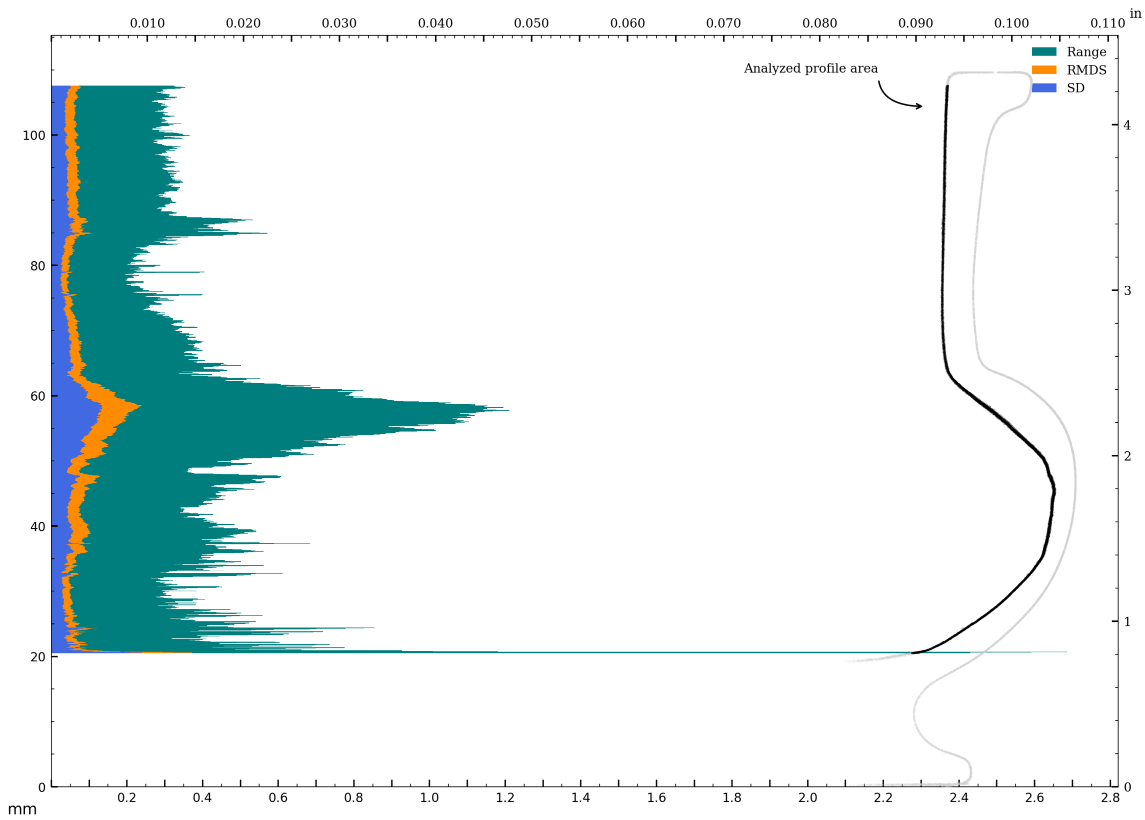


Figure 81: Circularity of interior surface - in z-plane.

Circularity analysis of interior surface, perpendicular to surface curvature, Standard Deviation and Root Mean Squared Deviation

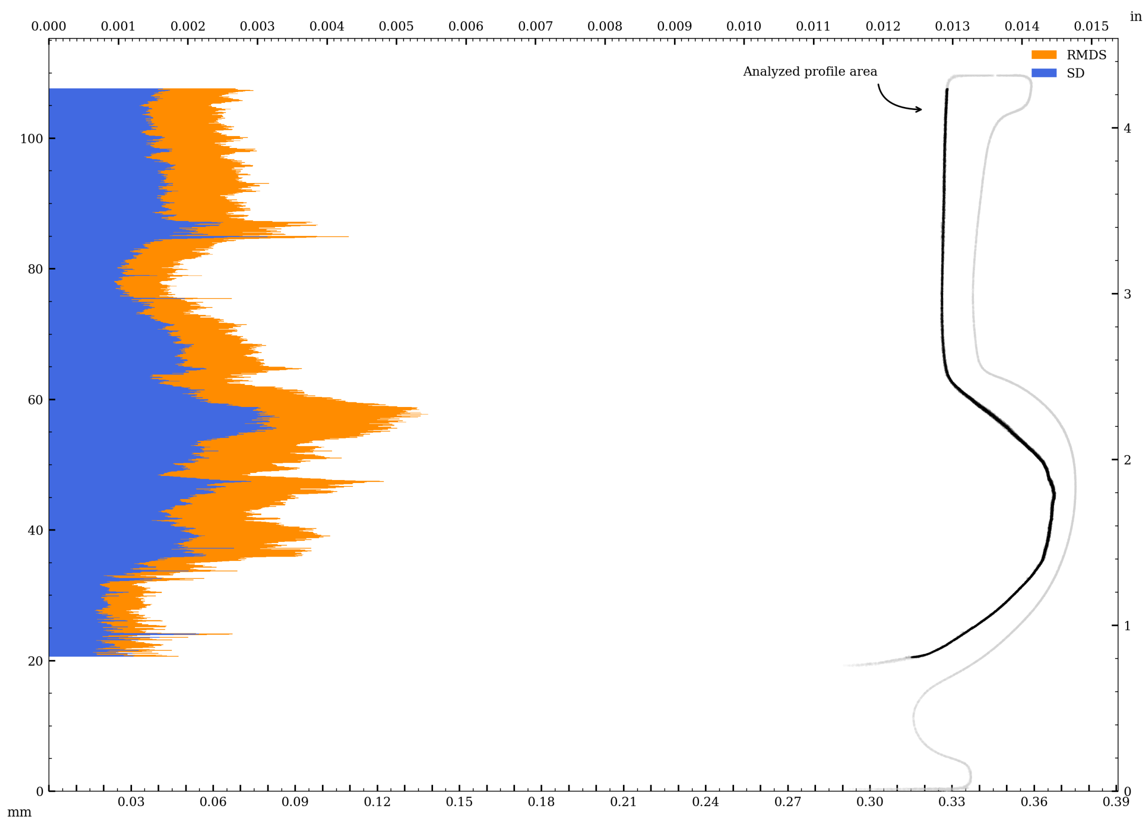


Figure 82: Vessel circularity of interior surface, perpendicular to surface curvature, standard deviation and median absolute deviation.

Circularity analysis of interior surface, in z-plane, Standard Deviation and Root Mean Squared Deviation

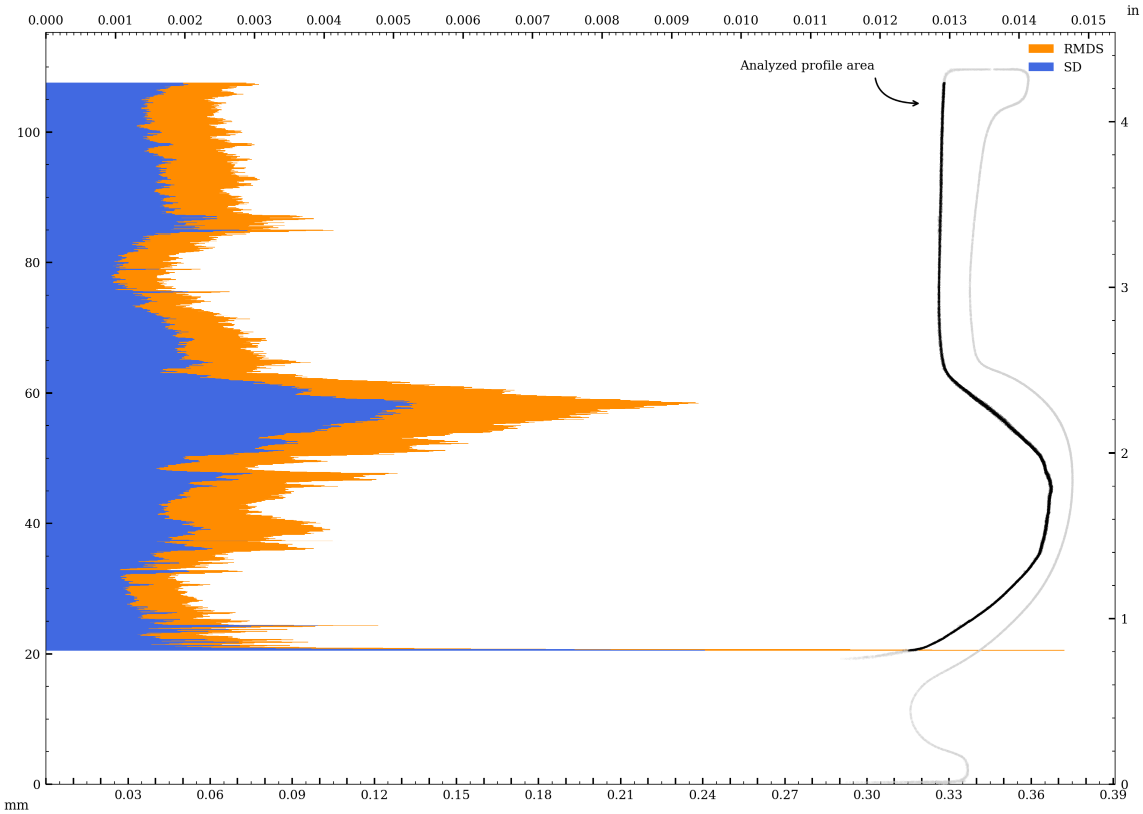


Figure 83: Vessel circularity of interior surface, in z-plane, standard deviation and median absolute deviation.

Circularity analysis of interior separately aligned surface - perpendicular to surface curvature

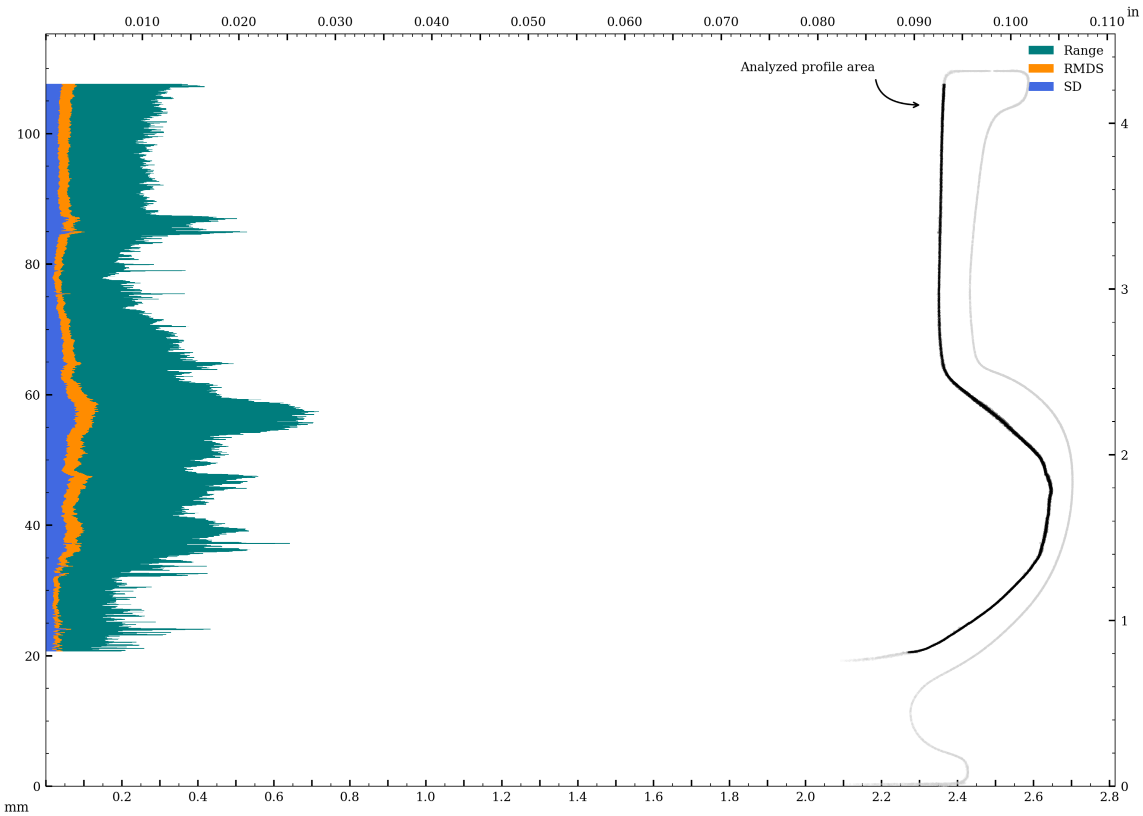


Figure 84: Circularity of interior_separate surface - perpendicular to surface curvature.

Circularity analysis of interior separately aligned surface - in z-plane

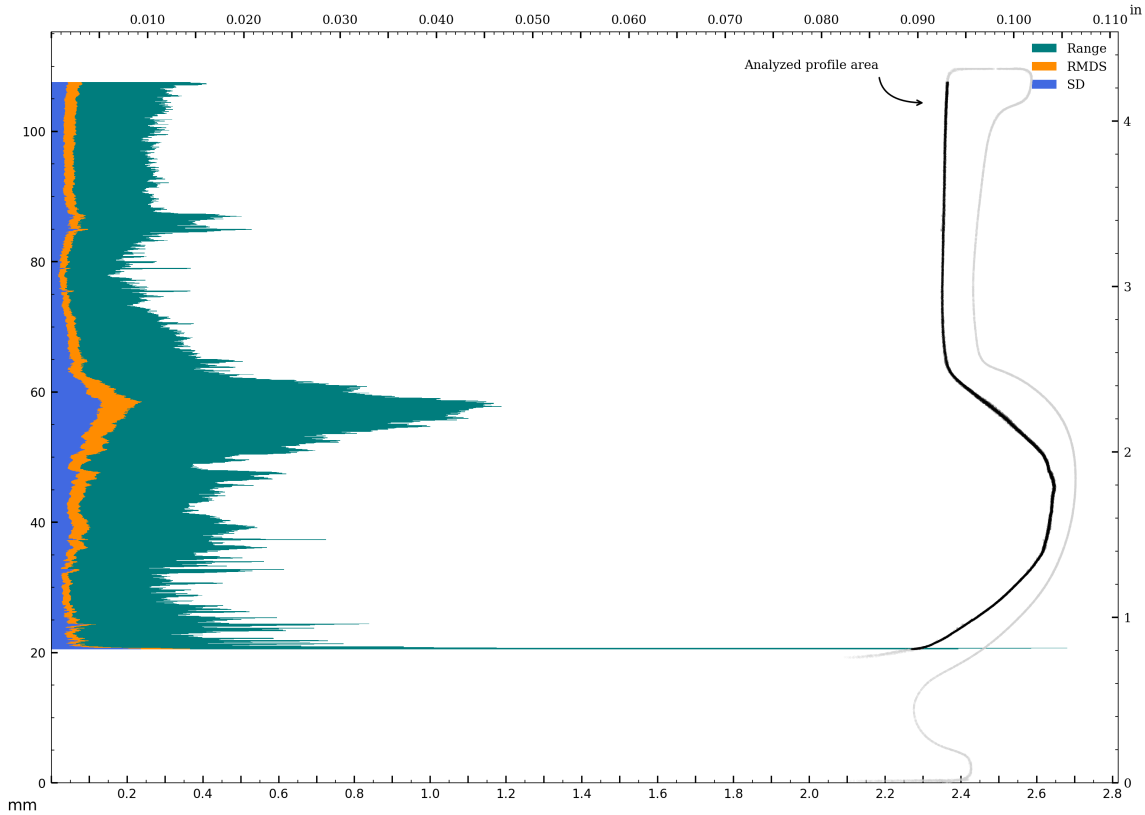


Figure 85: Circularity of interior_separate surface - in z-plane.

Circularity analysis of interior separately aligned surface, perpendicular to surface curvature, Standard Deviation and Root Mean Squared Deviation

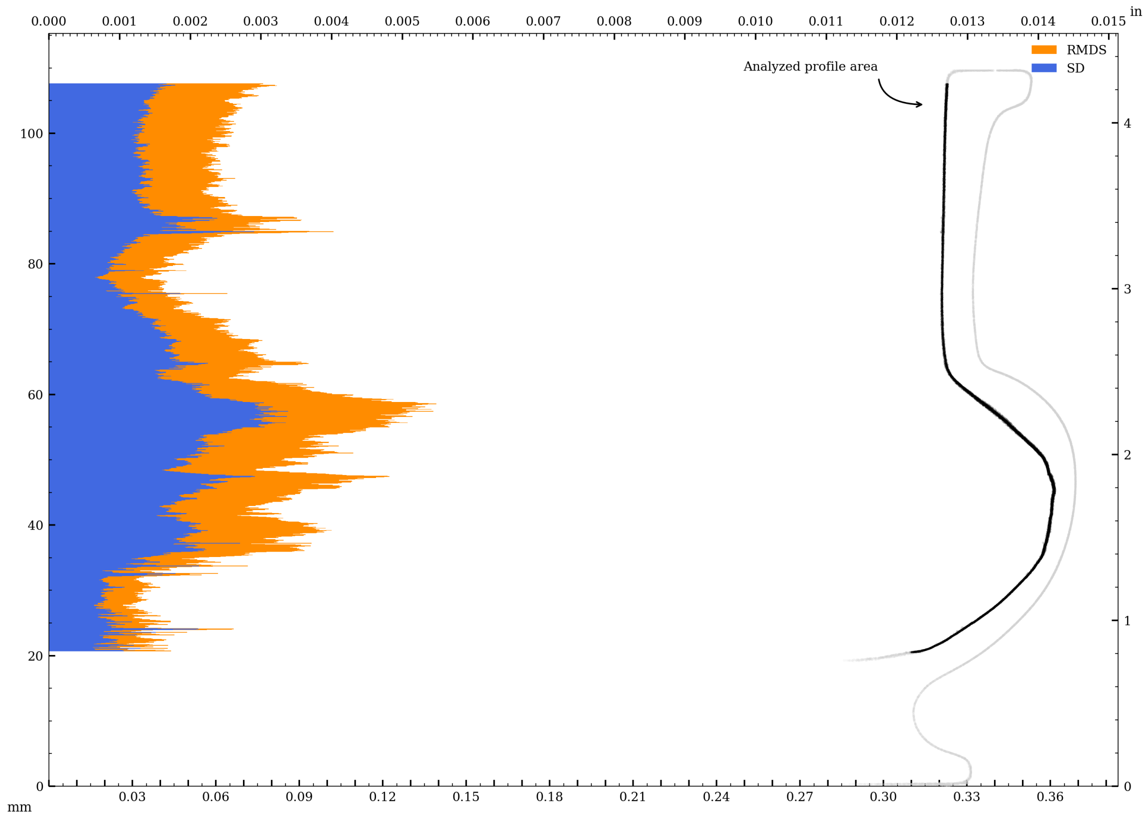


Figure 86: Vessel circularity of interior_separate surface, perpendicular to surface curvature, standard deviation and median absolute deviation.

Circularity analysis of interior separately aligned surface, in z-plane, Standard Deviation and Root Mean Squared Deviation

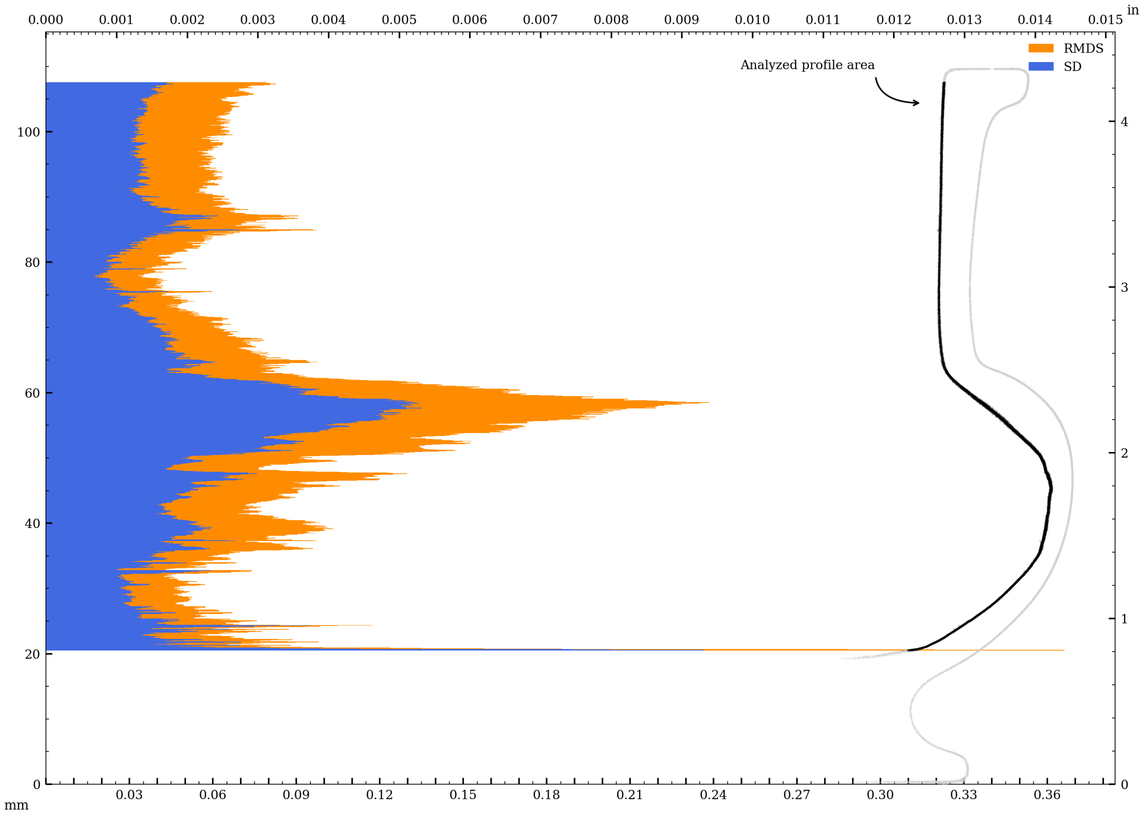


Figure 87: Vessel circularity of interior_separate surface, in z-plane, standard deviation and median absolute deviation.

Appendix B - Comparison Of Concentricity Measurements (Z-plane vs. surface-perpendicular)

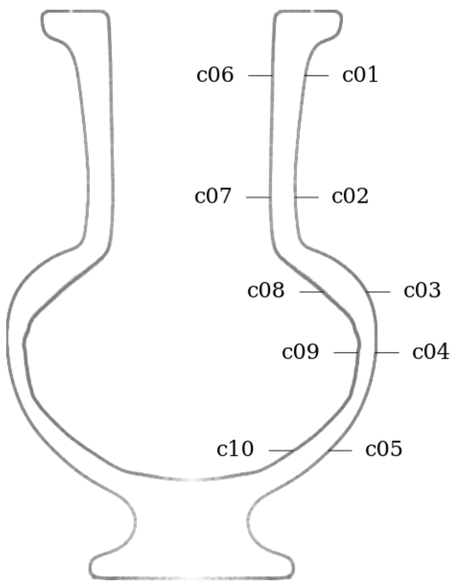


Figure 88: Circularity measurement sample locations, full mesh aligned to exterior surface

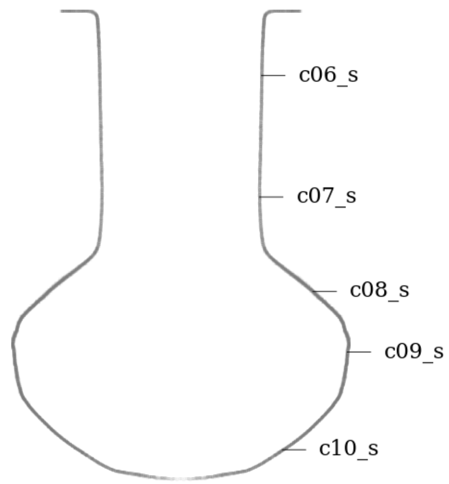


Figure 89: Circularity measurement sample location, separately aligned interior mesh

Concentricity measurements perpendicular to surface curvature

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.034	781	0.174	0.164	0.035	0.034	0.019	0.018	−34, −6
c02	z-axis	0.018	600	0.123	0.123	0.025	0.024	0.014	0.014	−15, 9
c03	z-axis	0.050	1349	0.246	0.223	0.048	0.047	0.027	0.027	−48, 12
c04	z-axis	0.007	1064	0.140	0.128	0.026	0.025	0.016	0.015	−0, −7
c05	z-axis	0.035	1274	0.332	0.293	0.061	0.061	0.033	0.032	11, −33
c06	z-axis	0.042	759	0.289	0.289	0.064	0.064	0.036	0.036	−42, −1
c06_s	z-axis	0.003	751	0.251	0.251	0.056	0.056	0.030	0.030	−2, 1
c07	z-axis	0.051	624	0.388	0.388	0.087	0.087	0.049	0.049	−29, −42
c07_s	z-axis	0.042	627	0.347	0.347	0.076	0.076	0.046	0.046	1, −42
c08	z-axis	0.095	2209	1.257	1.136	0.226	0.215	0.145	0.136	−33, −90
c08_s	z-axis	0.089	2208	1.228	1.119	0.219	0.207	0.140	0.130	−19, −87
c09	z-axis	0.026	1422	0.442	0.442	0.087	0.087	0.052	0.052	8, −25
c09_s	z-axis	0.036	1413	0.430	0.430	0.086	0.086	0.050	0.050	27, −24
c10	z-axis	0.009	1863	0.504	0.441	0.076	0.075	0.046	0.045	−5, −7
c10_s	z-axis	0.016	1857	0.477	0.427	0.076	0.074	0.046	0.044	15, −6
c01	c06	0.010								9, −5
c02	c07	0.053								13, 51
c03	c08	0.103								−15, 102
c04	c09	0.019								−8, 18
c05	c10	0.030								16, −26

Concentricity measurements in z-plane

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.034	781	0.174	0.164	0.035	0.034	0.019	0.018	−34, −6
c02	z-axis	0.018	600	0.123	0.123	0.025	0.024	0.014	0.014	−15, 9
c03	z-axis	0.050	1349	0.246	0.223	0.048	0.047	0.027	0.027	−48, 12
c04	z-axis	0.007	1064	0.140	0.128	0.026	0.025	0.016	0.015	−0, −7
c05	z-axis	0.035	1274	0.332	0.293	0.061	0.061	0.033	0.032	11, −33
c06	z-axis	0.042	759	0.289	0.289	0.064	0.064	0.036	0.036	−42, −1
c06_s	z-axis	0.003	751	0.251	0.251	0.056	0.056	0.030	0.030	−2, 1
c07	z-axis	0.051	624	0.388	0.388	0.087	0.087	0.049	0.049	−29, −42
c07_s	z-axis	0.042	627	0.347	0.347	0.076	0.076	0.046	0.046	1, −42
c08	z-axis	0.095	2209	1.257	1.136	0.226	0.215	0.145	0.136	−33, −90
c08_s	z-axis	0.089	2208	1.228	1.119	0.219	0.207	0.140	0.130	−19, −87
c09	z-axis	0.026	1422	0.442	0.442	0.087	0.087	0.052	0.052	8, −25
c09_s	z-axis	0.036	1413	0.430	0.430	0.086	0.086	0.050	0.050	27, −24
c10	z-axis	0.009	1863	0.504	0.441	0.076	0.075	0.046	0.045	−5, −7
c10_s	z-axis	0.016	1857	0.477	0.427	0.076	0.074	0.046	0.044	15, −6
c01	c06	0.010								9, −5
c02	c07	0.053								13, 51
c03	c08	0.103								−15, 102
c04	c09	0.019								−8, 18
c05	c10	0.030								16, −26

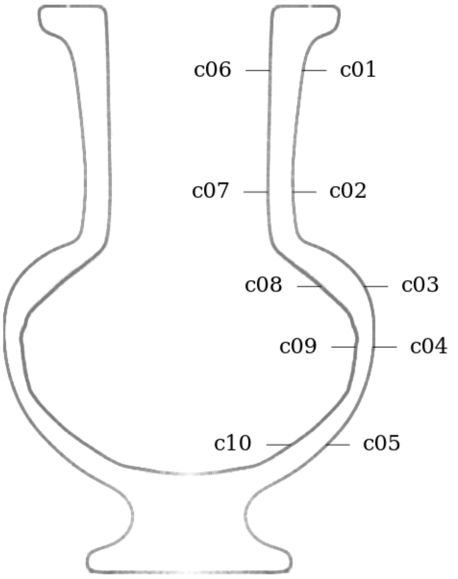


Figure 90: Circularity measurement sample locations, full mesh aligned to exterior surface

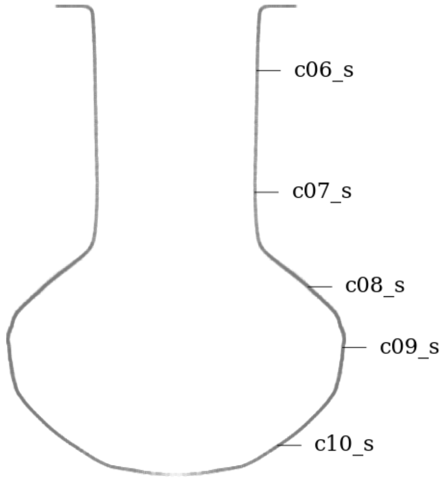


Figure 91: Circularity measurement sample location, separately aligned interior mesh

Concentricity measurements perpendicular to surface curvature

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0013	781	0.0068	0.0065	0.0014	0.0013	0.0007	0.0007	−1.3, −0.2
c02	z-axis	0.0007	600	0.0049	0.0049	0.0010	0.0009	0.0006	0.0005	−0.6, 0.3
c03	z-axis	0.0020	1349	0.0097	0.0088	0.0019	0.0019	0.0011	0.0011	−1.9, 0.5
c04	z-axis	0.0003	1064	0.0055	0.0050	0.0010	0.0010	0.0006	0.0006	−0.0, −0.3
c05	z-axis	0.0014	1274	0.0131	0.0115	0.0024	0.0024	0.0013	0.0013	0.4, −1.3
c06	z-axis	0.0017	759	0.0114	0.0114	0.0025	0.0025	0.0014	0.0014	−1.7, −0.0
c06_s	z-axis	0.0001	751	0.0099	0.0099	0.0022	0.0022	0.0012	0.0012	−0.1, 0.1
c07	z-axis	0.0020	624	0.0153	0.0153	0.0034	0.0034	0.0019	0.0019	−1.1, −1.7
c07_s	z-axis	0.0016	627	0.0137	0.0137	0.0030	0.0030	0.0018	0.0018	0.0, −1.6
c08	z-axis	0.0038	2209	0.0495	0.0447	0.0089	0.0085	0.0057	0.0053	−1.3, −3.5
c08_s	z-axis	0.0035	2208	0.0484	0.0440	0.0086	0.0081	0.0055	0.0051	−0.8, −3.4
c09	z-axis	0.0010	1422	0.0174	0.0174	0.0034	0.0034	0.0020	0.0020	0.3, −1.0
c09_s	z-axis	0.0014	1413	0.0169	0.0169	0.0034	0.0034	0.0020	0.0020	1.0, −1.0
c10	z-axis	0.0003	1863	0.0198	0.0174	0.0030	0.0030	0.0018	0.0018	−0.2, −0.3
c10_s	z-axis	0.0006	1857	0.0188	0.0168	0.0030	0.0029	0.0018	0.0017	0.6, −0.2
c01	c06	0.0004								0.4, −0.2
c02	c07	0.0021								0.5, 2.0
c03	c08	0.0041								−0.6, 4.0
c04	c09	0.0008								−0.3, 0.7
c05	c10	0.0012								0.6, −1.0

Concentricity measurements in z-plane

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0013	781	0.0068	0.0065	0.0014	0.0013	0.0007	0.0007	−1.3, −0.2
c02	z-axis	0.0007	600	0.0049	0.0049	0.0010	0.0009	0.0006	0.0005	−0.6, 0.3
c03	z-axis	0.0020	1349	0.0097	0.0088	0.0019	0.0019	0.0011	0.0011	−1.9, 0.5
c04	z-axis	0.0003	1064	0.0055	0.0050	0.0010	0.0010	0.0006	0.0006	−0.0, −0.3
c05	z-axis	0.0014	1274	0.0131	0.0115	0.0024	0.0024	0.0013	0.0013	0.4, −1.3
c06	z-axis	0.0017	759	0.0114	0.0114	0.0025	0.0025	0.0014	0.0014	−1.7, −0.0
c06_s	z-axis	0.0001	751	0.0099	0.0099	0.0022	0.0022	0.0012	0.0012	−0.1, 0.1
c07	z-axis	0.0020	624	0.0153	0.0153	0.0034	0.0034	0.0019	0.0019	−1.1, −1.7
c07_s	z-axis	0.0016	627	0.0137	0.0137	0.0030	0.0030	0.0018	0.0018	0.0, −1.6
c08	z-axis	0.0038	2209	0.0495	0.0447	0.0089	0.0085	0.0057	0.0053	−1.3, −3.5
c08_s	z-axis	0.0035	2208	0.0484	0.0440	0.0086	0.0081	0.0055	0.0051	−0.8, −3.4
c09	z-axis	0.0010	1422	0.0174	0.0174	0.0034	0.0034	0.0020	0.0020	0.3, −1.0
c09_s	z-axis	0.0014	1413	0.0169	0.0169	0.0034	0.0034	0.0020	0.0020	1.0, −1.0
c10	z-axis	0.0003	1863	0.0198	0.0174	0.0030	0.0030	0.0018	0.0018	−0.2, −0.3
c10_s	z-axis	0.0006	1857	0.0188	0.0168	0.0030	0.0029	0.0018	0.0017	0.6, −0.2
c01	c06	0.0004								0.4, −0.2
c02	c07	0.0021								0.5, 2.0
c03	c08	0.0041								−0.6, 4.0
c04	c09	0.0008								−0.3, 0.7
c05	c10	0.0012								0.6, −1.0