HV004

An Exploration of Precision
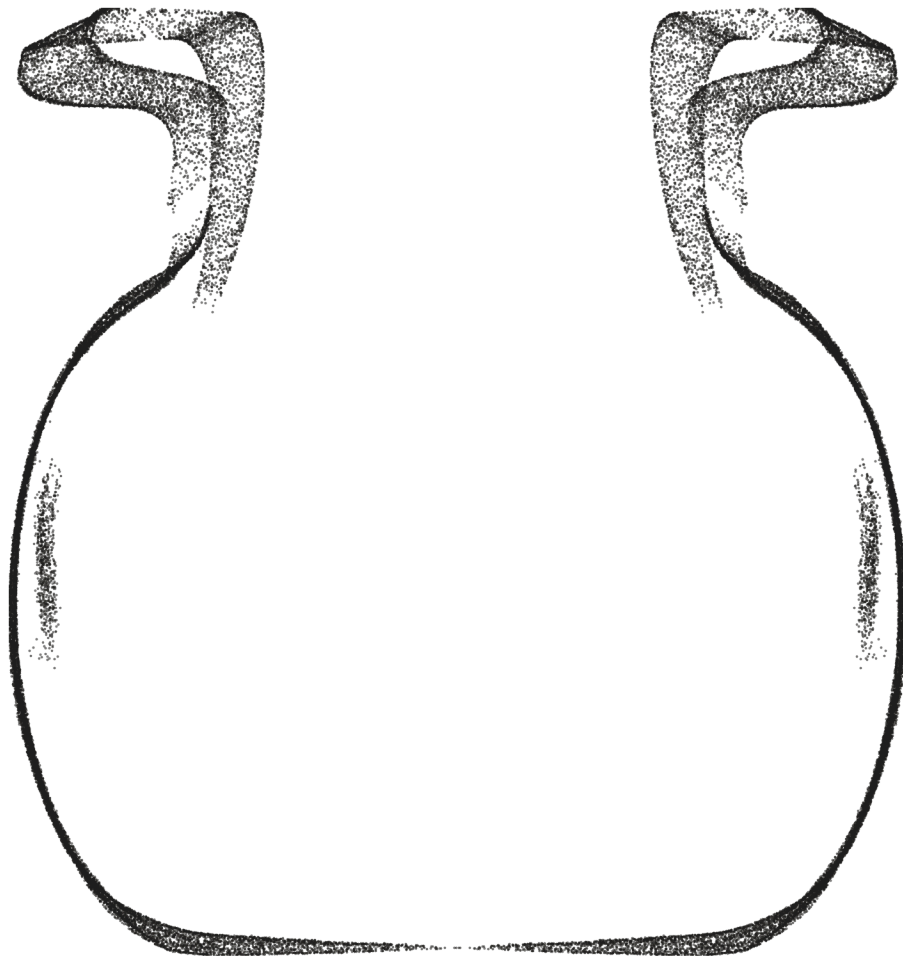
Image courtesy Max Fomitchec-Zamilov

Contents

Artifact Information

Artifact Data

Collection
Provenance[1]
Provenience[2]
Attribution

Art dealer information

Ref.   E4
Description
URL

Maijers vessel classification[3]

Short classification   Classification not set on object
Long classification   Classification not set on object

Physical properties

Precision score[4]   17
Height (approximate)
Width (approximate)   Material
                          Mohs Hardness[5]
Unknown (Unknown)   Weight

Scan information

Source   Max Fomitchev-Zamilov
Source file name   E4.stl
Scan method   CMM
Scanner   Keyence VL -500
Rated scan accuracy   10 μm | 0.41 thou
Scan date   2025
Scanned by   Max Fomitchev-Zamilov

Mesh decimation   None, raw scan file used in the analysis
Number of vertices   576 068
Mesh density[6]   168 μm | 6.60 thou
Max vertex distance   1295 μm | 50.973 thou
Min vertex distance   2 μm | 0.088 thou
Vertices per cm2   1342 (approximated)
Vertices per in2   8658 (approximated)

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[1] The verifiable chain of custody of an artifact

[2] The location or site where an artifact was recovered

[3] Vessel artifact classification developed by W. Arnold Maijer and described in his publication Masters of Stone, ISBN 978-90-829212-0-5

[4] The precision score metric is described in Precision Score Of The Artifact, p. 55

[5] The Mohs scale is an ordinal scale, from 1 to 10, describing the materials resistance to abrasion (the ability of harder material to scratch softer material)

[6] Median distance between vertices

Alignment In The Cartesian Coordinate System

For precise and valid measurements of the vessel's geometry to be possible, the points of the scanned dataset must first and foremost be placed optimally in a Cartesian coordinate system. Several alignment methods and algorithms have been tested on a number of different vessels to determine the best way to achieve optimal alignment.

Any misalignment of the artifact will increase the error of the precision measurements, due to the distortion/ wobble effect caused by the misaligned object. To visualize this distortion, we can consider a representation of the three-dimensional point cloud data, folded to a two-dimensional plane. This folded representation is obtained by rotating all scanned points around an assumed center axis to y = 0, x > 0, thus resulting in a two-dimensional profile representation of all scanned vertices in the object.

Figure 1 illustrates this effect on a ideal ellipsoid. In the first image, the ellipsoid is perfectly aligned, resulting in a narrow and precise two-dimensional folded profile. As misalignments are introduced, the two-dimensional profile increases in width, visually showing the distortion, causing the error in the precision measurements to increase. While easy to understand visually, this distortion can also be objectively quantified, and as such used to compare the fitness of different assumed center axes against each other, and further to create an automated and solid process for optimal Cartesian alignment of the scan data.

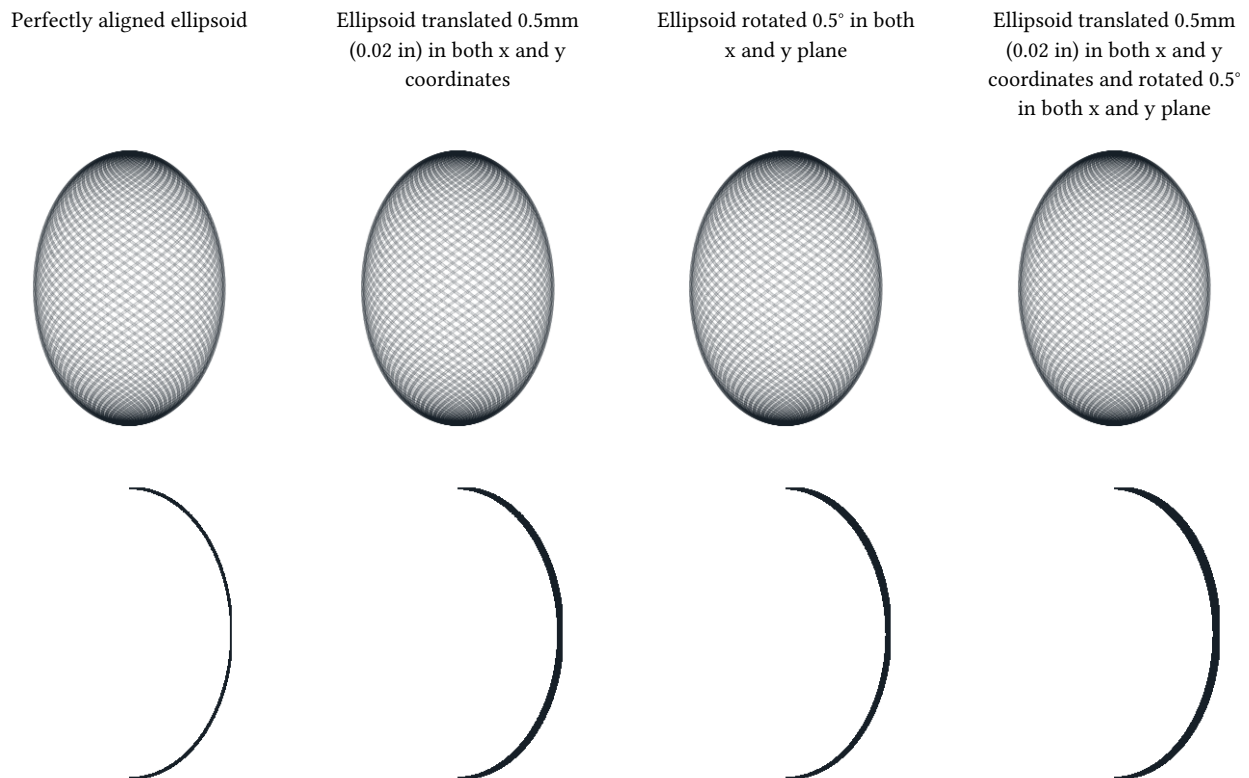| Perfectly aligned ellipsoid | Ellipsoid translated 0.5mm (0.02 in) in both x and y coordinates | Ellipsoid rotated 0.5° in both x and y plane | Ellipsoid translated 0.5mm (0.02 in) in both x and y coordinates and rotated 0.5° in both x and y plane |

Figure 1: Distortion caused by a misalignment of the artifact

In contemporary metrology analysis of modern production objects, it is common to align the object in a Cartesian coordinate system by fitting a flat surface of the object to a reference plane in the coordinate system, cylindrical features to an ideal cylinder etc., or by using specific markers placed on the object in the design process. This methodology, however, is inadequate for the ancient objects in question. Most scanned artifacts, do not have a valid flat surface which could be aligned to a plane in the Cartesian coordinate system; most surfaces seem to be curved. Some artifacts do have a flat base, however this is often a worn area of the artifact and practical tests have shown that alignment to such surfaces will not produce optimal alignment of the scan data.

As conventional methods of alignment do not always yield good results with these types of artifacts, a more adequate method of alignment has been developed to enable precise measurements and statistical analysis of the scan data.

To find the optimal position of the vessel in the coordinate system, a range of rotation and translation tests are carried out to find the best fit of the central axis.

Based on the assumption that the analyzed object was created using a rotational process, and thus have symmetry around a central axis, the alignment of the artifact is carried out in a two-step process. An overview of this process is given below.

The artifact is placed in a Cartesian coordinate system, in an initially unaligned state. The first step in the alignment process estimates the central rotational axis of the vessel, by analyzing the coaxiality of thin cross-section slices of the vessel. The slices will be as thin as possible based on the mesh density of the scan, while still ensuring enough data points in each slice to be statistically valid.

For each slice, circular regression[7] (estimate of best fit circle) is used to estimate the center point of this slice. Combined over the total Z-axis range of the vessel, these center points provide us with an indicator of the incline and position of the vessel's central axis.

The next step will optimize the center axis alignment by progressively minimizing the deviation (perpendicular to the surface curvature) of the two-dimensional profile, see Figure 1. By ascertaining and comparing the resulting fit of many thousands of different potential rotations, the best fit alignment of the scan data can be estimated, and an optimal center axis (in relation to the data points) can be reconstructed. The actual three-dimensional point-cloud is then aligned to this axis, by rotating and translating the scanned data points to match the Z-axis of the Cartesian coordinate system.

To enable extensive analysis of the full surface of the artifact, the mesh is split into exterior and interior surfaces. The exterior surface is aligned independently of interior data points, providing a baseline for exterior quality assessment. The interior surface is represented by two alignments:
- Aligned with the exterior mesh to analyze concentricity, and
- Aligned separately to assess its precision and compare the true tilt/displacement between interior and exterior surfaces.

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[7]Circle regression algorithm used: Kenichi Kanatani, Prasanna Rangarajan, "Hyper least squares fitting of circles and ellipses" Computational Statistics & Data Analysis, Vol. 55, pages 2197-2208, (2011)

Statistics used throughout the report

This section provides an overview of the key statistical and model-evaluation metrics employed throughout the report to analyze dataset variability, model fit, and predictive accuracy.

Each measure is introduced with its mathematical formulation, practical interpretation, and explicit reference to how it is calculated in the context of the evaluated models and residuals. Together, these metrics quantify:

- Data variability (e.g., MAD, Standard Deviation, Range).
- Model accuracy (e.g., MSD, RMSD).
- Robustness vs. sensitivity to extreme values and central tendencies.

Mean Squared Deviation (MSD), also known as Mean Squared Error (MSE).

$$\text{MSD} = \frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}$$

The Mean Squared Deviation (MSD) measures the average magnitude of squared differences between observed (y_i) and predicted ($\hat{y}$) values, calculated as the mean of squared residuals, and is a used as a measure of discrepancy in regression and model-fitting contexts.

This measure amplifies the influence of larger deviations through squaring, emphasizes imperfections in the observed data, but retains sensitivity to outliers.

Root Mean Squared Deviation (RMSD), also known as Root Mean Squared Error (RMSE).

$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y})^2}{n}}$$

The Root Mean Square Deviation (RMSD) measures the magnitude of differences between observed (y_i) and predicted ($\hat{y}$) values by calculating the square root of the average of squared residuals.

RMSD is a commonly used measure of discrepancy in regression and model-fitting contexts. It quantifies the average magnitude of residuals while retaining sensitivity to larger deviations (via squaring), making it particularly useful for evaluating model accuracy.

Standard Deviation (SD)

$$s = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}}$$

The Standard Deviation measures the spread of data (y_i) around the mean ($\bar{y}$) by calculating the square root of the average of squared differences between each value and the mean.

It is sensitive to outliers as it amplifies their influence through squaring, in contrast to MAD.

Throughout this report, the Standard Deviation is calculated using the absolute residuals from regression models.

Median Absolute Deviation (MedianAD)

$$\text{MedianAD} = \text{median}(|y_i - \text{median}(y)|)$$

The Median Absolute Deviation (MAD) measures the spread of data around the median by calculating the median of absolute differences between each value and the median.

MAD is a robust measure of spread, analogous to the interquartile range (a robust measure centered on the middle 50% of data), and differs from the standard deviation in that it minimizes the impact of outliers.

Throughout this report, the MAD is calculated using the absolute values of residuals from regression models.

Range

$$\max(y_i) - \min(y_i)$$

The Range measures the spread of a dataset by calculating the difference between the maximum and minimum values.

The Range is a simple measure of spread, capturing the full extent of variability. Range is very sensitive to extreme values, as it is entirely determined by the two most extreme data points.

Throughout this report, the Range is calculated using the full range of residuals from regression models.

Precision

To explore the manufacturing precision of the artifact in depth, the following analysis have been carried out:

- Circularity around the axis of symmetry is examined in detail at selected cross-sections.
- Overall circularity around the axis of symmetry is measured for the full height of the vessel (areas of the vessel with extensive damage are not taken into account for this metric).
- Concentricity of the vessel between selected cross-sections are examined in detail to determine if the existence of an axis of rotation in the manufacture of the object can be established.
- The coaxiality of the vessel is analyzed to explore the precision of the central axis of the object.
- The surface variability is analyzed and visualized on through a heatmap.

Circularity

Circularity is the measurement of how round the surface of an object is, optionally in reference to a datum axis. The *circularity tolerance* is the radial distance of two circles, each with their centers in the datum axis, and each of them conforming, respectively, to the minimum and maximum deviations of the data-set to a true circle, see Figure 2.
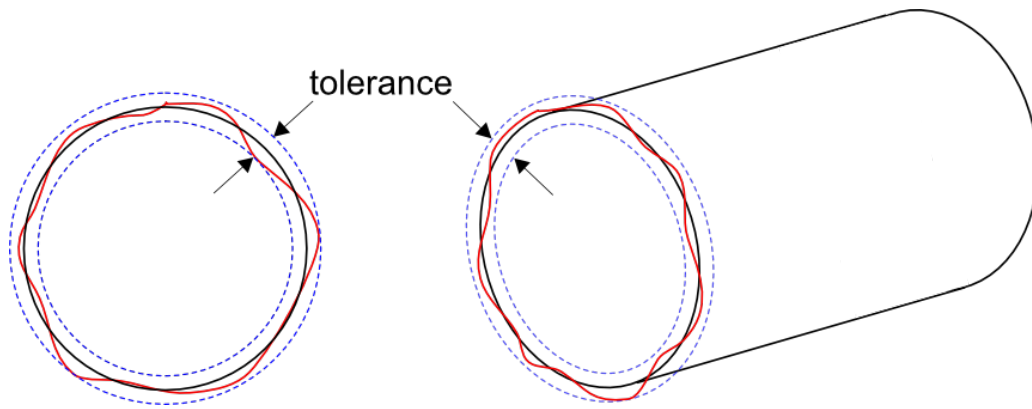
Figure 2: Circularity tolerance.

Circularity is examined at different cross-sections of the vessel, using the established Z-axis as the datum axis (axis of symmetry). The distance between the scanned points in the local datum plane is measured to determine the range between the two concentric circles encompassing the measured points, see Figure 3.

Referencing all of the individual circularity measurements to the global (reconstructed) axis of symmetry of the object, allows us to ascertain not only circularity of local features of the object, but how well circularity was *maintained* over the entire manufacturing process. This is an important distinction, which may be able to provide valuable insights into requirements of the construction methods. For reference, and seeing that the variance in local circularity also holds interest, measurements of circularity of the vessel without reference to the axis of symmetry can additionally be found in the Concentricity, p. 31.
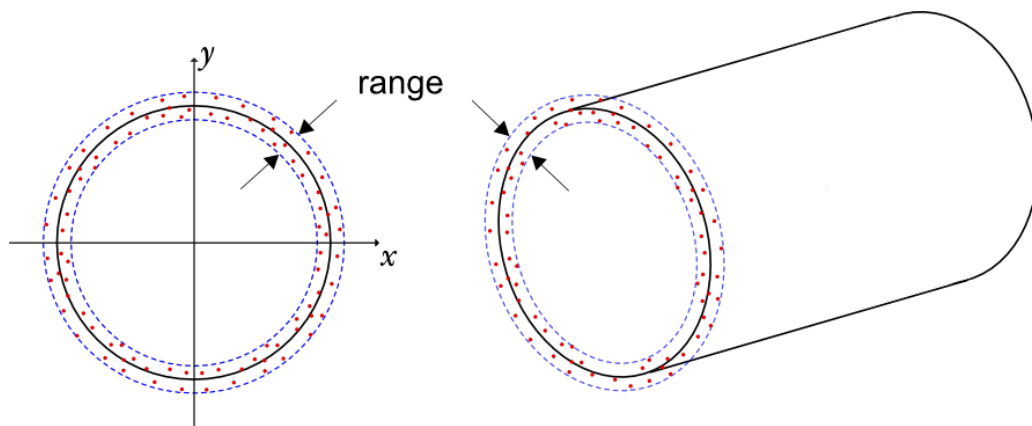
Figure 3: Circularity measurements.

If the circularity is determined from slices of the vessel exclusively in the *Z-plane* (actually measuring the cylindricity of a very thin slices of the vessel, in an attempt to approximate circularity), this would - in some areas - introduce significant distortion (increasing measurement errors) in the samples, due to the curvature of the vessel's surface.

Each sample slice of the vessel is therefore obtained perpendicular to the surface curvature, see Figure 6 to Figure 14. The measurements are taken conservatively without filtration of potential outliers.

To explore the potential distortion caused by obtaining samples in the Z-plane only, please refer to Appendix A, where measurements in the Z-plane and measurements perpendicular to surface curvature are compared side by side.

Detailed circularity measurements of selected points

Circularity measurements across a range of selected slices of the vessel (see Table 1) have been analyzed in-depth, and detailed plots of each measurement is provided. Furthermore, full circularity measurements are shown for each available scanned surface including a detailed plot to visualize the circularity of all areas of the vessel.
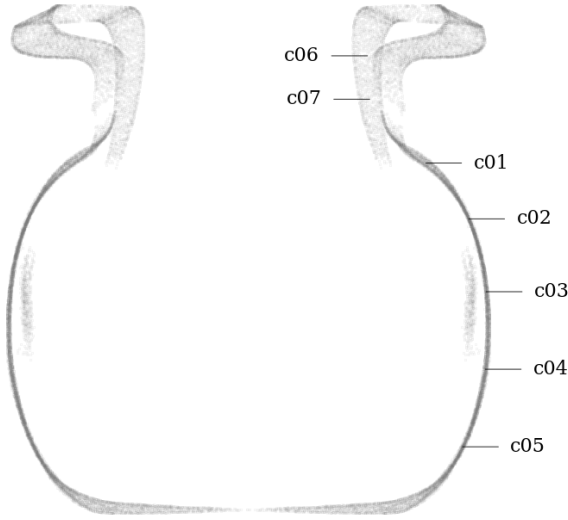
Figure 4: Circularity measurement sample locations,
full mesh aligned with exterior surface
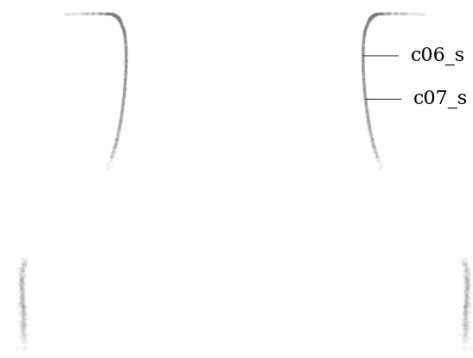
Figure 5: Circularity measurement sample location,
separately aligned interior mesh

Metric

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø65.457±0.923	1.590	0.467	0.162	0.222	430	0.200	64.210	32.728
c02	exterior	Ø81.326±0.435	0.715	0.129	0.048	0.089	764	0.200	53.902	40.663
c03	exterior	Ø87.791±0.399	0.719	0.176	0.059	0.083	716	0.200	40.434	43.896
c04	exterior	Ø87.422±0.508	0.971	0.205	0.086	0.117	706	0.200	26.104	43.711
c05	exterior	Ø79.051±0.374	0.718	0.170	0.067	0.106	687	0.200	11.775	39.526
c06	interior	Ø44.051±2.731	5.261	1.812	0.628	0.815	365	0.200	84.031	22.026
c06_s	interior sep.	Ø43.738±0.200	0.337	0.064	0.025	0.036	332	0.200	84.031	21.869
c07	interior	Ø44.874±2.105	4.175	1.456	0.490	0.627	332	0.200	76.021	22.437
c07_s	interior sep.	Ø44.666±0.189	0.375	0.084	0.036	0.052	324	0.200	76.021	22.333

Imperial

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø2.5770±0.0363	0.0626	0.0184	0.0064	0.0087	430	0.0079	2.5280	1.2885
c02	exterior	Ø3.2018±0.0171	0.0282	0.0051	0.0019	0.0035	764	0.0079	2.1221	1.6009
c03	exterior	Ø3.4564±0.0157	0.0283	0.0069	0.0023	0.0033	716	0.0079	1.5919	1.7282
c04	exterior	Ø3.4418±0.0200	0.0382	0.0081	0.0034	0.0046	706	0.0079	1.0277	1.7209
c05	exterior	Ø3.1123±0.0147	0.0283	0.0067	0.0026	0.0042	687	0.0079	0.4636	1.5561
c06	interior	Ø1.7343±0.1075	0.2071	0.0713	0.0247	0.0321	365	0.0079	3.3083	0.8672
c06_s	interior sep.	Ø1.7220±0.0079	0.0133	0.0025	0.0010	0.0014	332	0.0079	3.3083	0.8610
c07	interior	Ø1.7667±0.0829	0.1644	0.0573	0.0193	0.0247	332	0.0079	2.9930	0.8834
c07_s	interior sep.	Ø1.7585±0.0075	0.0148	0.0033	0.0014	0.0020	324	0.0079	2.9930	0.8792

Table 1: Detailed circularity measurements at selected samples of HV004.

Figure 6 to Figure 14 shows a detailed plots of each circularity measurement.

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[8] Sample diameter Ø± maximum measured deviation from measured radius

[9] Root mean square deviation (RMSD) also called Root mean square error (RMSE)

[10] Median absolute deviation

[11] Median sample radius from z-axis

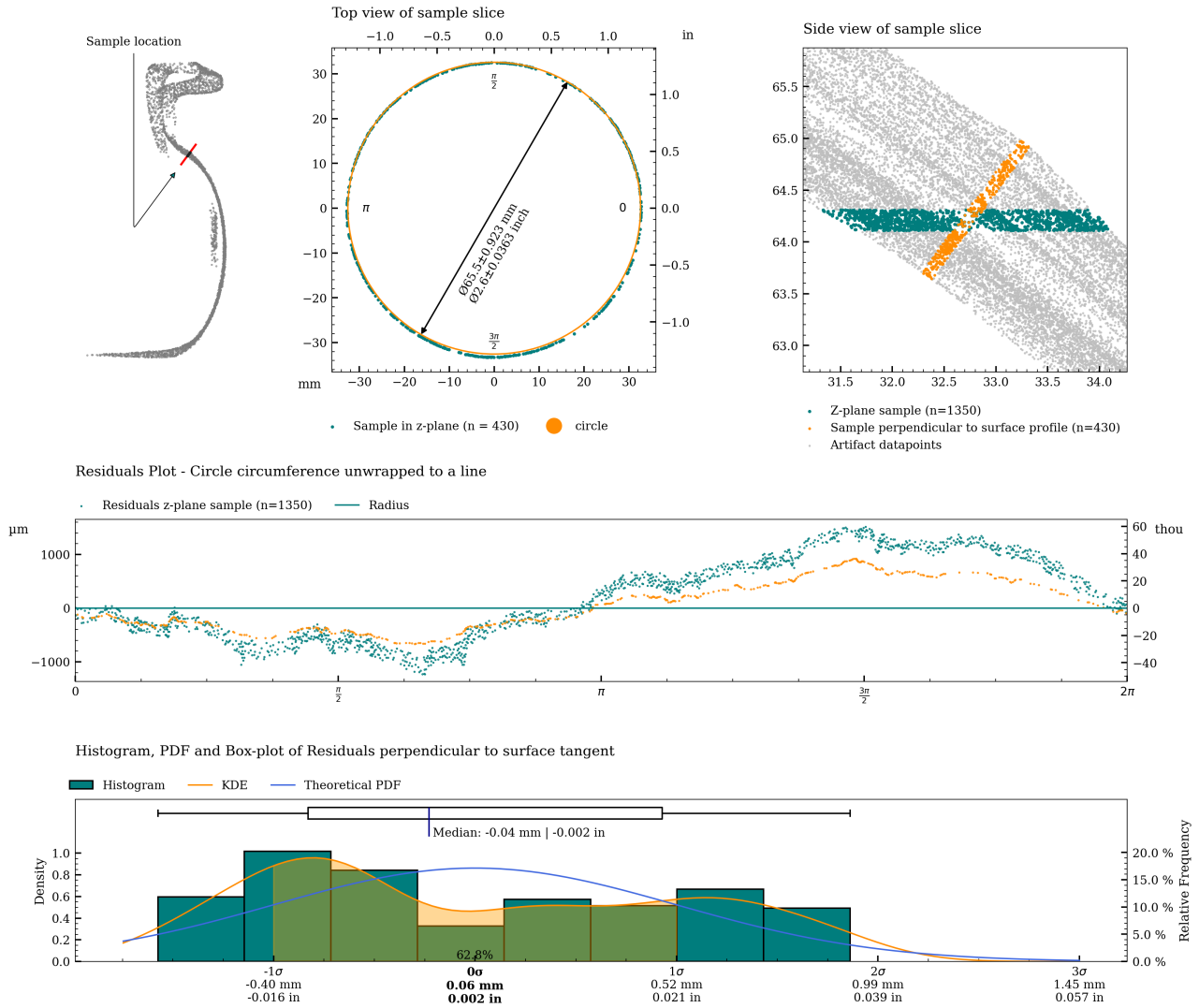# Graphical overview of circularity measurement c01

Sample location

Top view of sample slice

Ø65.5±0.923 mm
Ø2.6±0.0363 inch

- Sample in z-plane (n = 430)
- circle

Side view of sample slice

- Z-plane sample (n=1350)
- Sample perpendicular to surface profile (n=430)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Residuals z-plane sample (n=1350) — Radius

Histogram, PDF and Box-plot of Residuals perpendicular to surface tangent

- Histogram — KDE — Theoretical PDF

Median: -0.04 mm | -0.002 in

62.8%

-1σ	0σ	1σ	2σ	3σ	
-0.40 mm	**0.06 mm**	0.52 mm	0.99 mm	1.45 mm	
-0.016 in	**0.002 in**	0.021 in	0.039 in	0.057 in	

Figure 6: Charts with statistics for the measurement of c01.

Graphical overview of circularity measurement c02
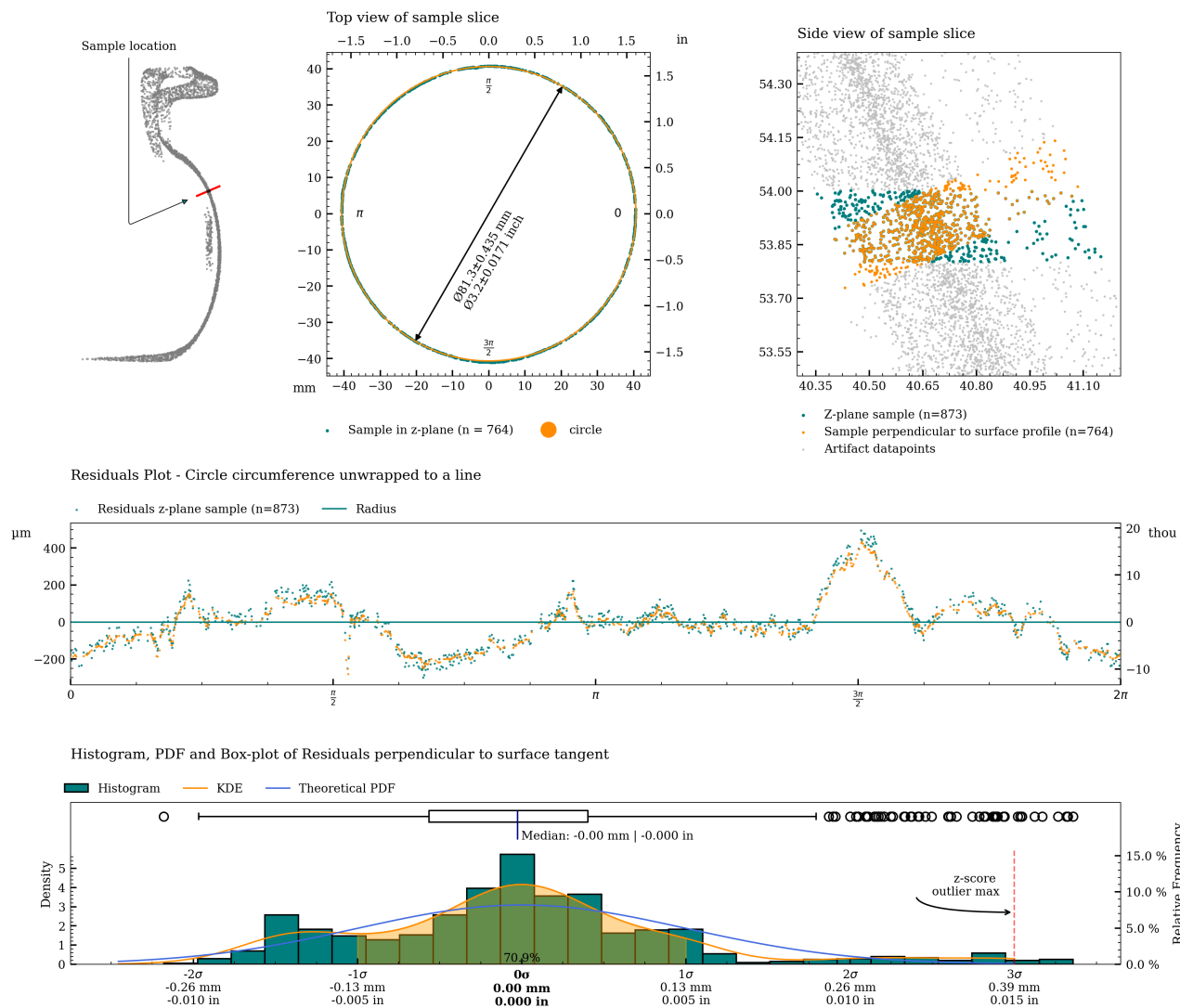
Figure 7: Charts with statistics for the measurement of c02.

Graphical overview of circularity measurement c03

Sample location

Top view of sample slice

Ø87.8±0.399 mm
Ø3.5±0.0157 inch

- Sample in z-plane (n = 716)
- circle

Side view of sample slice

- Z-plane sample (n=730)
- Sample perpendicular to surface profile (n=716)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Residuals z-plane sample (n=730)
- Radius

Histogram, PDF and Box-plot of Residuals perpendicular to surface tangent

- Histogram
- KDE
- Theoretical PDF

Median: -0.05 mm | -0.002 in

58.2%

	-2σ	-1σ	0σ	1σ	2σ	3σ
	-0.35 mm	-0.17 mm	**0.00 mm**	0.18 mm	0.36 mm	0.53 mm
	-0.014 in	-0.007 in	**0.000 in**	0.007 in	0.014 in	0.021 in

Figure 8: Charts with statistics for the measurement of c03.

Graphical overview of circularity measurement c04
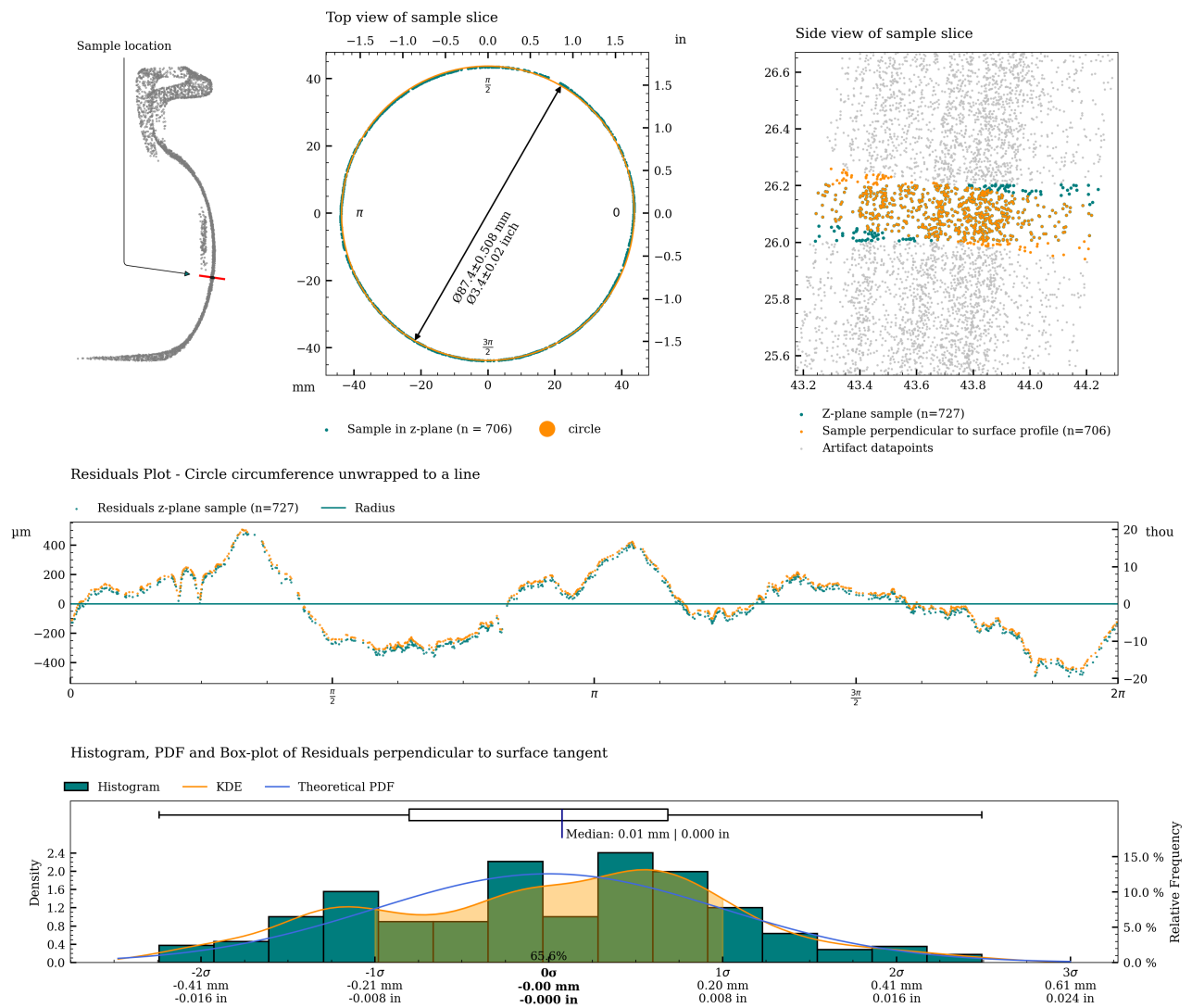
Figure 9: Charts with statistics for the measurement of c04.

Graphical overview of circularity measurement c05

Sample location

Top view of sample slice

Side view of sample slice

Ø79.1±0.374 mm
Ø3.1±0.0147 inch

- Sample in z-plane (n = 687)
- circle

- Z-plane sample (n=858)
- Sample perpendicular to surface profile (n=687)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Residuals z-plane sample (n=858)
- Radius

Histogram, PDF and Box-plot of Residuals perpendicular to surface tangent

- Histogram
- KDE
- Theoretical PDF

Median: -0.00 mm | -0.000 in

66.7%

-2σ	-1σ	0σ	1σ	2σ	3σ
-0.35 mm	-0.18 mm	**-0.01 mm**	0.16 mm	0.33 mm	0.50 mm
-0.014 in	-0.007 in	**-0.000 in**	0.006 in	0.013 in	0.020 in

Figure 10: Charts with statistics for the measurement of c05.

Graphical overview of circularity measurement c06
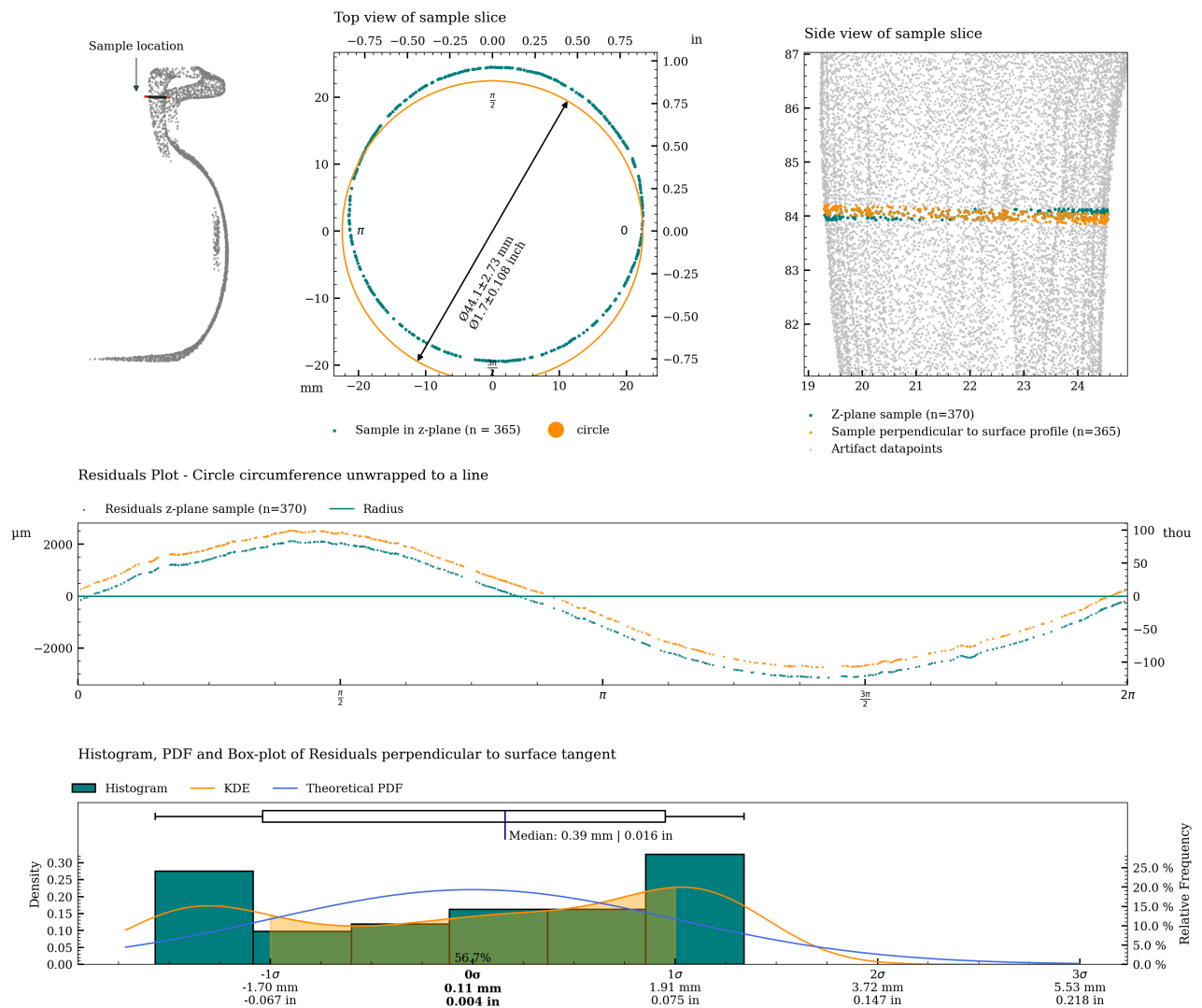
Figure 11: Charts with statistics for the measurement of c06.

Graphical overview of circularity measurement c06_s
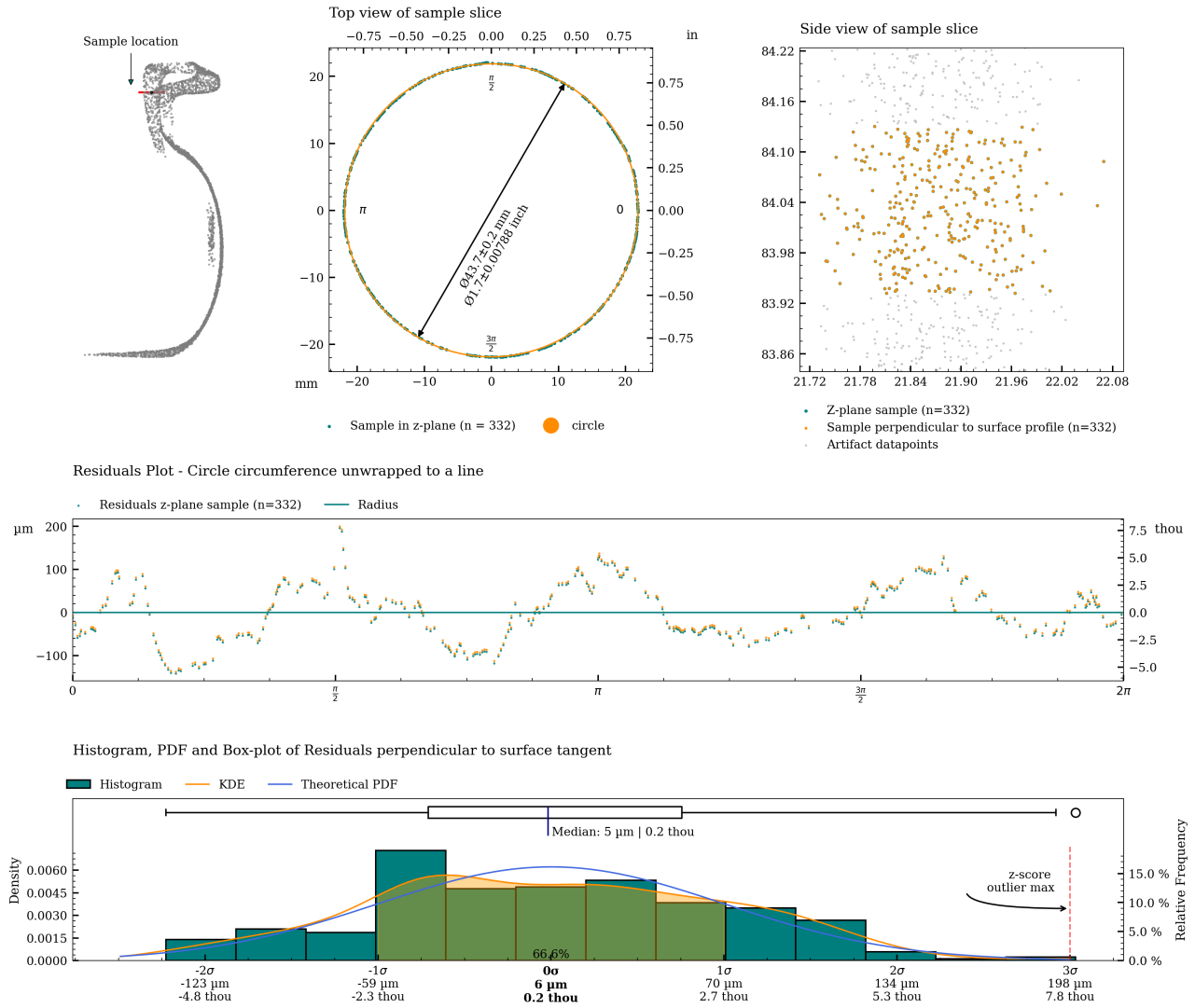
Figure 12: Charts with statistics for the measurement of c06_s.

Graphical overview of circularity measurement c07
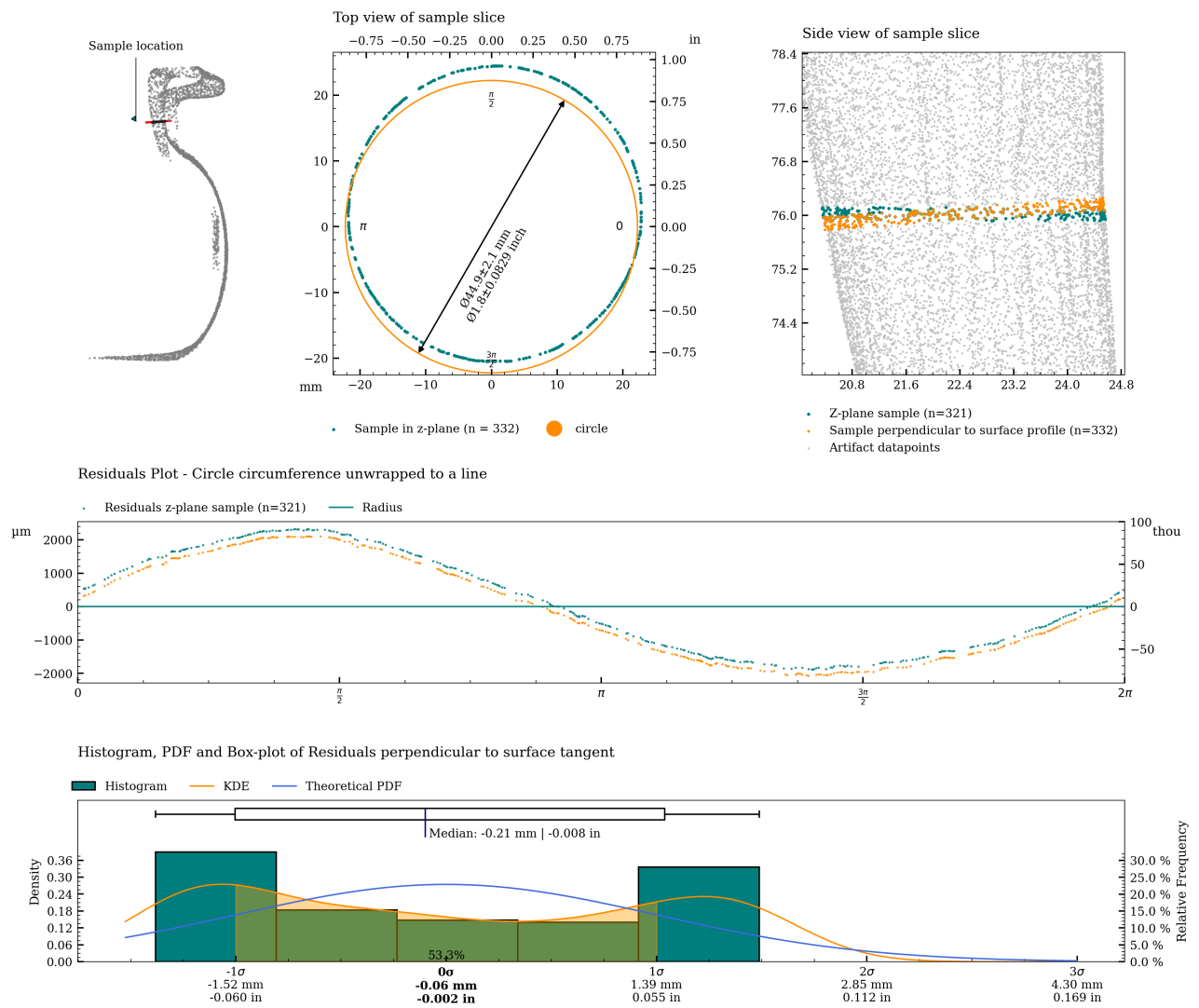
Figure 13: Charts with statistics for the measurement of c07.

Graphical overview of circularity measurement c07_s
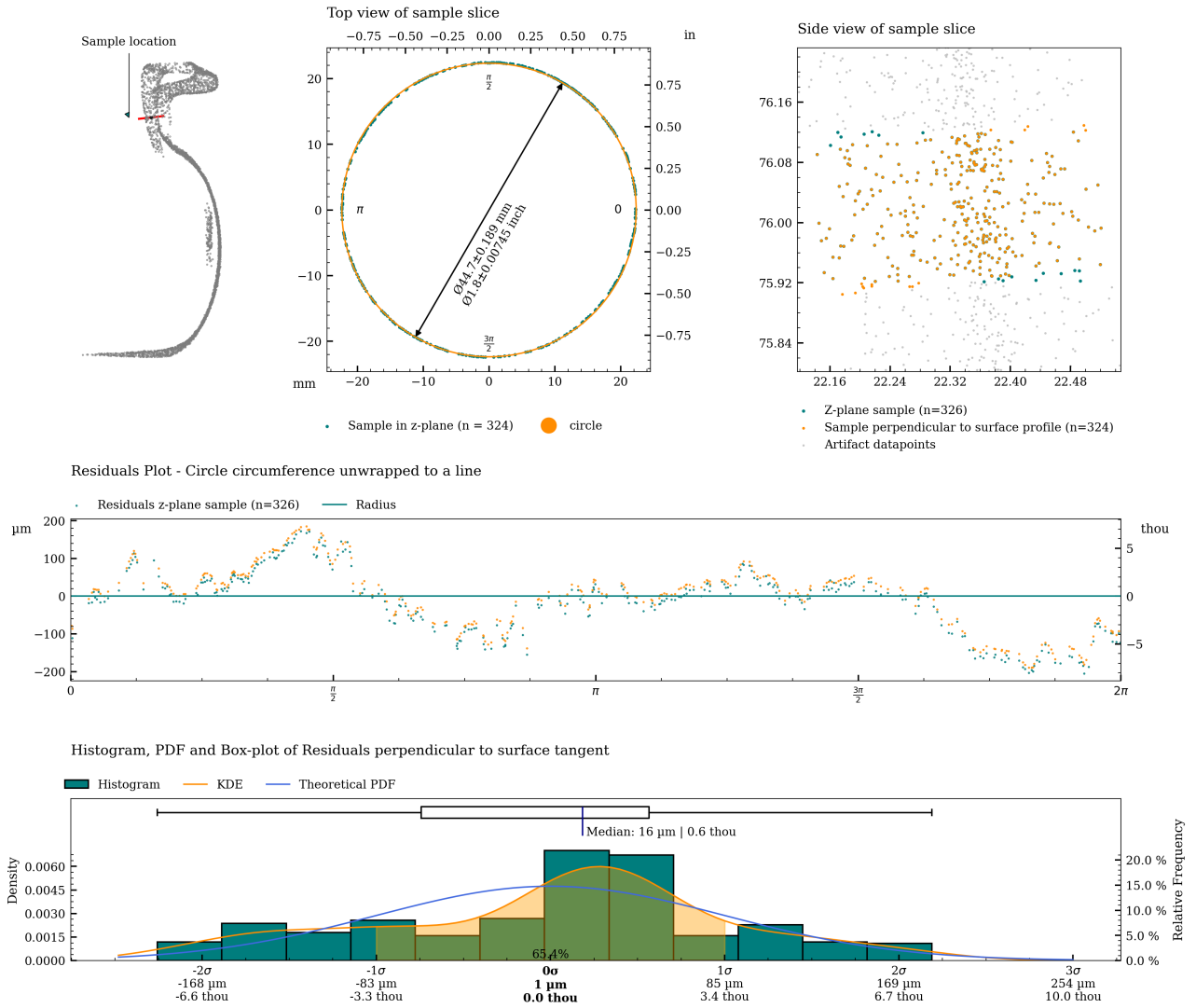
Figure 14: Charts with statistics for the measurement of c07_s.

Table 2 shows statistical measures of the circularity of the vessel, measured along the full height (areas on the artifact scan containing damaged parts have been removed to the best extent possible to reduce the influence of the measurement).

Metric

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.829	0.665	1.386	0.106	0.081	0.194	0.198	0.123	0.416	326	0.200
Interior	4.324	1.923	6.604	0.649	0.253	1.017	1.454	0.492	2.325	110	0.200
Interior separate	0.392	0.281	0.777	0.050	0.034	0.085	0.084	0.063	0.138	110	0.200

Imperial

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.829	0.665	1.386	0.106	0.081	0.194	0.198	0.123	0.416	326	0.200
Interior	4.324	1.923	6.604	0.649	0.253	1.017	1.454	0.492	2.325	110	0.200
Interior separate	0.392	0.281	0.777	0.050	0.034	0.085	0.084	0.063	0.138	110	0.200

Table 2: Perpendicular Circularity analysis of HV004.

Circularity analysis of exterior surface
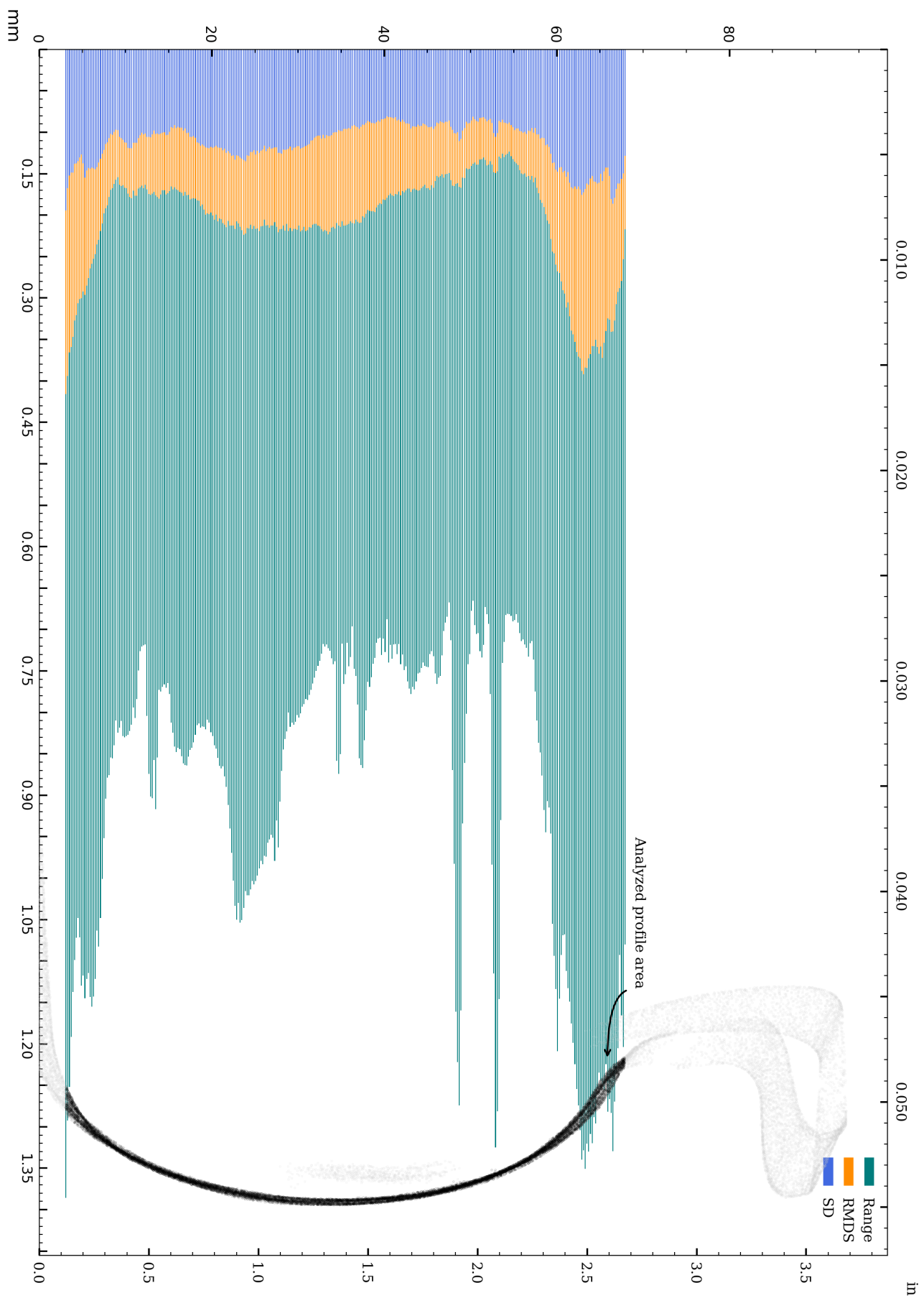
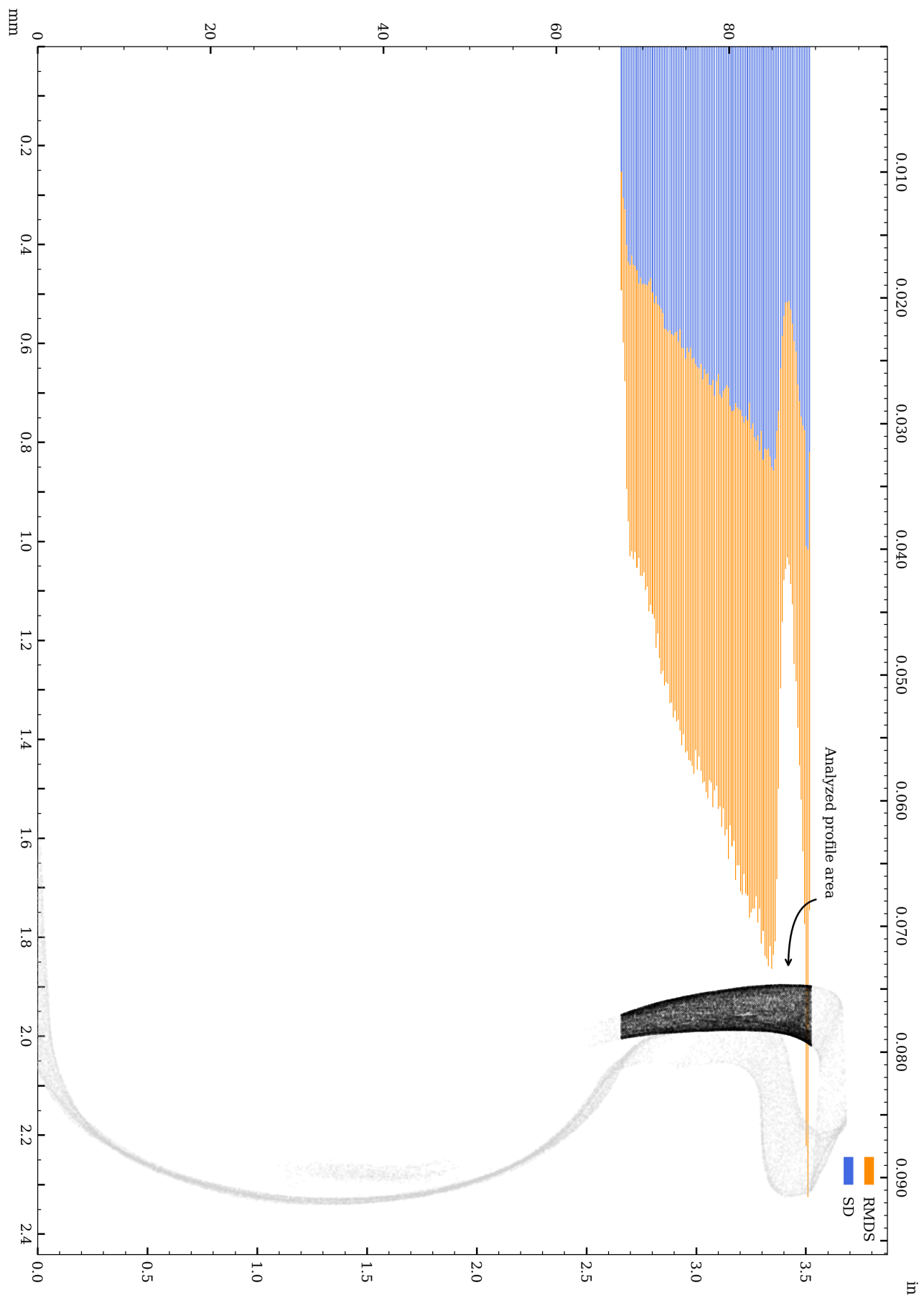Figure 15: Circularity of exterior surface.
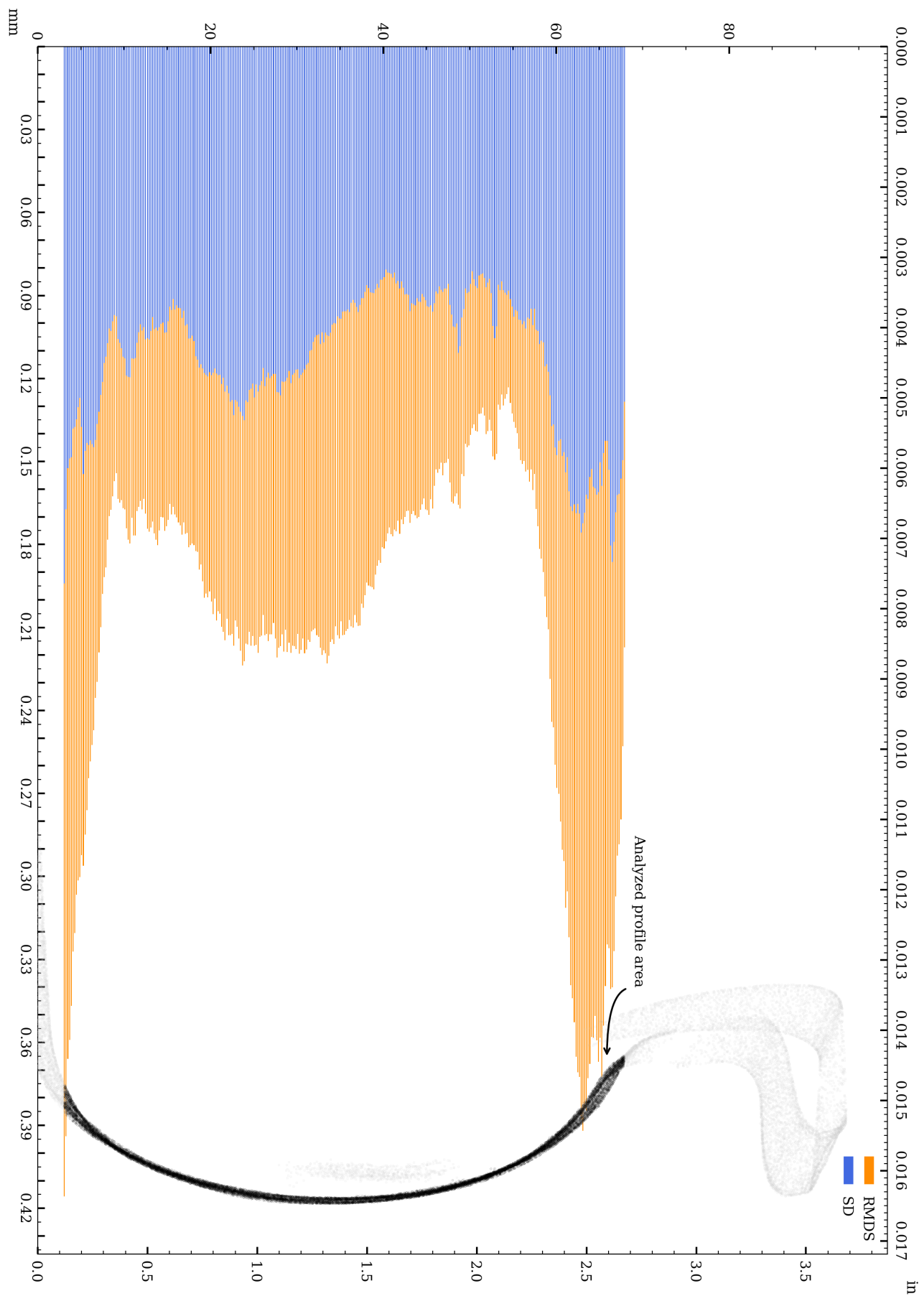
Figure 16: Vessel circularity of exterior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 326 slices of the exterior surface are shown below.

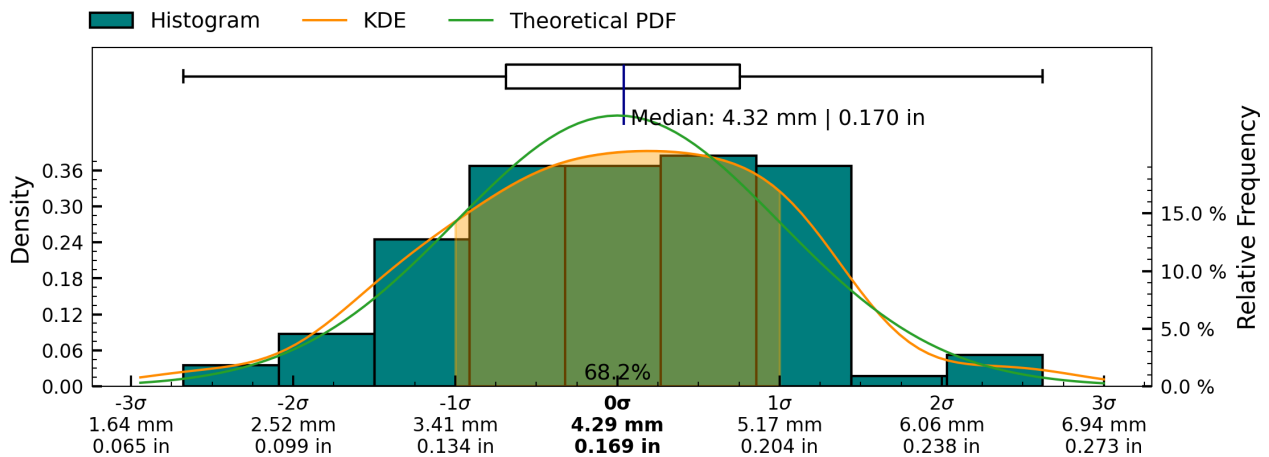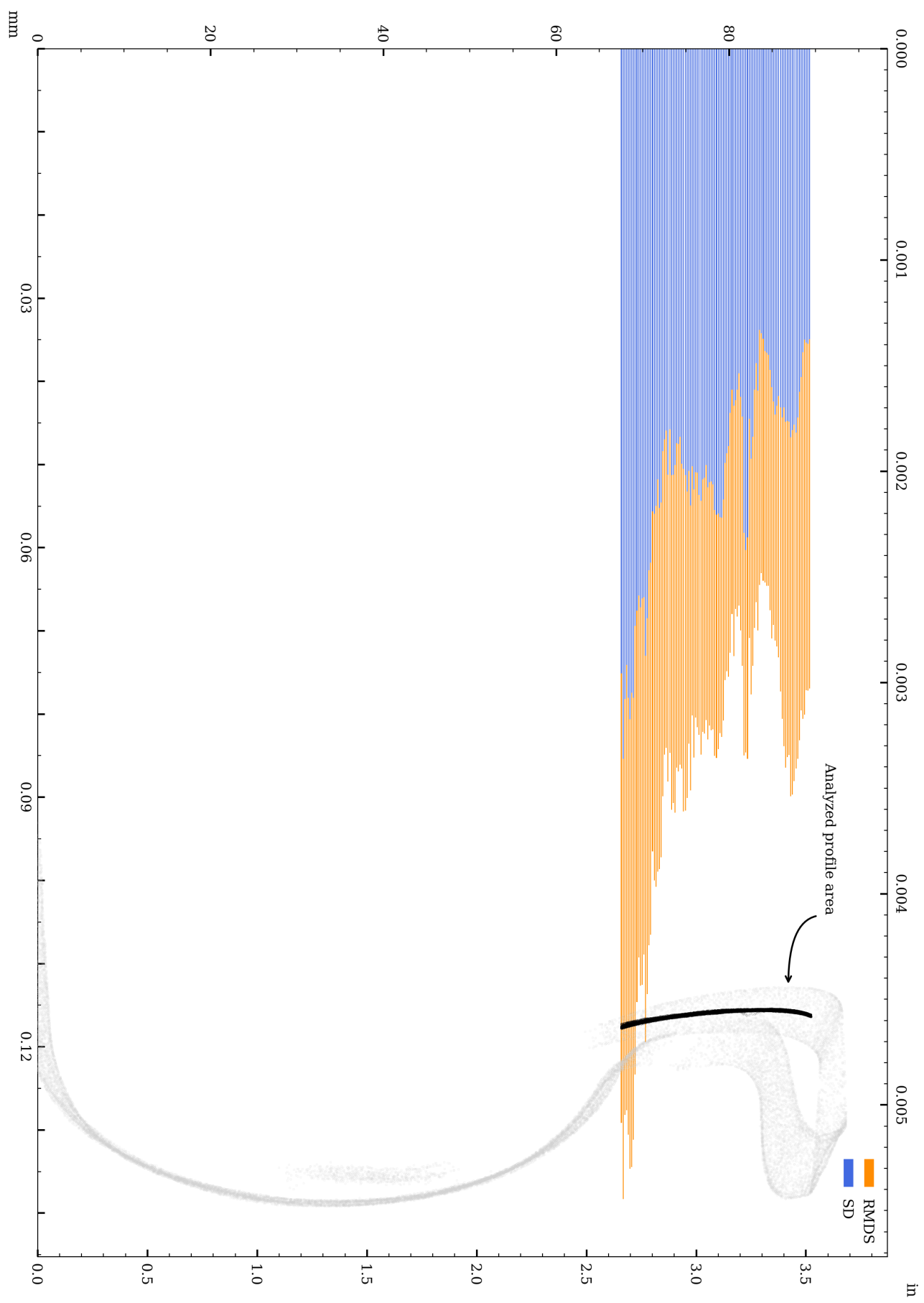**Range measurement distribution across 326 slices of exterior surface**
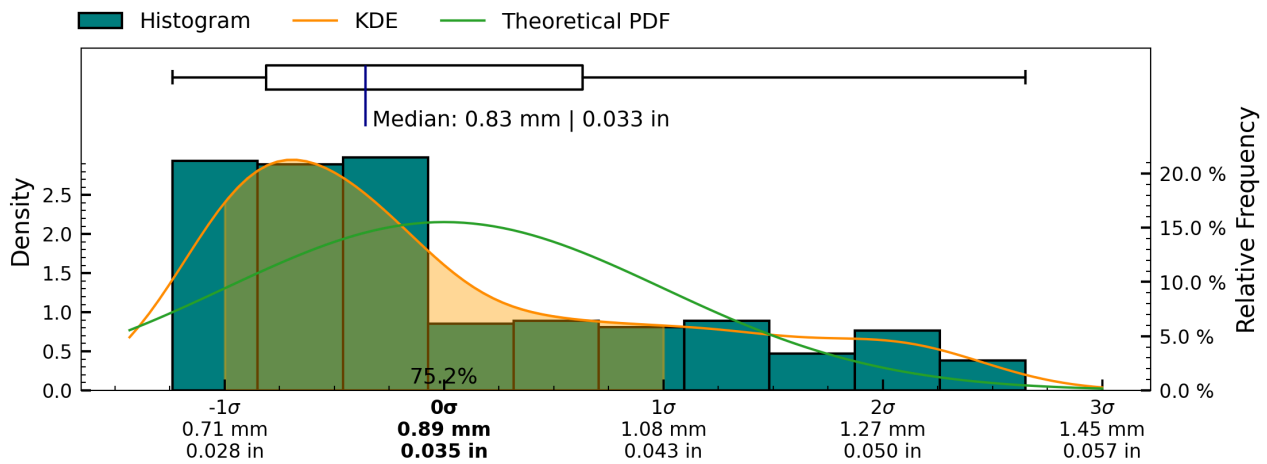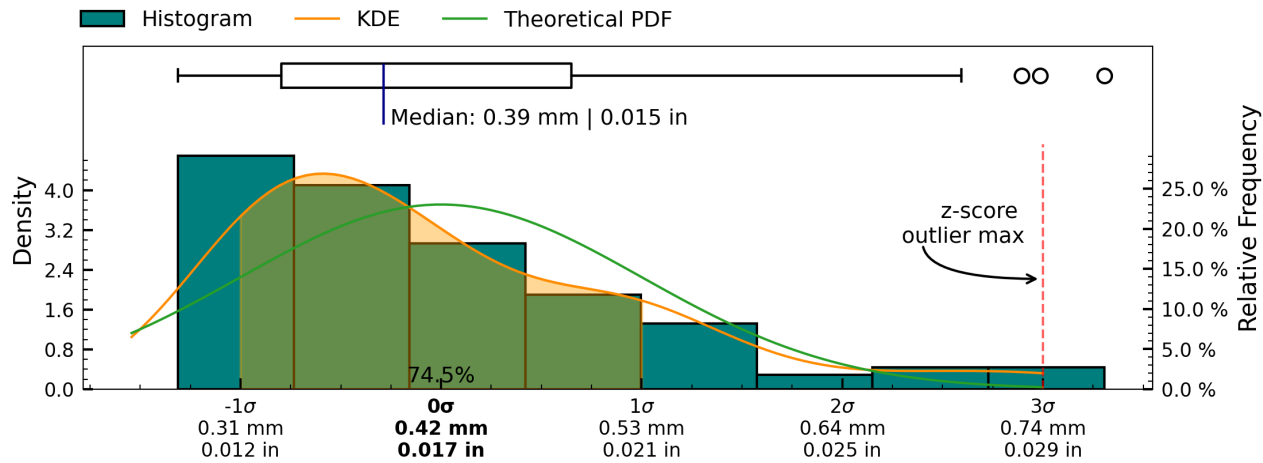

Figure 17: Range measurement distribution across measured slices of exterior surface

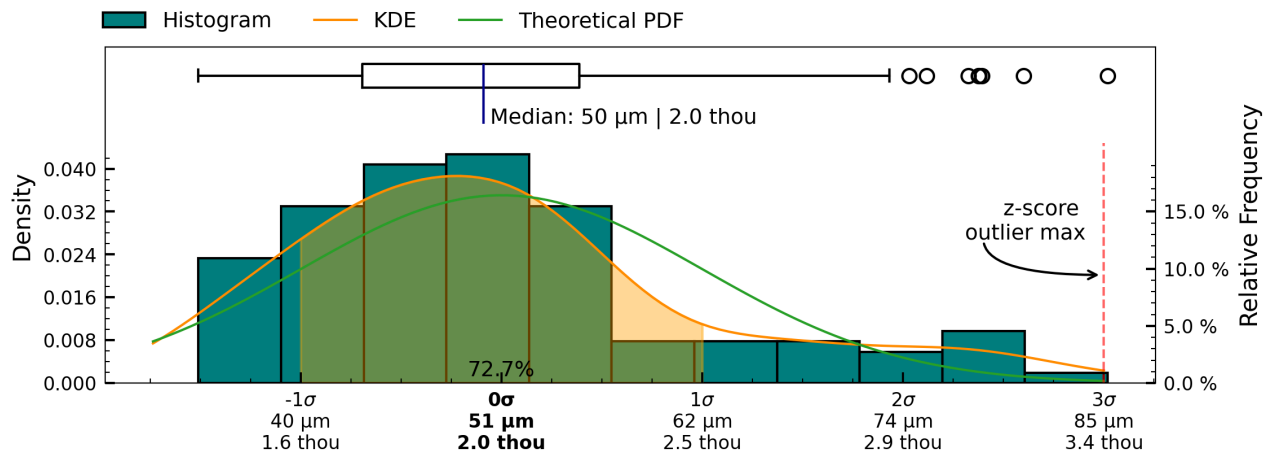**Standard Deviation measurement distribution across 326 slices of exterior surface**
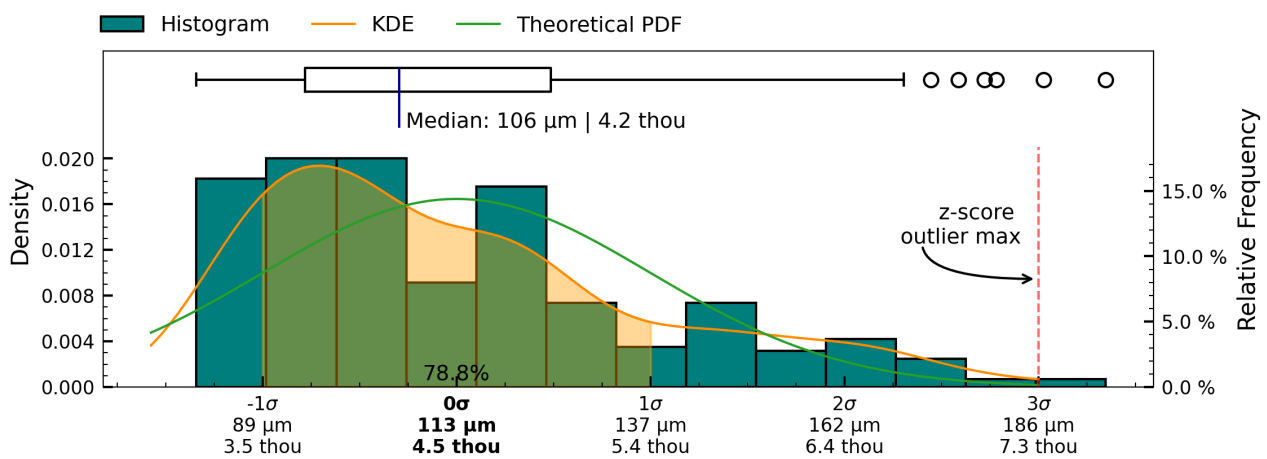
Figure 18: Standard Deviation measurement distribution across measured slices of " + exterior + " surface

Root Mean Squared Deviation measurement distribution across 326 slices of exterior surface
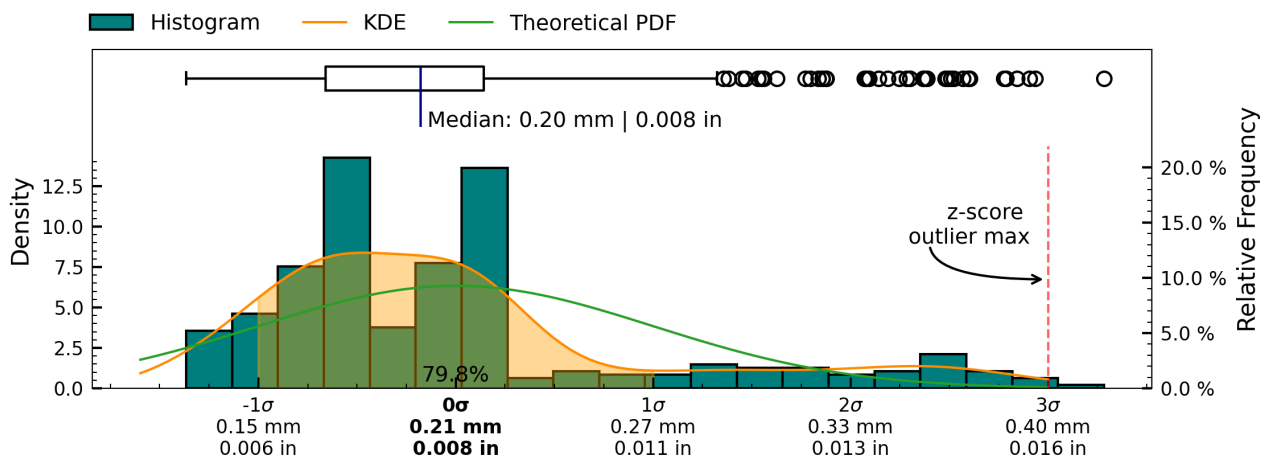
Figure 19: Root Mean Squared Deviation measurement distribution across measured slices of exterior surface
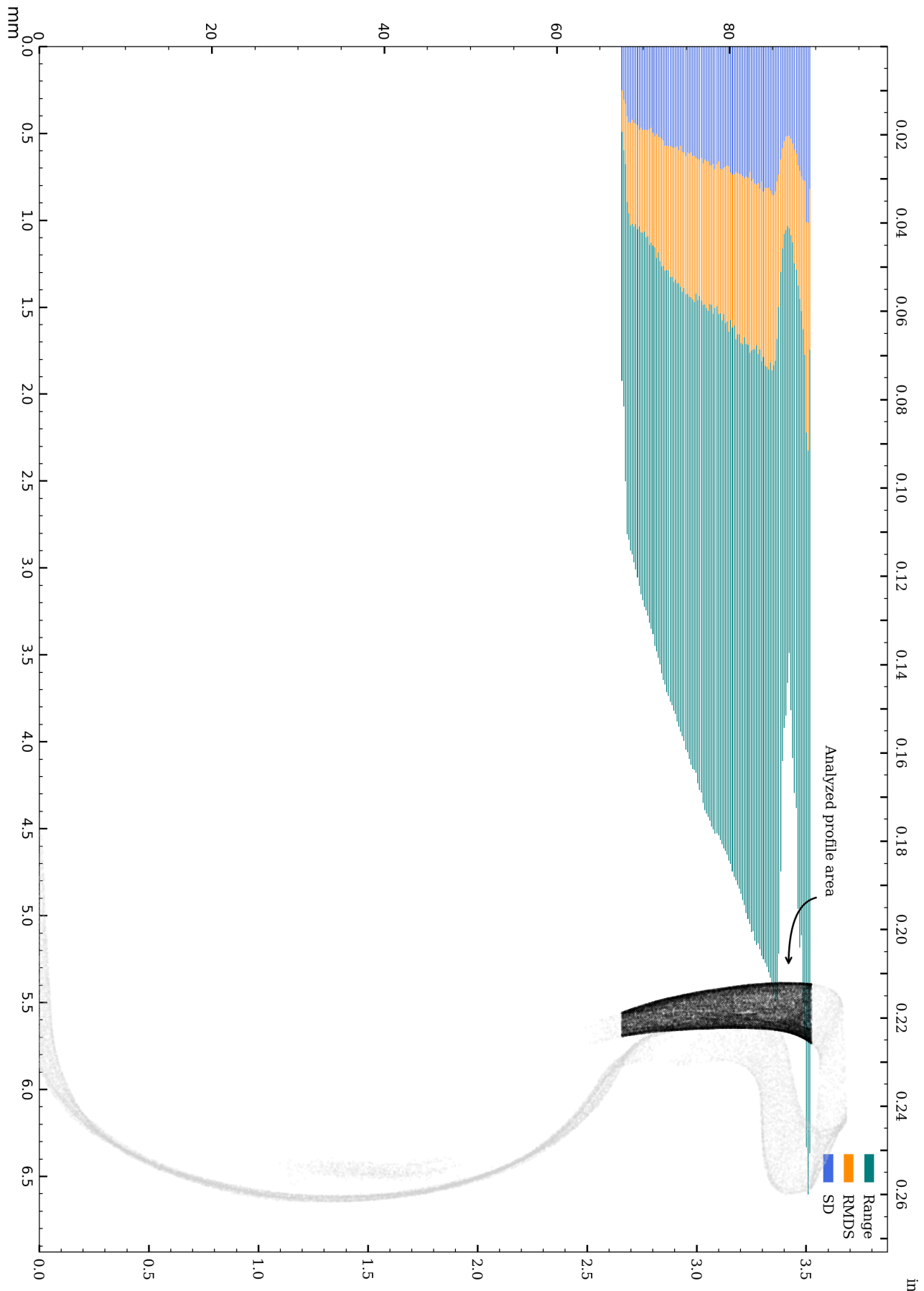
Figure 20: Circularity of interior surface.

Circularity analysis of interior surface, Standard Deviation and Root Mean Squared Deviation

Figure 21: Vessel circularity of interior surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 110 slices of the interior surface are shown below.

Range measurement distribution across 110 slices of interior surface

Figure 22: Range measurement distribution across measured slices of interior surface

Standard Deviation measurement distribution across 110 slices of interior surface
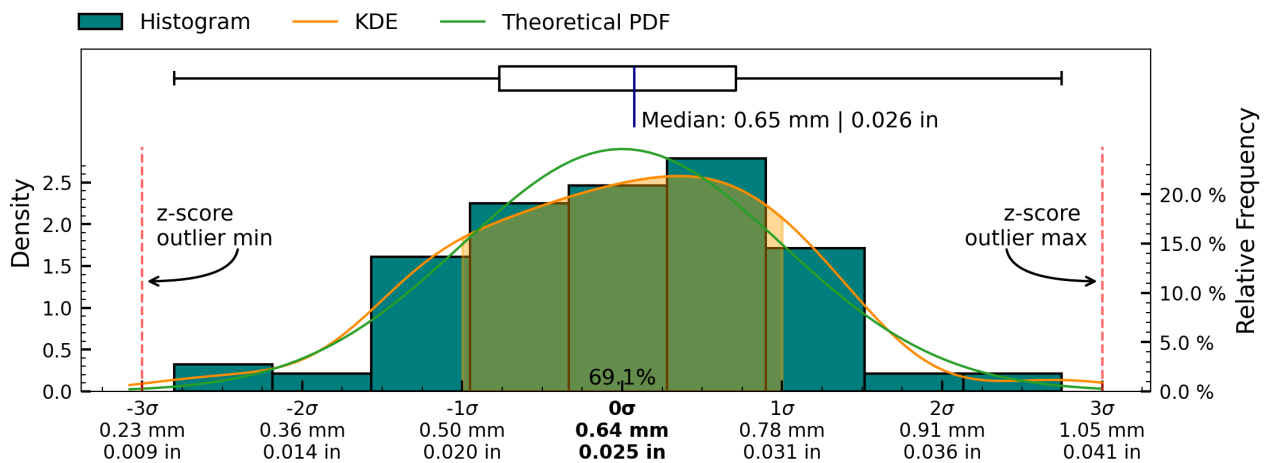
Figure 23: Standard Deviation measurement distribution across measured slices of " + interior + " surface

Root Mean Squared Deviation measurement distribution across 110 slices of interior surface
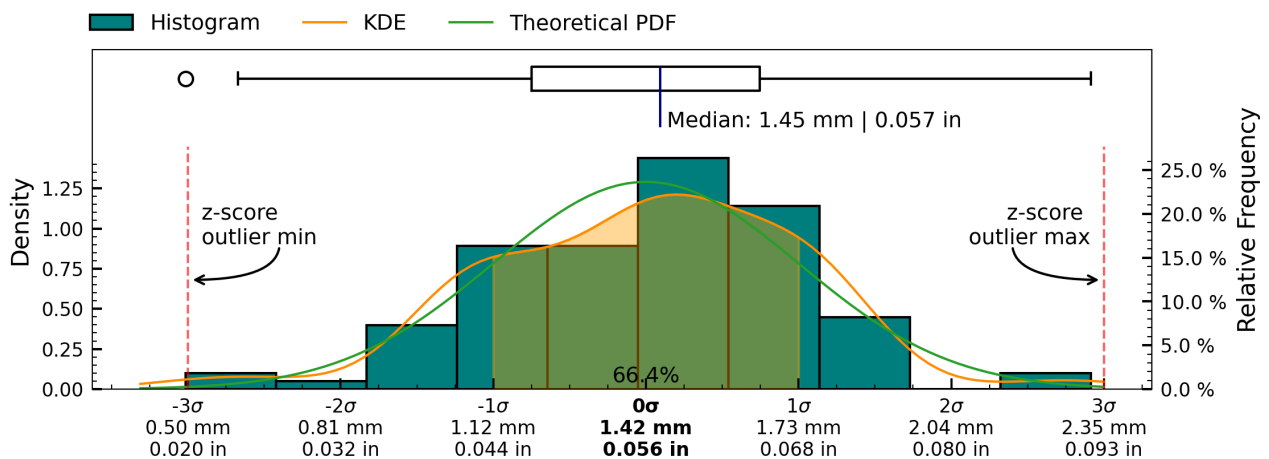
Figure 24: Root Mean Squared Deviation measurement distribution across measured slices of interior surface

Circularity analysis of interior separately aligned surface
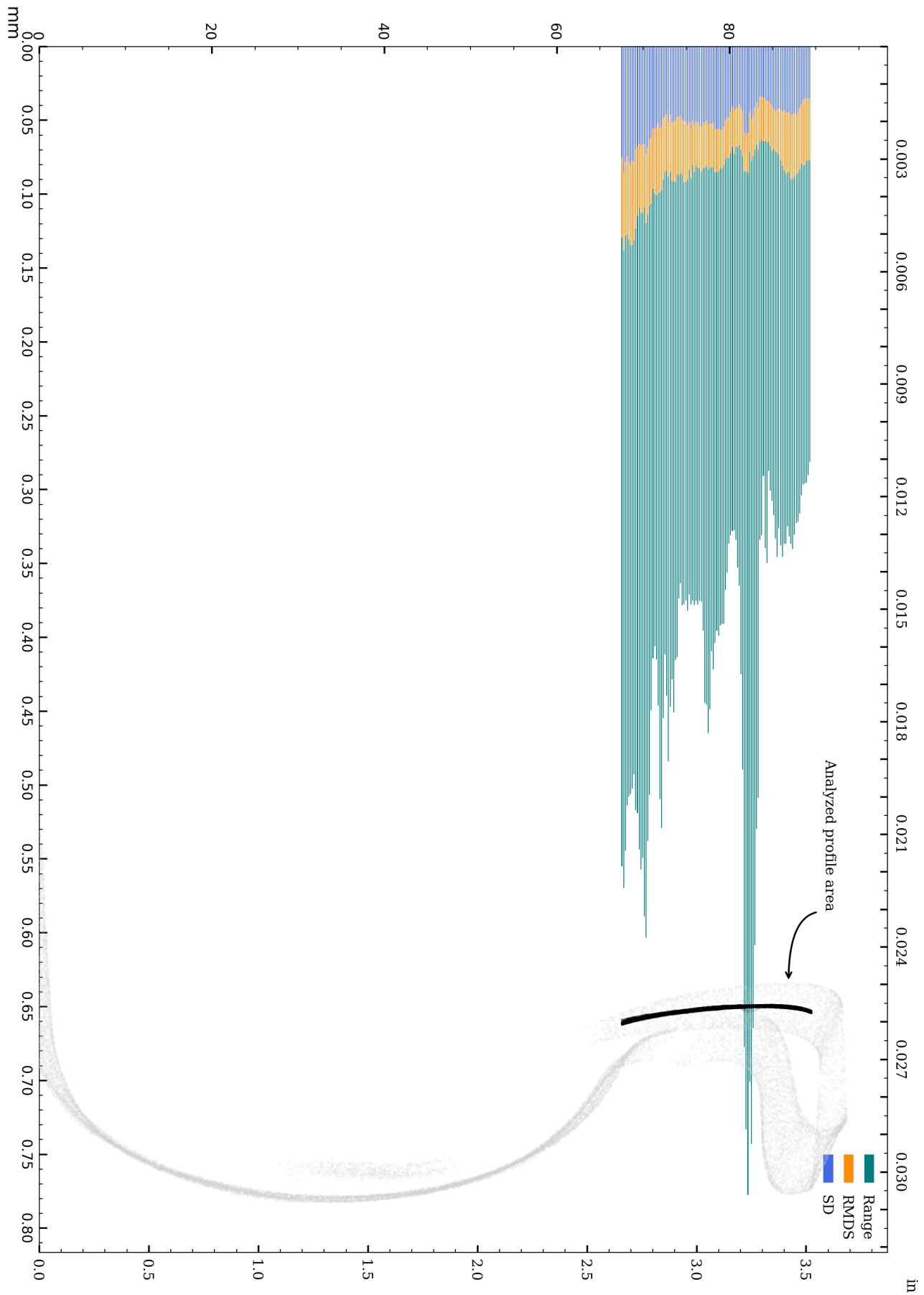
Figure 25: Circularity of interior_separate surface.

Figure 26: Vessel circularity of interior_separate surface, standard deviation and median absolute deviation.

The distributions of the circularity measurements across 110 slices of the interior_separate surface are shown below.

Range measurement distribution across 110 slices of interior separately aligned surface

Figure 27: Range measurement distribution across measured slices of interior_separate surface

Standard Deviation measurement distribution across 110 slices of interior separately aligned surface

Figure 28: Standard Deviation measurement distribution across measured slices of " + interior_separate + " surface

Root Mean Squared Deviation measurement distribution across 110 slices of interior separately aligned surface
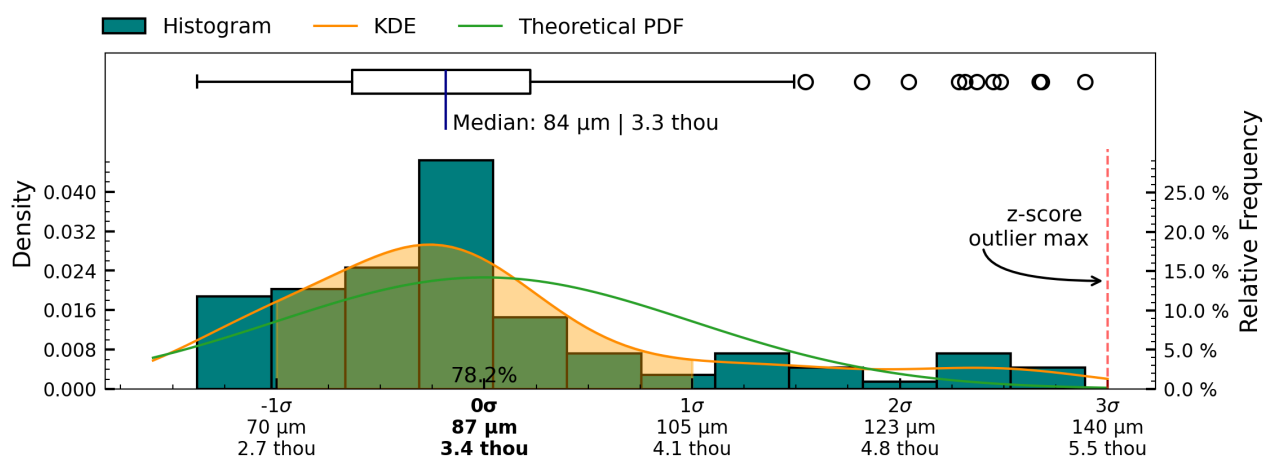
Figure 29: Root Mean Squared Deviation measurement distribution across measured slices of interior separately aligned surface

Concentricity

The concentricity metric describes the deviation in the center-point of the referenced features. As such, it is a measure to determine if several features of the object share the same center point/axis, and how closely. See Figure 30 for a visual representation of this metric.
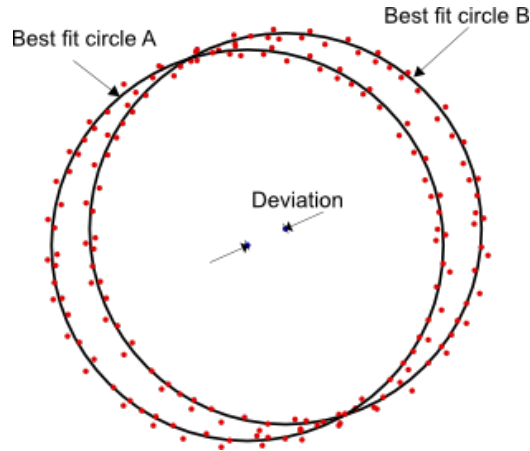
Figure 30: Concentricity measures the deviation (distance) between the center of two circles.

Determination of concentricity has been carried out by establishing the best fit circles of sample slices, using RANSAC (Random sample consensus) algorithm for outlier detection of a least squares circle regression on the scanned data-points at each cross-section, to estimate centers of each cross-section.

The concentricity between both the interior and exterior circular cross-sections is explored for cross-section measurements with the same Z-coordinates.

Additionally, the concentricity between each cross-section measurement defined in Figure 4 and the datum axis $(x, y) = (0, 0)$ has been calculated to establish the deviation of the feature center from the datum axis.
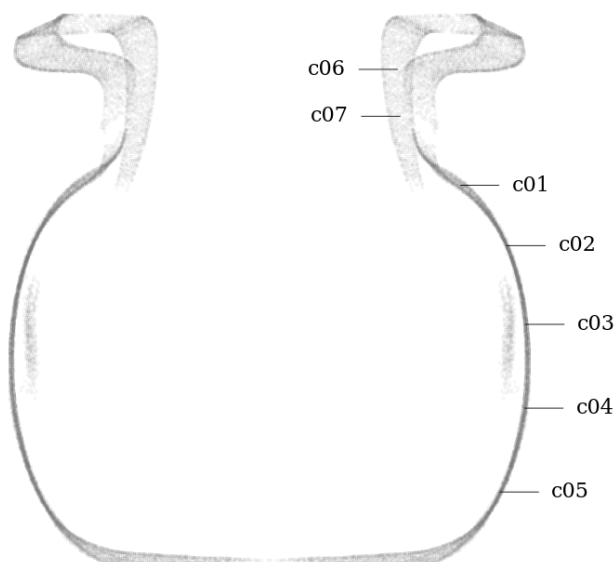
Figure 31: Circularity measurement sample locations, full mesh aligned to exterior surface

Figure 32: Circularity measurement sample location, separately aligned interior mesh

Metric

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	µm
c01	z-axis	0.375	430	2.014	2.014	0.648	0.648	0.259	0.259	17, −374
c02	z-axis	0.054	764	0.809	0.797	0.150	0.145	0.098	0.093	4, −54
c03	z-axis	0.078	716	0.926	0.926	0.210	0.210	0.111	0.111	12, 77
c04	z-axis	0.048	706	0.952	0.952	0.209	0.209	0.120	0.120	−43, −21
c05	z-axis	0.022	687	0.735	0.735	0.172	0.172	0.105	0.105	5, −22
c06	z-axis	2.556	365	11.768	11.768	4.199	4.199	1.810	1.810	552, 2495
c06_s	z-axis	0.015	332	0.344	0.310	0.067	0.066	0.041	0.040	−3, −15
c07	z-axis	2.039	332	9.369	9.369	3.340	3.340	1.478	1.478	516, 1973
c07_s	z-axis	0.038	324	0.404	0.404	0.097	0.097	0.059	0.059	−13, 36
c06	c06_s	2.571								555, 2510
c07	c07_s	2.007								529, 1937

Imperial

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0148	430	0.0793	0.0793	0.0255	0.0255	0.0102	0.0102	0.7, −14.7
c02	z-axis	0.0021	764	0.0318	0.0314	0.0059	0.0057	0.0039	0.0037	0.1, −2.1
c03	z-axis	0.0031	716	0.0365	0.0365	0.0083	0.0083	0.0044	0.0044	0.5, 3.0
c04	z-axis	0.0019	706	0.0375	0.0375	0.0082	0.0082	0.0047	0.0047	−1.7, −0.8
c05	z-axis	0.0009	687	0.0289	0.0289	0.0068	0.0068	0.0042	0.0042	0.2, −0.9
c06	z-axis	0.1006	365	0.4633	0.4633	0.1653	0.1653	0.0713	0.0713	21.7, 98.2
c06_s	z-axis	0.0006	332	0.0135	0.0122	0.0027	0.0026	0.0016	0.0016	−0.1, −0.6
c07	z-axis	0.0803	332	0.3688	0.3688	0.1315	0.1315	0.0582	0.0582	20.3, 77.7
c07_s	z-axis	0.0015	324	0.0159	0.0159	0.0038	0.0038	0.0023	0.0023	−0.5, 1.4
c06	c06_s	0.1012								21.9, 98.8
c07	c07_s	0.0790								20.8, 76.2

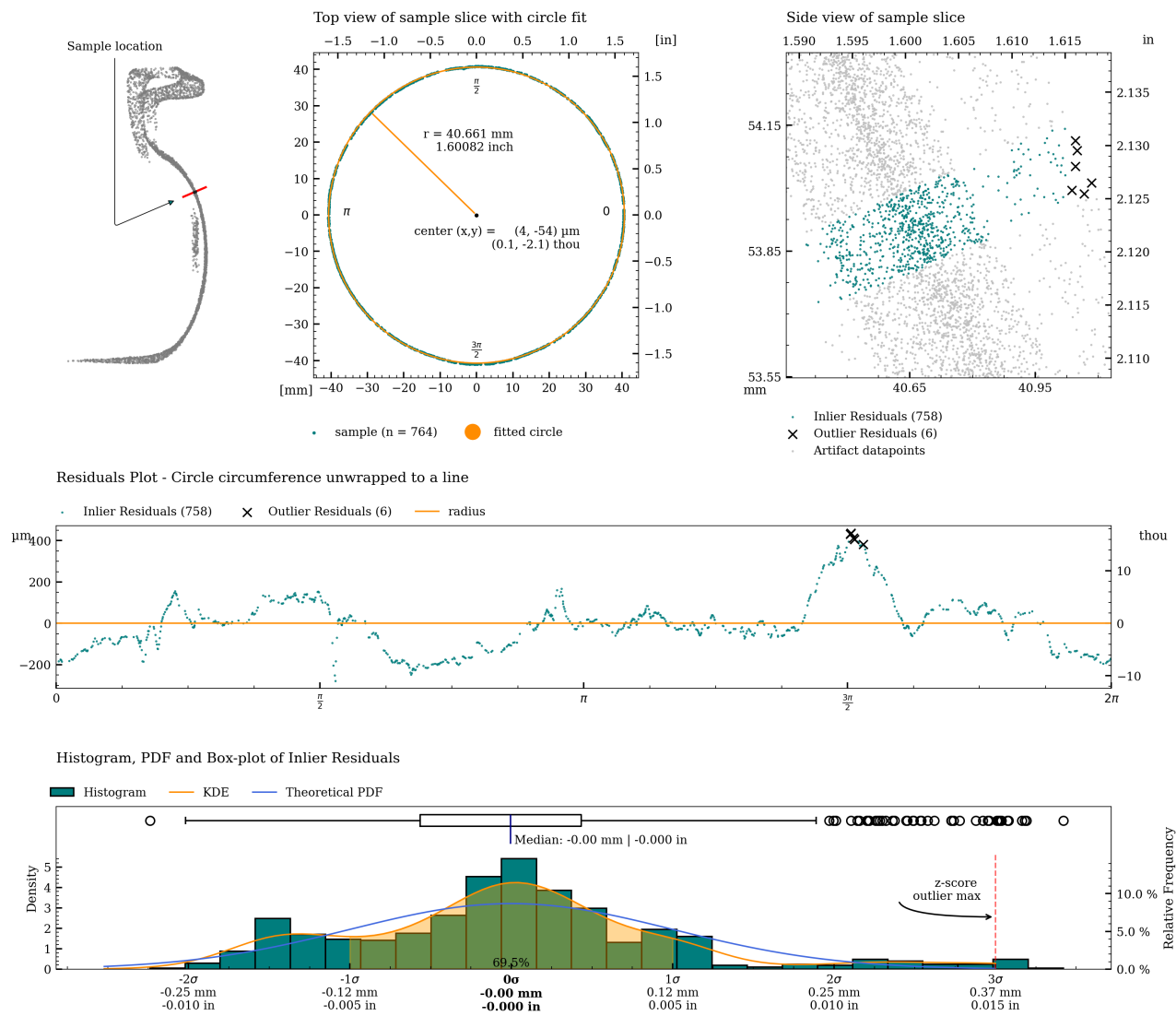Table 3: Concentricity analysis of HV004.

Concentricity analysis of c01

Sample location

Top view of sample slice with circle fit

r = 32.759 mm
1.28974 inch

center (x,y) =   (17, -374) µm
(0.7, -14.7) thou

sample (n = 430)   fitted circle

Side view of sample slice

Inlier Residuals (430)
Outlier Residuals (0)
Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

Inlier Residuals (430)   Outlier Residuals (0)   radius

Histogram, PDF and Box-plot of Inlier Residuals

Histogram   KDE   Theoretical PDF

Median: -0.06 mm | -0.002 in

62.8%

-1σ	0σ	1σ	2σ	3σ
-0.42 mm	**0.04 mm**	0.51 mm	0.97 mm	1.43 mm
-0.017 in	**0.002 in**	0.020 in	0.038 in	0.056 in

Figure 33: Detailed plot of concentricity measurement for c01.

Concentricity analysis of c02

Sample location

Top view of sample slice with circle fit

r = 40.661 mm
1.60082 inch

center (x,y) = (4, -54) µm
(0.1, -2.1) thou

sample (n = 764)   fitted circle

Side view of sample slice

Inlier Residuals (758)
Outlier Residuals (6)
Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

Inlier Residuals (758)   Outlier Residuals (6)   radius

Histogram, PDF and Box-plot of Inlier Residuals

Histogram   KDE   Theoretical PDF

Median: -0.00 mm | -0.000 in

69.5%

z-score
outlier max

	-2σ	-1σ	0σ	1σ	2σ	3σ
	-0.25 mm	-0.12 mm	**-0.00 mm**	0.12 mm	0.25 mm	0.37 mm
	-0.010 in	-0.005 in	**-0.000 in**	0.005 in	0.010 in	0.015 in

Figure 34: Detailed plot of concentricity measurement for c02.

Concentricity analysis of c03

Sample location

Top view of sample slice with circle fit

r = 43.897 mm
1.72824 inch

center (x,y) =   (12, 77) μm
(0.5, 3.0) thou

sample (n = 716)   fitted circle

Side view of sample slice

Inlier Residuals (716)
Outlier Residuals (0)
Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

Inlier Residuals (716)   Outlier Residuals (0)   radius

Histogram, PDF and Box-plot of Inlier Residuals

Histogram   KDE   Theoretical PDF

Median: -0.05 mm | -0.002 in

58.2%

-2σ	-1σ	0σ	1σ	2σ	3σ
-0.35 mm	-0.17 mm	**0.00 mm**	0.18 mm	0.36 mm	0.53 mm
-0.014 in	-0.007 in	**0.000 in**	0.007 in	0.014 in	0.021 in

Figure 35: Detailed plot of concentricity measurement for c03.

Concentricity analysis of c04

Figure 36: Detailed plot of concentricity measurement for c04.
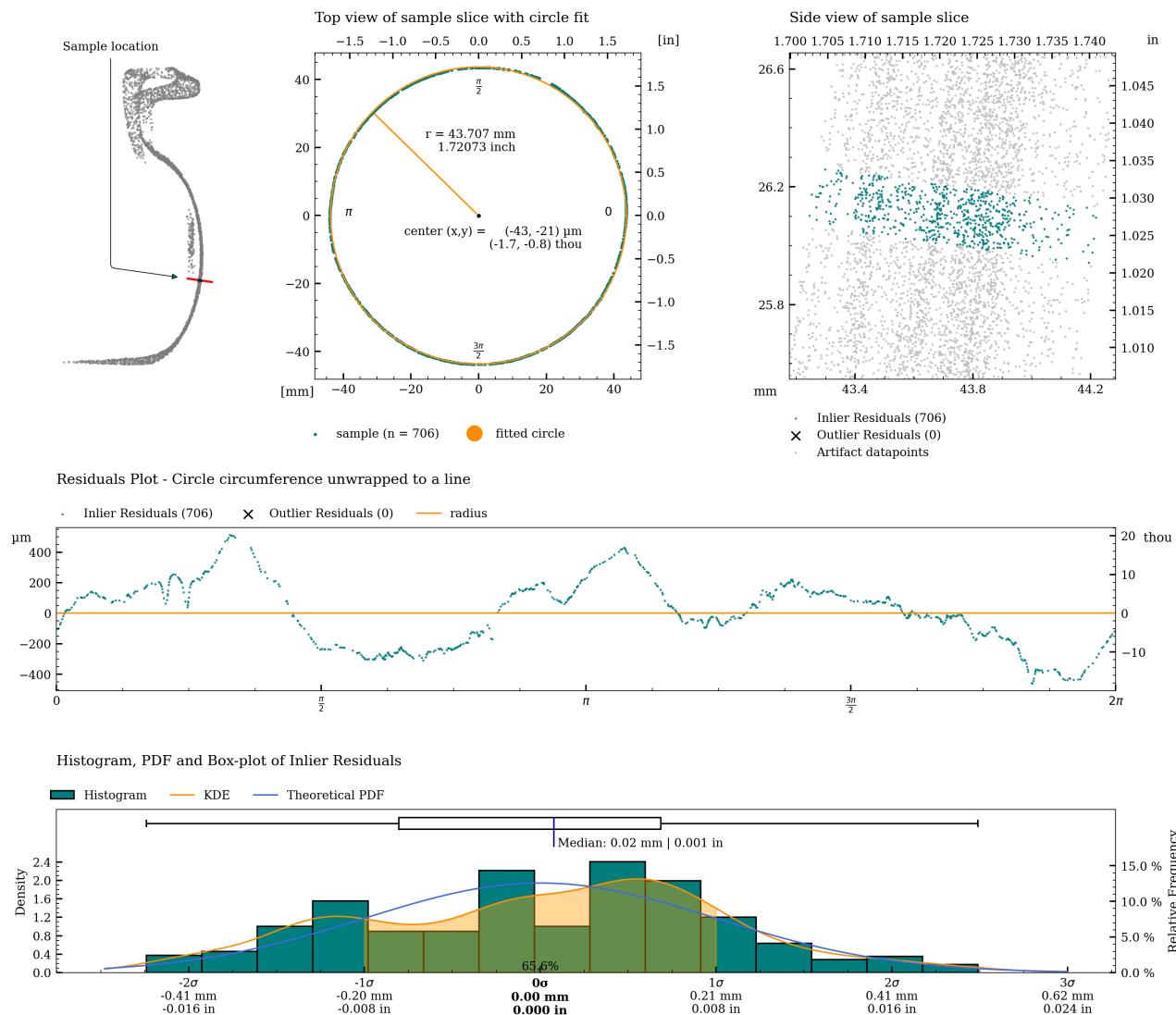
Concentricity analysis of c05
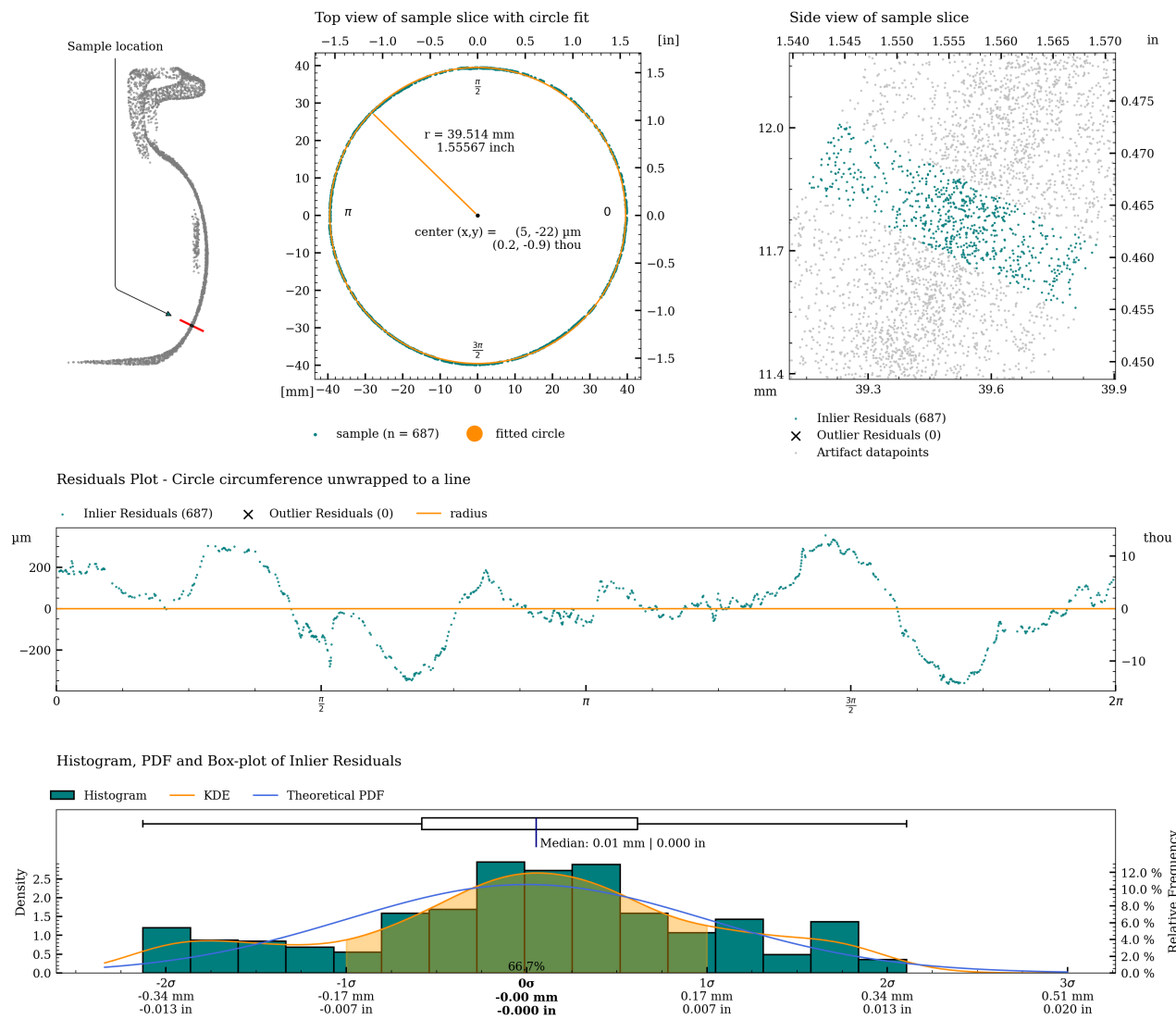
Figure 37: Detailed plot of concentricity measurement for c05.

Concentricity analysis of c06

Top view of sample slice with circle fit

Sample location

r = 21.874 mm
0.86120 inch

center (x,y) = (552, 2495) µm
(21.7, 98.2) thou

· sample (n = 365)   ● fitted circle

Side view of sample slice

· Inlier Residuals (365)
✕ Outlier Residuals (0)
· Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

· Inlier Residuals (365)   ✕ Outlier Residuals (0)   — radius

Histogram, PDF and Box-plot of Inlier Residuals

■ Histogram   — KDE   — Theoretical PDF

Median: 0.55 mm | 0.021 in

56.7%

	-1σ	0σ	1σ	2σ	3σ
	-1.55 mm	0.26 mm	2.07 mm	3.87 mm	5.68 mm
	-0.061 in	0.010 in	0.081 in	0.153 in	0.224 in

Figure 38: Detailed plot of concentricity measurement for c06.

Concentricity analysis of c06_s
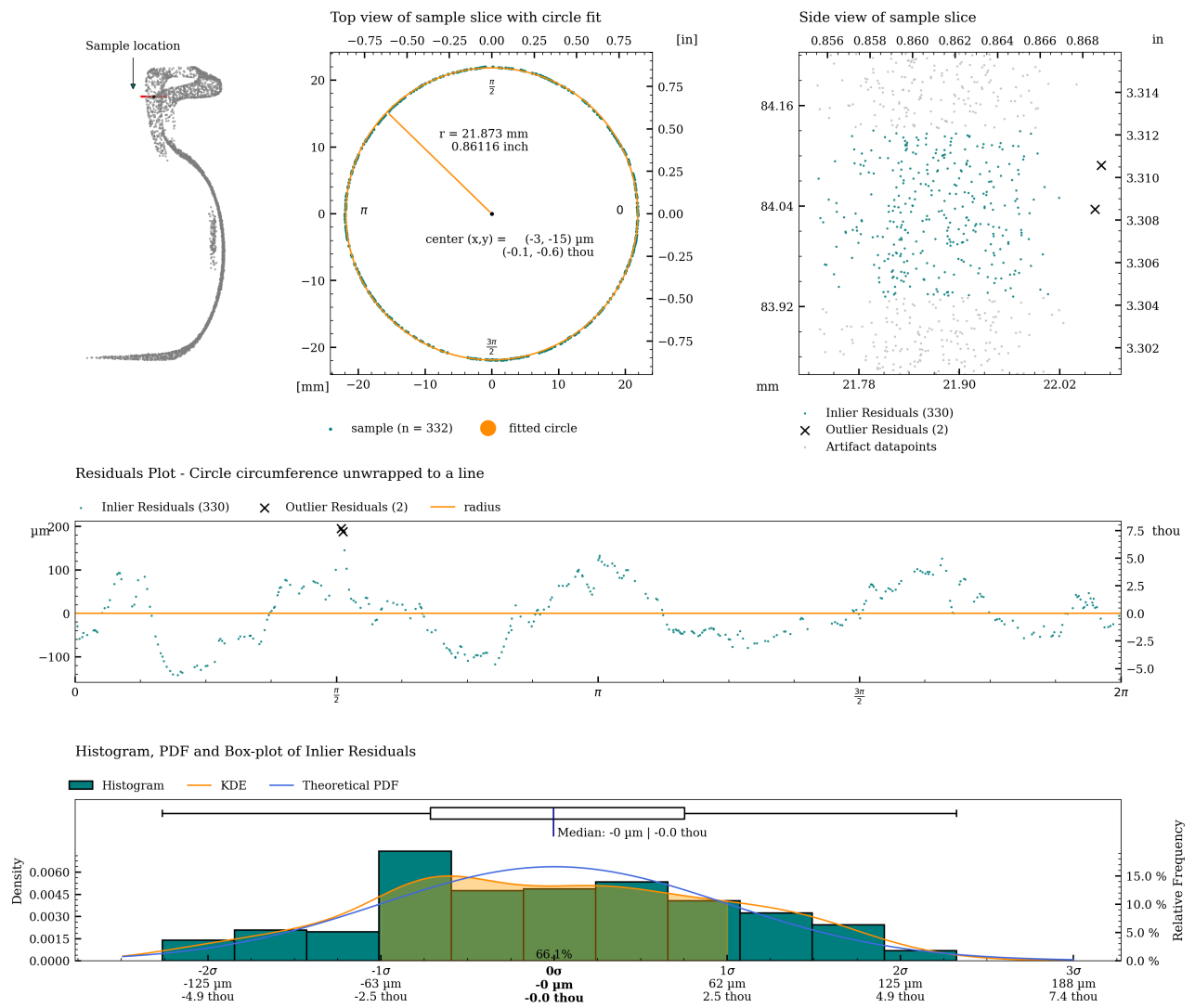
Figure 39: Detailed plot of concentricity measurement for c06_s.

Concentricity analysis of c07
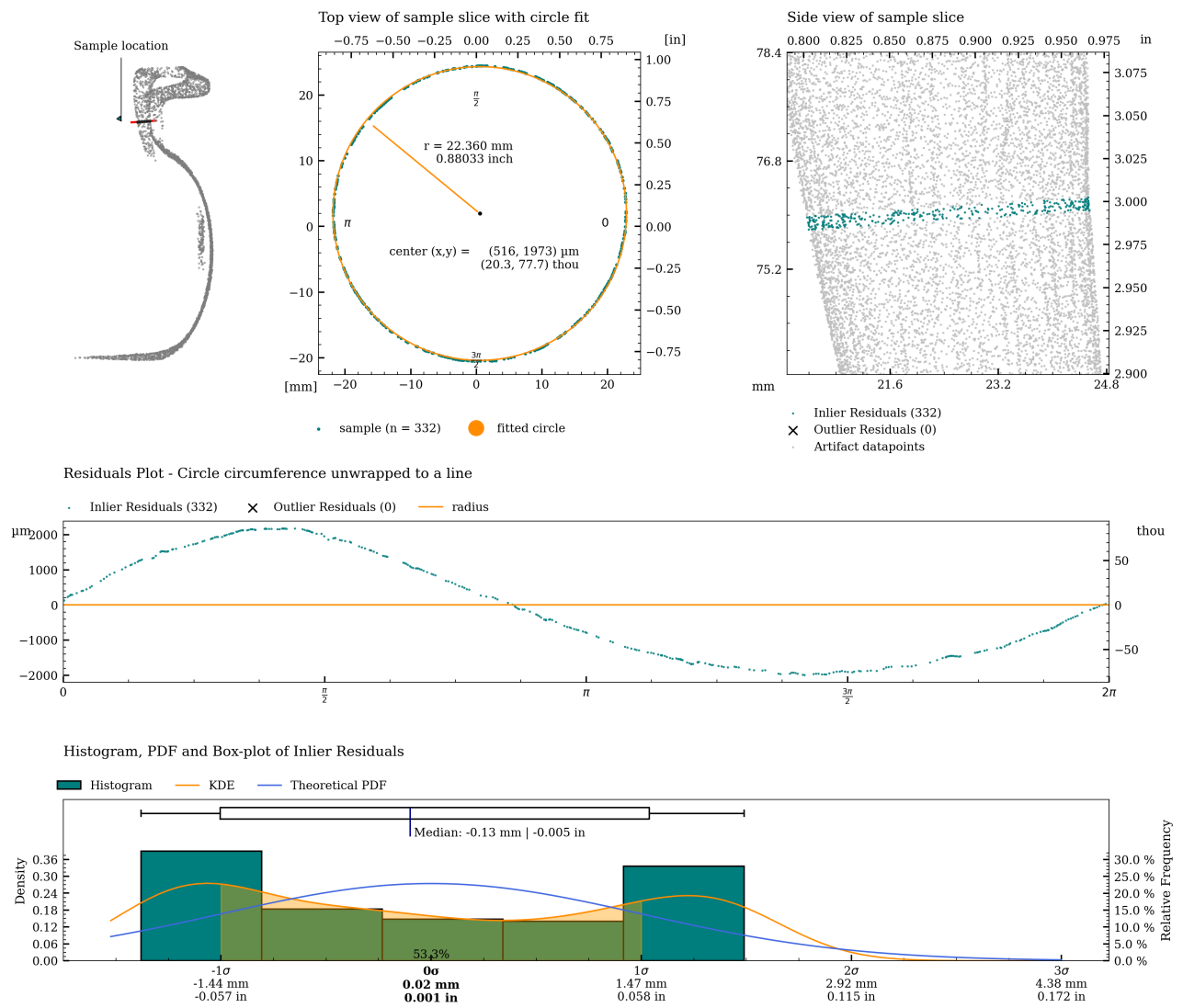
Figure 40: Detailed plot of concentricity measurement for c07.

Concentricity analysis of c07_s

Sample location

Top view of sample slice with circle fit

r = 22.331 mm
0.87919 inch

center (x,y) =   (-13, 36) µm
(-0.5, 1.4) thou

- sample (n = 324)
- fitted circle

Side view of sample slice

- Inlier Residuals (324)
- × Outlier Residuals (0)
- Artifact datapoints

Residuals Plot - Circle circumference unwrapped to a line

- Inlier Residuals (324)   × Outlier Residuals (0)   — radius

Histogram, PDF and Box-plot of Inlier Residuals

- Histogram   — KDE   — Theoretical PDF

Median: 18 µm | 0.7 thou

65.4%

-2σ	-1σ	0σ	1σ	2σ	3σ
-166 µm	-82 µm	2 µm	86 µm	171 µm	255 µm
-6.5 thou	-3.2 thou	0.1 thou	3.4 thou	6.7 thou	10.0 thou

Figure 41: Detailed plot of concentricity measurement for c07_s.

Coaxiality

Coaxiality refers to the straightness and consistency of a central line running through the center of the vase. It measures how aligned the core of the vase remains along its vertical axis.

The coaxiality measurements are calculated using RANSAC (Random sample consensus) algorithm for outlier detection on least squares circle regression on cross-sections of the vessel (excluding potential handles), to estimate the best fit circle centers for each slice of the vessel. A best-fit line connects these centers, showing whether the vessels's shape twists or remains straight. This concept helps describe the symmetry and structural uniformity in a visual and analytical way.

Coaxiality is measured for:
- The exterior surface (excluding handles)
- The interior surface

	Exterior		Interior		Interior separate	
Analyzed Slices	326		110		110	
Median sample size	703		334		352	
Slice Height	200 µm	7.9 thou	200 µm	7.9 thou	200 µm	7.9 thou
Statistics with Z-axis as Reference						
Median Absolute Deviation (MAD)	70 µm	2.7 thou	2203 µm	86.7 thou	46 µm	1.8 thou
Standard Deviation (SD)	101 µm	4.0 thou	484 µm	19.1 thou	23 µm	0.9 thou
Root Mean Square Deviation (RMSD)	146 µm	5.8 thou	2210 µm	87.0 thou	50 µm	2.0 thou
Statistics with Best Fit Central Axis as Reference						
Best fit Central Axis Equation (in metric coordinate system with unit [mm])	x = -0.017 + t-0.00062 y = 0.164 + t0.00491 z = 0.000 + t-0.99999		x = 0.403 + t0.00172 y = -3.755 + t0.07429 z = 0.000 + t0.99724		x = 0.079 + t-0.00090 y = -0.076 + t0.00094 z = 0.000 + t1.00000	
Axis tilt	−0.036°		0.091°		−0.051°	
Median Absolute Deviation (MAD)	100 µm	4.0 thou	55 µm	2.1 thou	49 µm	1.9 thou
Standard Deviation (SD)	48 µm	1.9 thou	70 µm	2.7 thou	19 µm	0.8 thou
Root Mean Square Deviation (RMSD)	111 µm	4.4 thou	101 µm	4.0 thou	49 µm	1.9 thou

Table 4: Coaxiality analysis of vessel HV004.

Coaxiality plots, exterior surface
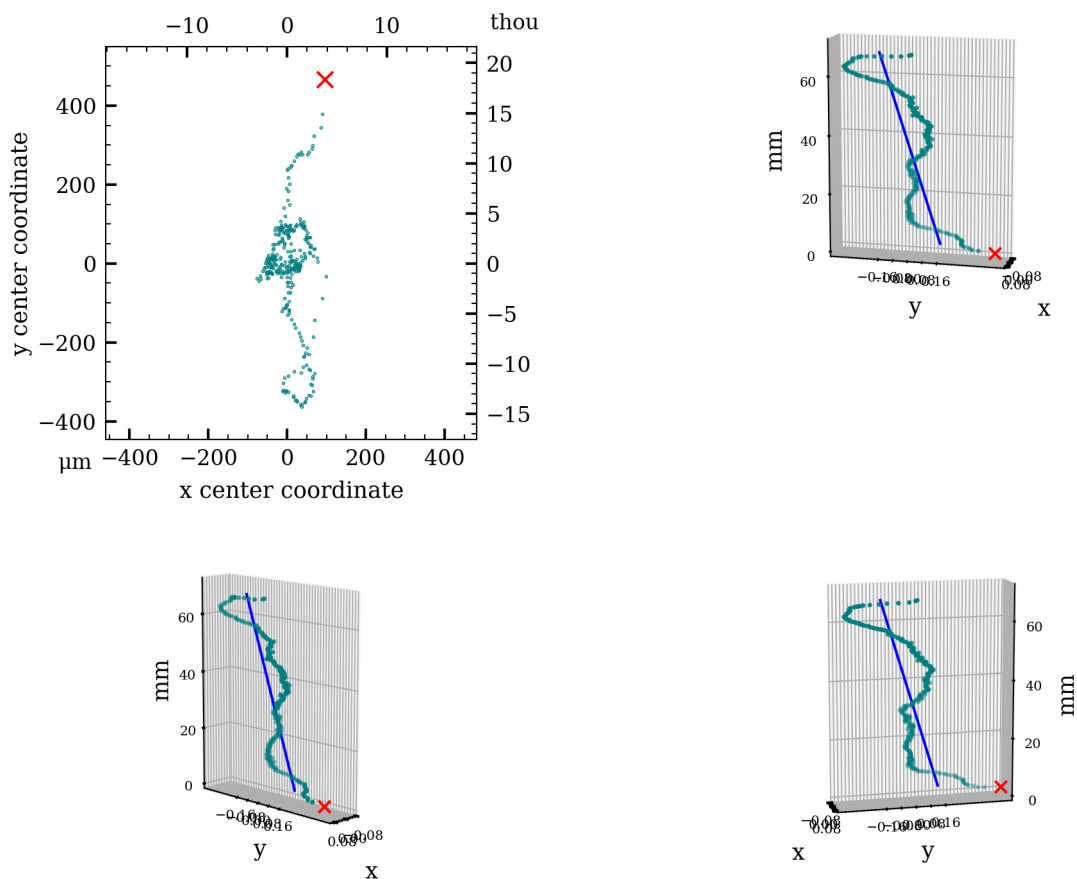
Coaxiality residuals from fitted axis, exterior surface
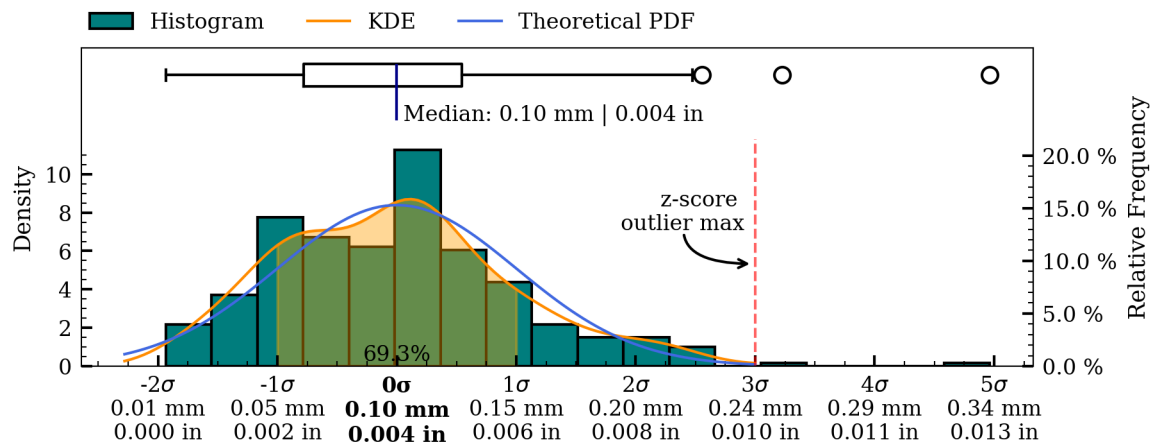
Figure 42: Coaxiality residual plots of exterior surface, HV004.

Coaxiality plots, interior surface
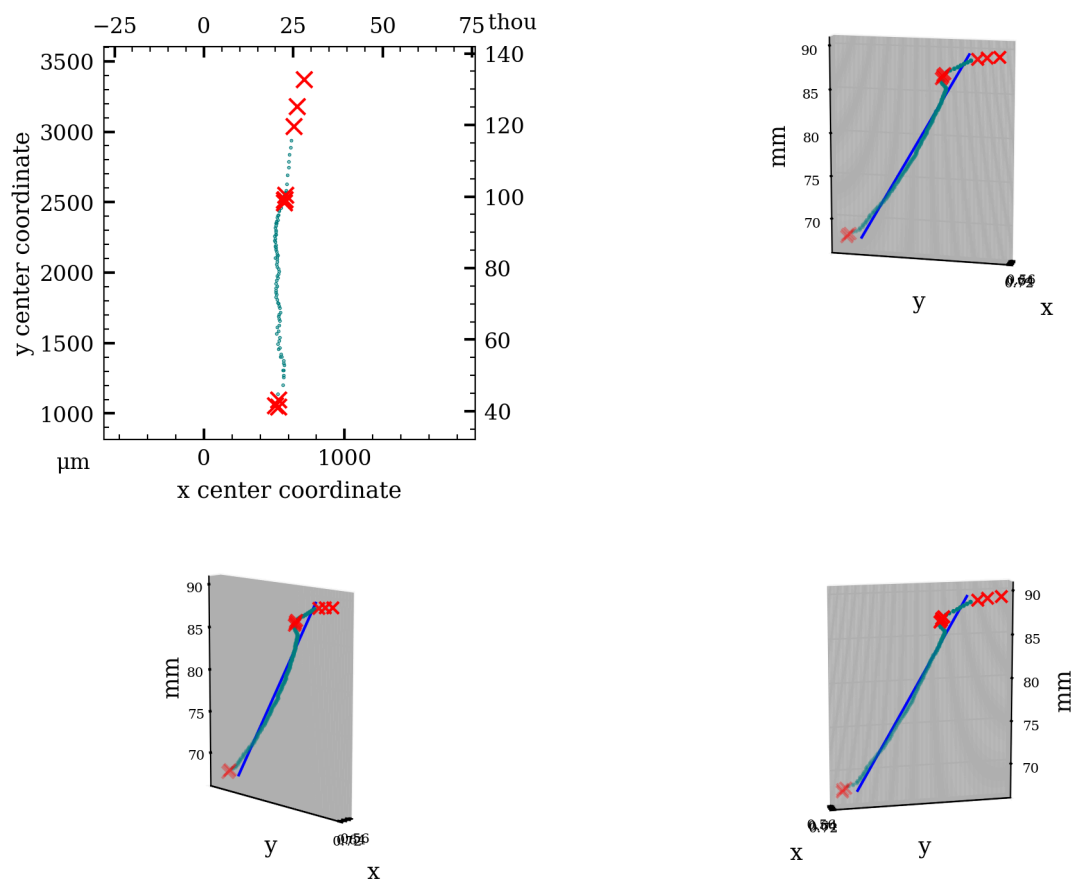
Coaxiality residuals from fitted axis, interior surface
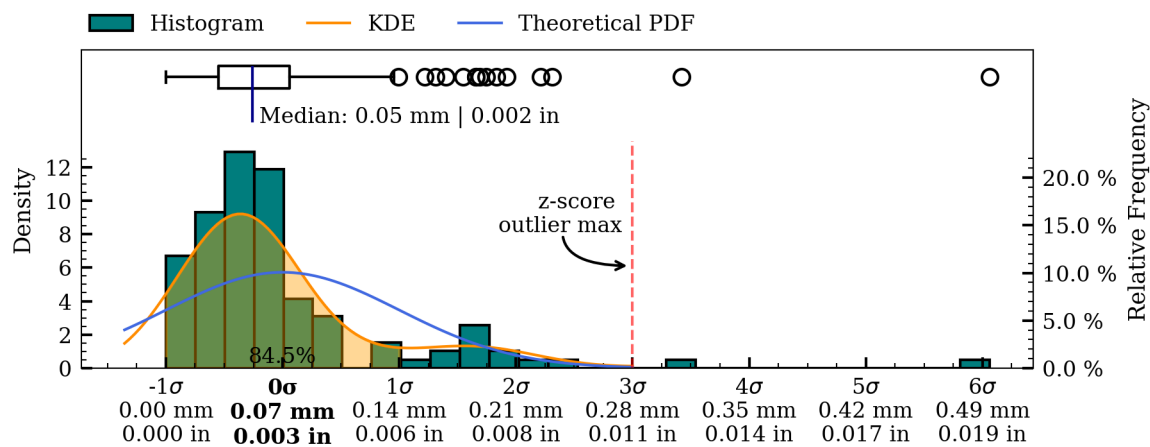
Figure 43: Coaxiality residual plots of interior surface, HV004.

Coaxiality plots, interior separately aligned surface
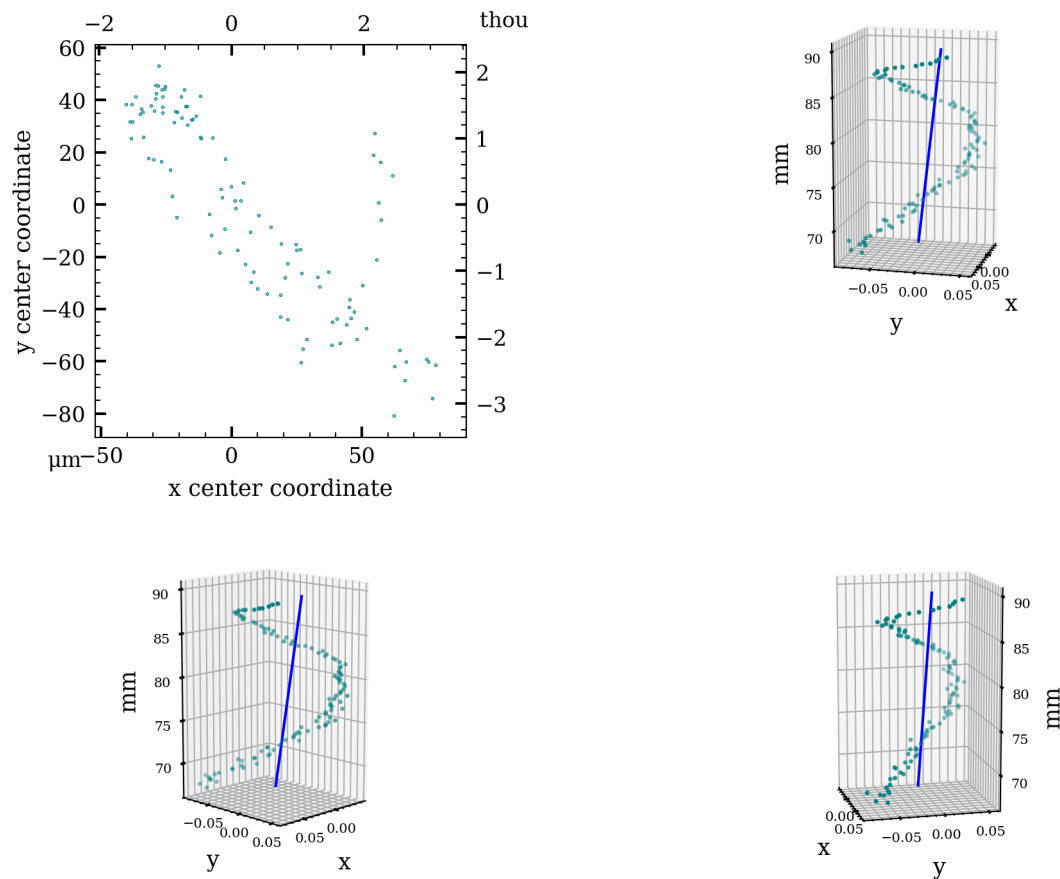
Coaxiality residuals from fitted axis, interior separately aligned surface
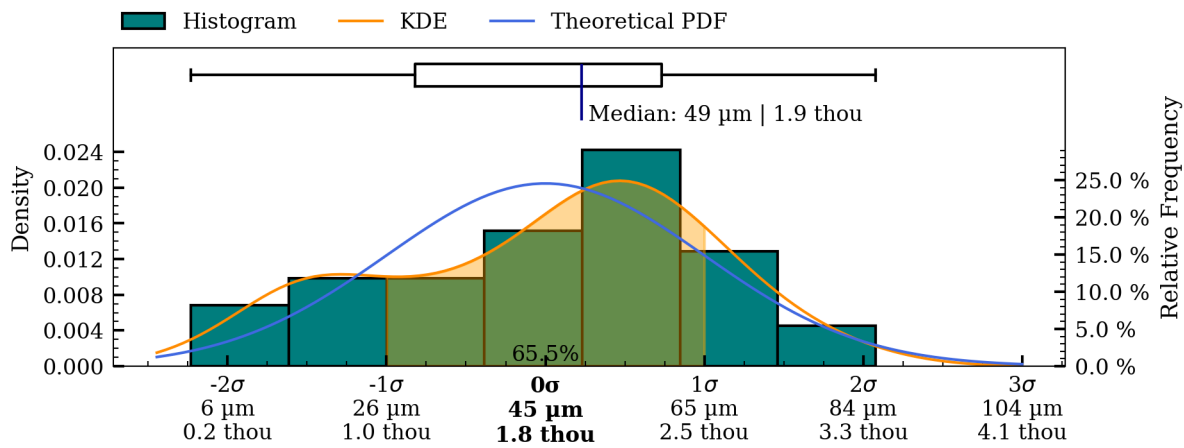
Figure 44: Coaxiality residual plots of interior_separate surface, HV004.

Surface Variability

To illustrate the overall surface deviations of the object, a surface variability heatmap has been created. This heatmap provides an accessible overview of the topography of the manufacturing precision and surface structure of the object.

The surface variability measurements are created by fitting a number of higher-order polynomials to the two-dimensional folded profile of the scan data. This process creates an idealized mathematical representation of actual surface curvature of object, and as such provides a continuous model representation of the actual object. It is important to note that only such a non-discretized representation is sufficient to avoid introducing inconsistently varying errors in the mapping of the final surface deviation results, that the rendered heatmaps are based on.

To produce the final surface variability map, the distance from each scanned vertex to the fitted polynomial is calculated and used as the mapping function input, for applying colours to the surface of the object.

It is important to note that this variability map does not describe deviations from the original *intended* shape of the artifact (if any), as this shape (the *intended design*, so to speak) will have been lost to time. It does however provide a very informative visualization of the texture and structure of the surface and very importantly, *does* hightlight potential manufacturing-relevant patterns in the surface texture (if present). Such patterns are, as an example, clearly evident on the interior surface of artifact PV001.

Exterior surface
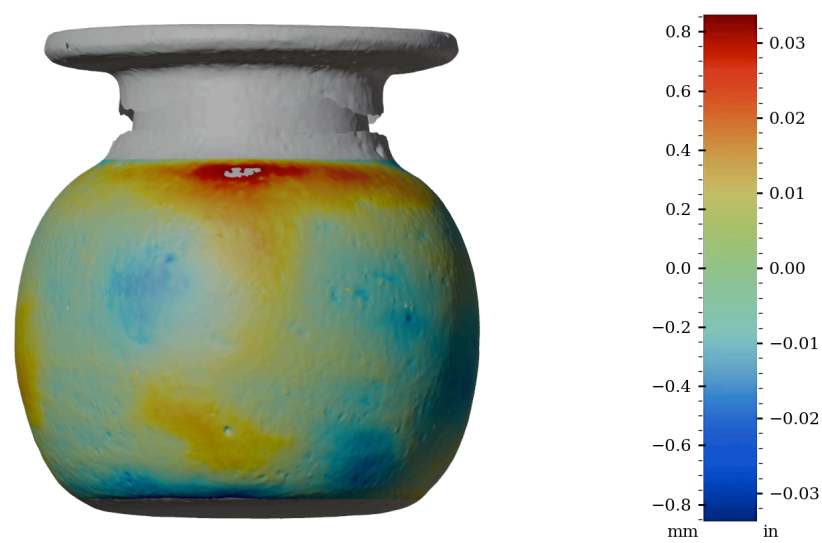
Figure 45: Surface variability heatmap of HV004, front view
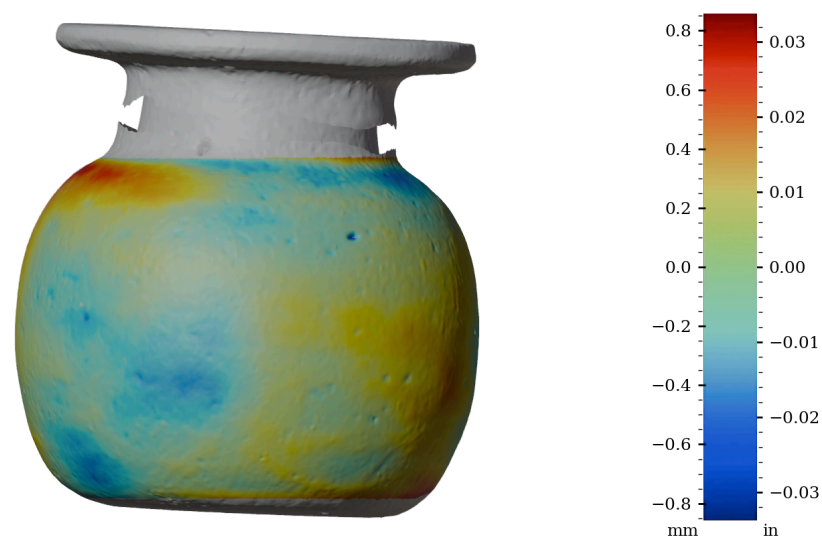
Figure 46: Surface variability heatmap of HV004, rotated 90°
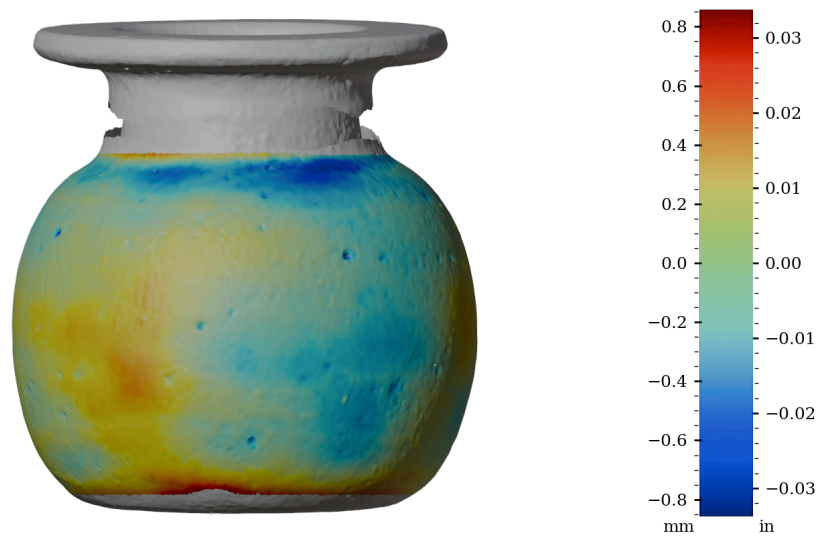
Figure 47: Surface variability heatmap of HV004, rotated 180°

Figure 48: Surface variability heatmap of HV004, rotated 270°

Interior surface
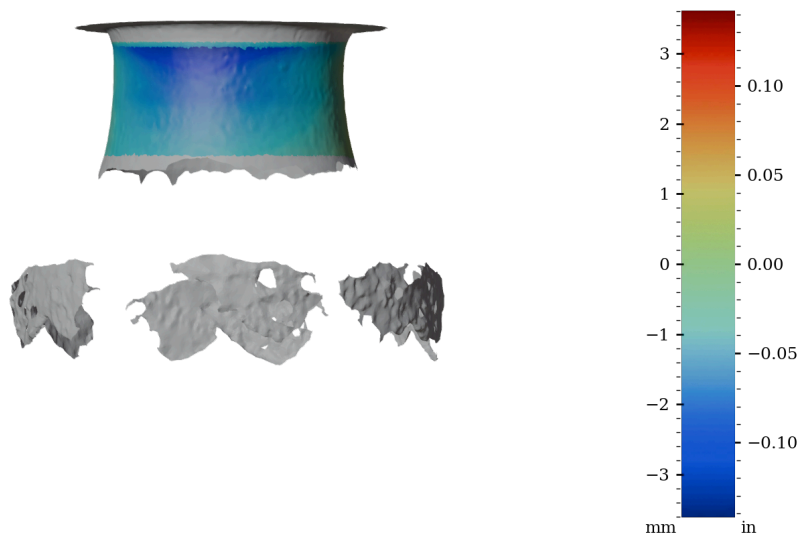
Figure 49: Surface variability heatmap of HV004, front view
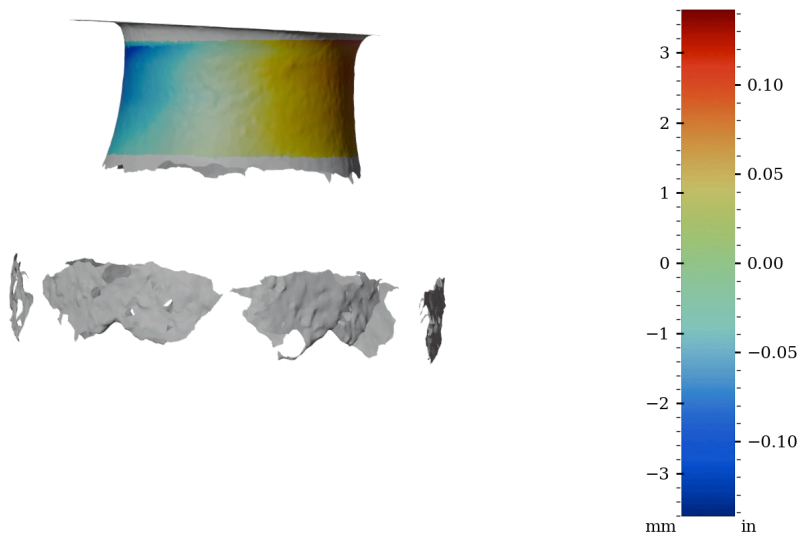
Figure 50: Surface variability heatmap of HV004, rotated 90°
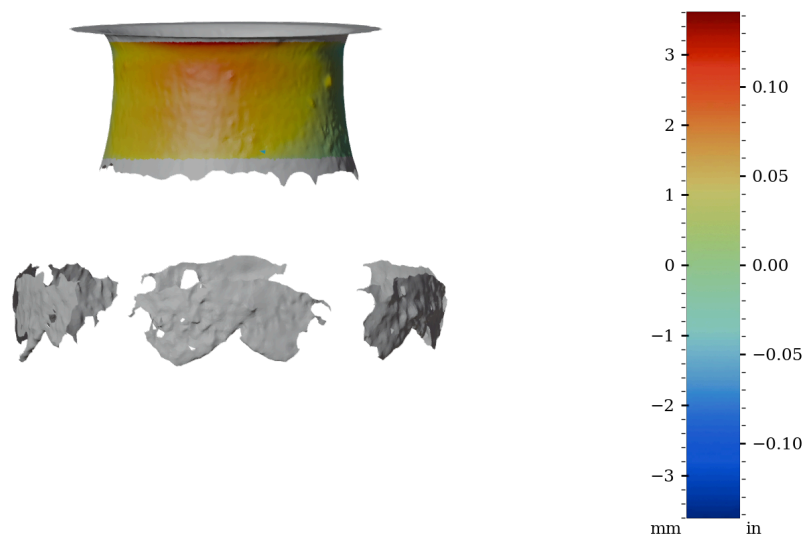
Figure 51: Surface variability heatmap of HV004, rotated 180°
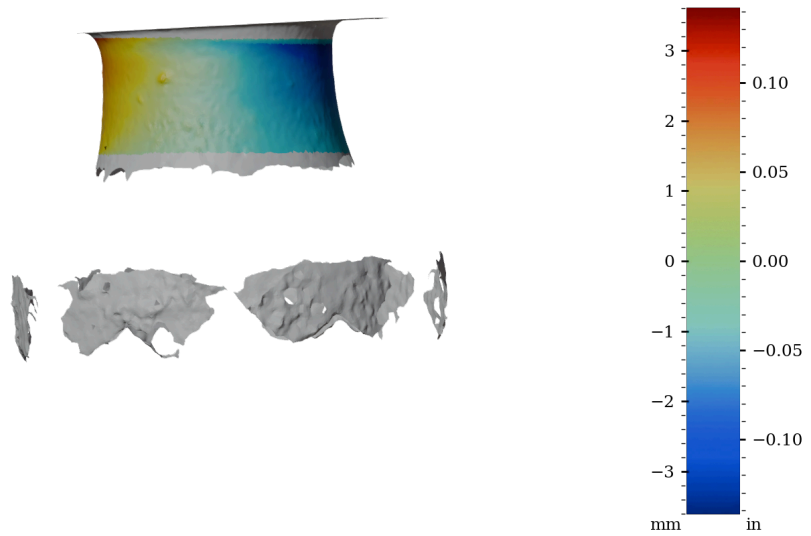
Figure 52: Surface variability heatmap of HV004, rotated 270°

Interior surface aligned separately
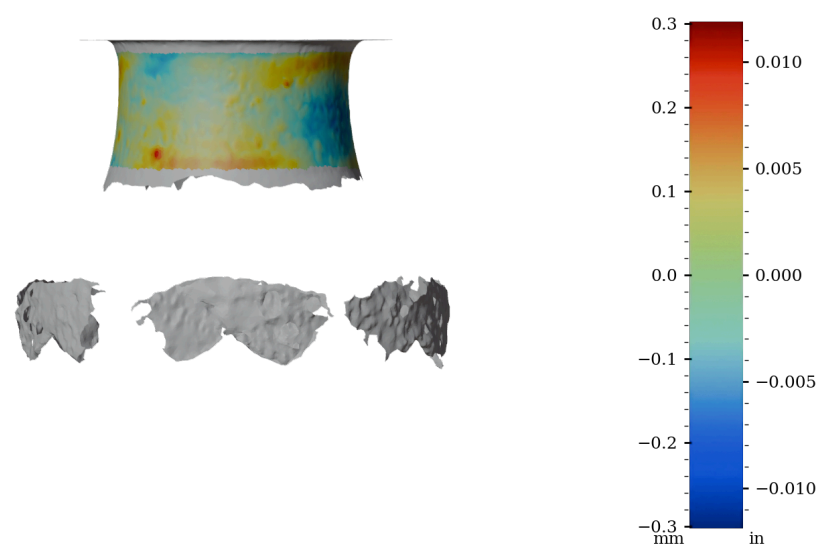
Figure 53: Surface variability heatmap of HV004, front view
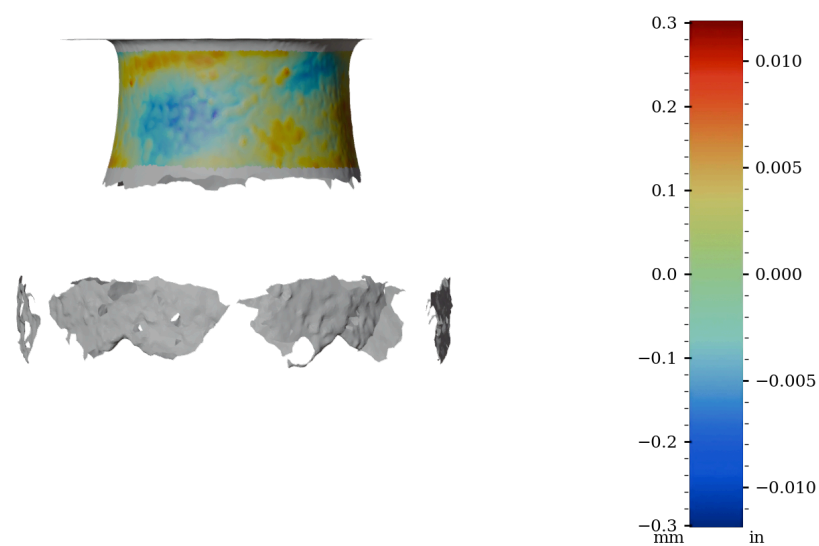
Figure 54: Surface variability heatmap of HV004, rotated 90°
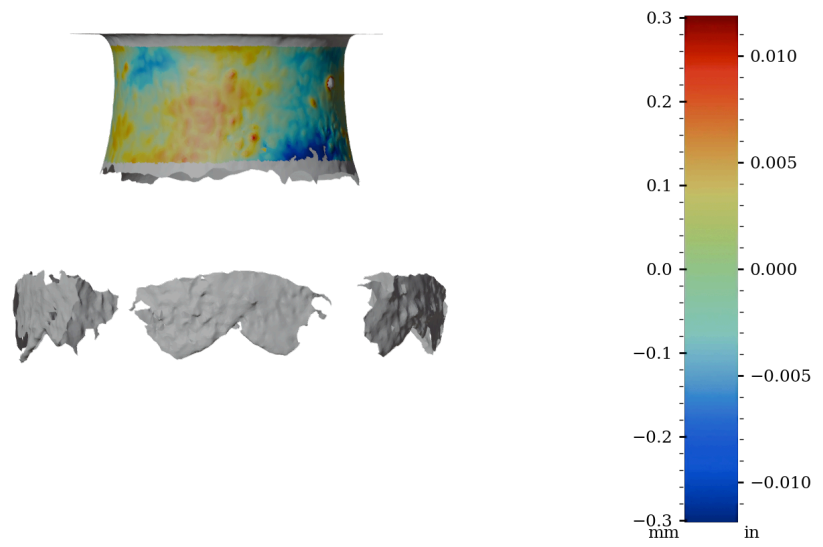
Figure 55: Surface variability heatmap of HV004, rotated 180°
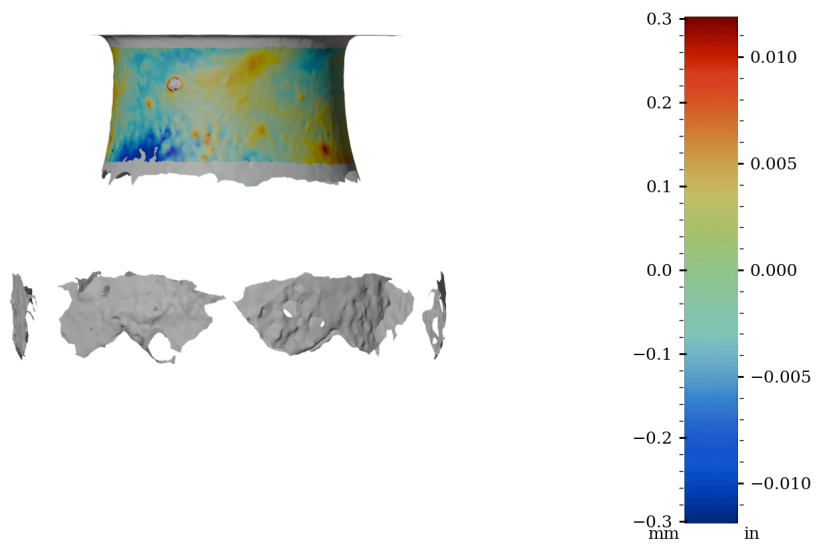
Figure 56: Surface variability heatmap of HV004, rotated 270°

Surface variability statistics

Area	MSD	RMSD	SD	Median AD	Range	Min	Max	Sample size
	mm^2	mm	mm	mm	mm	mm	mm	
Exterior	0.0600	0.245	0.155	0.096	2.558	−1.351	1.207	284607
Interior	2.5167	1.586	0.759	0.582	6.800	−3.600	3.199	39839
Interior separate	0.0075	0.086	0.052	0.034	0.993	−0.359	0.633	39580
	in^2	in	in	in	in	in	in	
Exterior	0.000093	0.0096	0.0061	0.0038	0.1007	−0.0532	0.0475	284607
Interior	0.003901	0.0625	0.0299	0.0229	0.2677	−0.1417	0.1260	39839
Interior separate	0.000012	0.0034	0.0020	0.0013	0.0391	−0.0141	0.0249	39580

Table 5: Surface variability statistics, HV004

Table 5 shows the statistics of the distance from the scan vertices to the best fit object model. These statistics are briefly explained below.

Histogram, KDE and Box-plot of measured surface variability - exterior surface
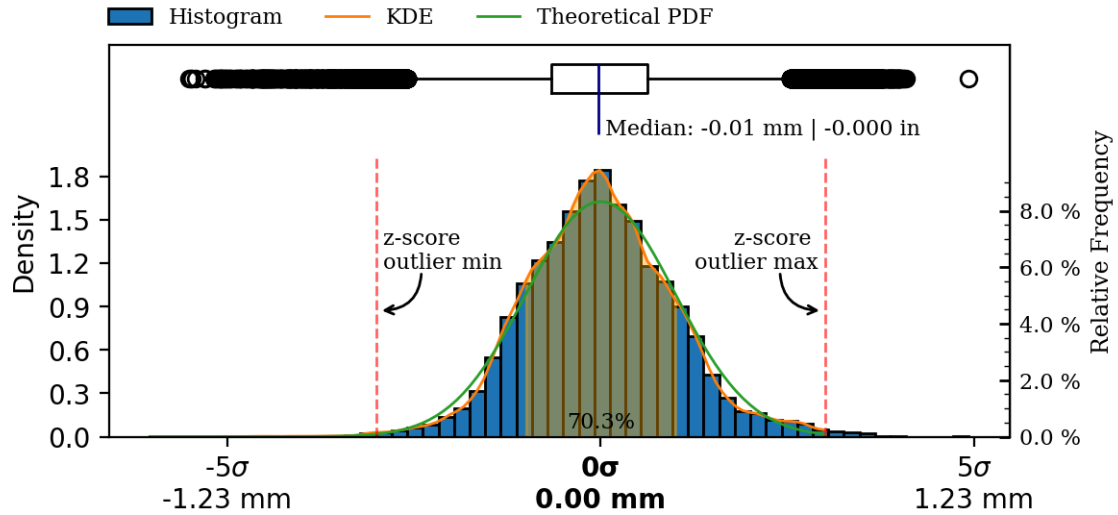
Figure 57: Exterior surface variability boxplot, kds and histogram.

Histogram, KDE and Box-plot of measured surface variability - interior surface
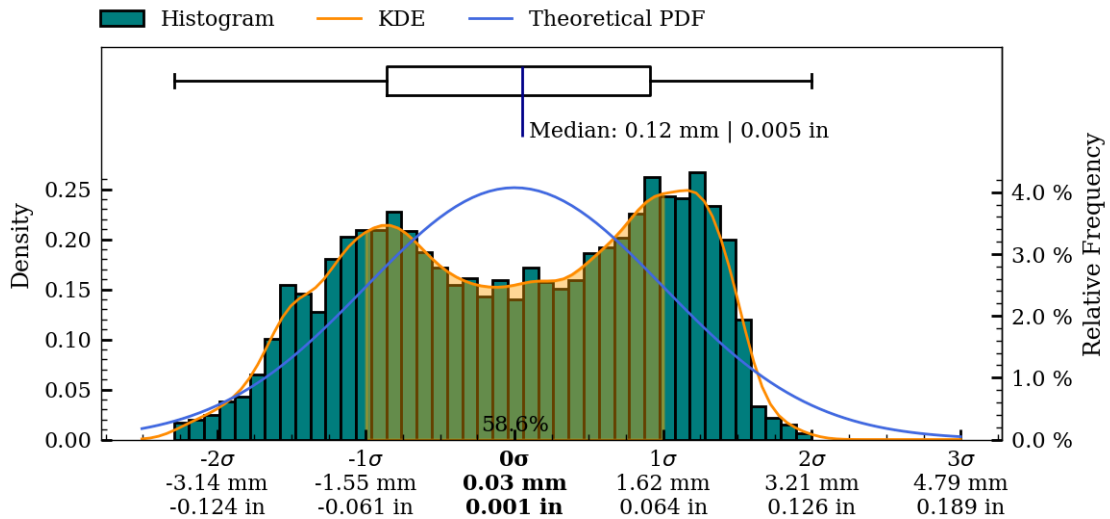
Figure 58: Interior surface variability boxplot, kds and histogram.

Histogram, KDE and Box-plot of measured surface variability - interior separately aligned surface
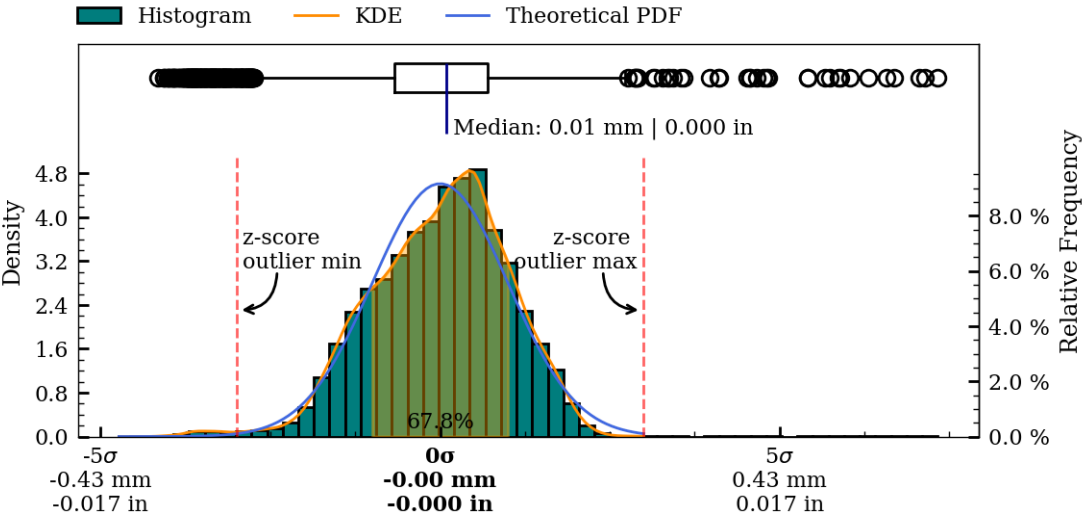
Figure 59: Interior separately aligned surface variability boxplot, kds and histogram.

Precision Score Of The Artifact

To enable valid comparison of the manufacturing precision of different artifacts, a metric that robustly quantifies the overall precision of the object is required. The considerations for such a metric will be explored in this section.

Based on these considerations, a *Precision Score* metric will be defined.

For an object to be described as having been manufactured with high precision, several qualities must be present *concurrently*, and throughout the *entire* geometry of the final object. A given object may exhibit high levels of one or more *components* of precision, but be lacking in others. For example:

- An object may present high levels of coaxiality, but lack circularity.
- An object may exhibit good circularity, but show imperfections in the surface structure.
- An object may be smoothed to perfection *without* any circularity or coaxiality.
- An object may exhibit high levels of all of the above metrics in *some* areas, but not in others.

Therefore, a precision score metric **must** account for *all* aspects of the individual, underlying precision metrics (circularity, concentricity, coaxiality and surface variability) throughout the *entire* surface area of the object.

The composite high order polynomial model, used to generate the surface variability map (described in Surface Variability, p. 46) is the best continuous mathematical representation of the object available to us (lacking any original design plans, as would normally be available in metrological analysis). This idealized model encompasses all of the above component metrics.

In the creation of the model, all scan data-points are taken into account (excluding areas with extensive damage), making it the best possible idealized representation we can achieve. When this model has been accurately created, the deviation between the model and the scanned data-points can be calculated over the non-discretized polynomials, *without* the need for an "original" CAD model (and importantly, unless such a CAD model *actually* corresponded to the original design intent, it would be an insufficient comparison basis).

Within the context of defining a valid, overall precision metric, this approach satisfies the incorporation of all of the necessary metrics:

- **Circularity**: Because the reconstructed polynomial model is revolved around the Z-plane, the idealized representation is perfectly circular, and thus incorporates the circularity component.
- **Concentricity and coaxiality**: Because the Z-axis (datum axis) is the center axis of the model, it incorporates the concentricity and coaxiality components.
- **Surface variability**: Because the model is continuous and non-discretized, it can be used accurately for all points of the scan data, and incorporates the surface variability component.

The level of precision ultimately achieved in a physical object does not share a linear relationship with its manufacturing requirements. Since continuously higher levels of final precision becomes progressively harder to achieve, an overall precision metric must take this relationship into account.

A robust statistical metric that satisfies this requirement is the *Mean Squared Deviation* (MSD or MSE). Here specifically, we can utilize the mean square of the deviations between the model ($\hat{y}$) and the data-points (y_i).

Combining all of the above considerations, we can express a well-defined *Precision Score* metric, that provides an immediately accessible way to understand the overall precision of an object, while being statistically valid. Since the Mean Squared Deviation tends towards zero as the overall precision increases, the inverse of the Mean Squared Deviation is taken to obtain a precision score metric that increases as precision increases[12]:

$$\text{Precision Score} = \frac{n}{\sum_{i=1}^{n} (y_i - \hat{y})^2}$$
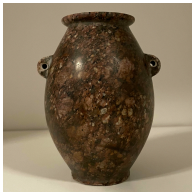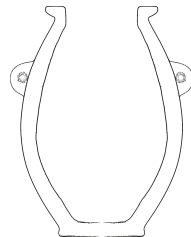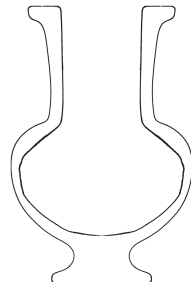
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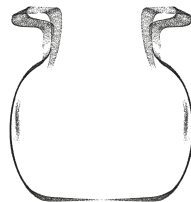
[12]The precision score unit is $\frac{1}{\text{mm}^2}$

The precision score of HV004 have been calculated separately for:

- Precision score, exterior surface: 17
- Precision score, separately aligned interior surface: 134
- Precision score, interior surface: 0.40
- Precision score, full surface: 19

The precision score of a Zeiss 1.00000 inch reference sphere have been calculated to 43,943 (RMSE = 0.00477 mm / 0.00010 in). The scan was obtained by Max Fomitchev-Zamilov using a Keyence VL −500 scanner with a rated accuracy of 10 microns. The precision analysis of the reference sphere scan indicates at the maximum possible precision score obtainable.

Table 6 shows the precision score of this artifact (HV004), compared to the two most precise, and the two least precise vessels currently analyzed.

Artifact			Material	Precision Score	Link to Report
		PV001	Red Granite	**1980** Full: 1177 Exterior: 1980 Interior separate: 798 Interior: 722	Report Publication
		PV003		**1092** Full: 272 Exterior: 1092 Interior separate: 167 Interior: 163	Report Publication
Artifact image pending		HV004		**17** Full: 19 Exterior: 17 Interior separate: 134 Interior: 0.40	Report Publication
		RV003	Marble breccia	**1.46** Full: 1.49 Exterior: 1.46 Interior separate: 1.53 Interior: 0.54	Report Publication
		MV010	Calcite (Egyptian Alabaster)	**1.17** Full: 1.32 Exterior: 1.17 Interior separate: 11 Interior: 0.17	Report Publication

Analysis Roadmap

While the current iteration of this work already provides valuable results, continued future additions and improvements will enhance their utility further. This section details planned iterative updates and improvements, to both the reports themselves, and to the underlying methodology and software they are created with.

Alignment Section
- Detailed exploration of different circle regression algorithms
- If handles are present on the vessel, exploring alignment of the vessels so the handle positions match each other
- Add optimization of the perpendicular surface deviation, with the best results of the coaxial alignment
- Align by minimizing circularity results (of rotated sample slice, to compensate for sample height distortions)

Measurements of Precision
- Section detailing how measurements perpendicular to the surface curvature are obtained
- Detailed surface area analysis, exploring the residual patterns throughout subsequent sample slices of the artifact surface
- Wall thickness deviation color map
- Robust outlier identification on circularity, to better handle analysis of damaged areas of the artifacts in addition to removal of interior crystalline structure points present in CT scans
- Layout updates to the charts and tables

Visibility of Outliers and Damaged Sections
- Identification and marking of damaged parts
- Visualization of outliers on the artifact surface

Exploration of Mathematical Primitives
- Analysis of selected curvatures and flat surfaces on the vessel in both the horizontal and vertical planes
  - ‣ Circles
  - ‣ Parabolas
  - ‣ Ellipsoids
  - ‣ Hyperbolas
  - ‣ Cones
- Implementation of robust regressions models suitable for this domain, based on RANSAC.

Metrics on Primary Features
- Measurements of features in the horizontal plane
- Measurements of features in the vertical plane
- Measurements of angles
- Measurements of volume

Exploration of Potential Design Ratios
- π, φ, e, 1, 2, 3, 4 etc.

Raw Dataset Attachments
- Including all measurement and sample coordinates as CSV-files embedded in the report
- Including an STL file of the aligned object alongside the report, for easier external replication and validation of the research results

Appendix A - Comparison Of Circularity Measurements (Z-plane vs. surface-perpendicular)
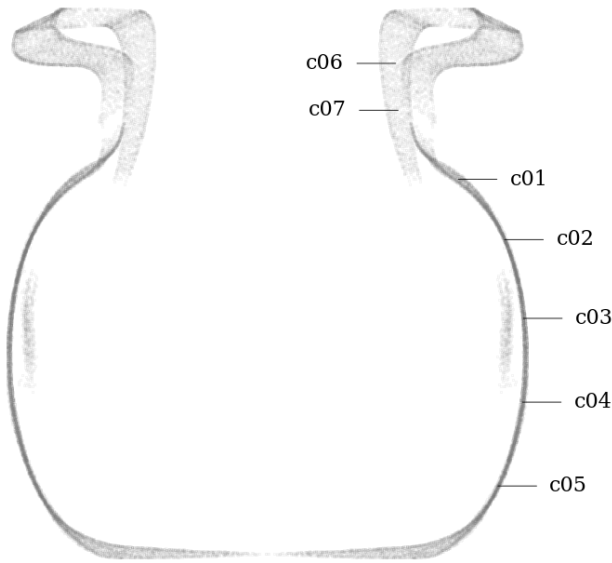
Figure 60: Circularity measurement sample locations, full mesh aligned to exterior surface
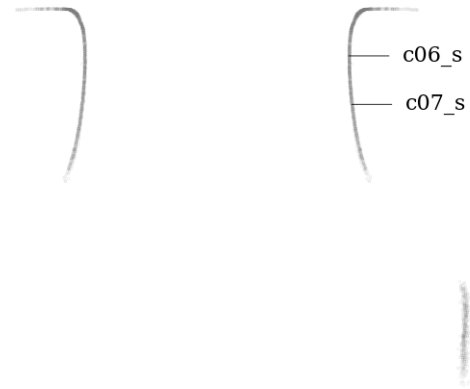
Figure 61: Circularity measurement sample location, separately aligned interior mesh

Samples perpendicular to the surface curvature

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø65.457±0.923	1.590	0.467	0.162	0.222	430	0.200	64.210	32.728
c02	exterior	Ø81.326±0.435	0.715	0.129	0.048	0.089	764	0.200	53.902	40.663
c03	exterior	Ø87.791±0.399	0.719	0.176	0.059	0.083	716	0.200	40.434	43.896
c04	exterior	Ø87.422±0.508	0.971	0.205	0.086	0.117	706	0.200	26.104	43.711
c05	exterior	Ø79.051±0.374	0.718	0.170	0.067	0.106	687	0.200	11.775	39.526
c06	interior	Ø44.051±2.731	5.261	1.812	0.628	0.815	365	0.200	84.031	22.026
c06_s	interior sep.	Ø43.738±0.200	0.337	0.064	0.025	0.036	332	0.200	84.031	21.869
c07	interior	Ø44.874±2.105	4.175	1.456	0.490	0.627	332	0.200	76.021	22.437
c07_s	interior sep.	Ø44.666±0.189	0.375	0.084	0.036	0.052	324	0.200	76.021	22.333

Table 7: Detailed circularity measurements at selected samples in z-plane, vessel HV004.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		mm	mm	mm	mm	mm		mm	mm	mm
c01	exterior	Ø65.121±1.517	2.745	0.766	0.283	0.373	1350	0.200	64.210	32.561
c02	exterior	Ø81.307±0.495	0.795	0.140	0.051	0.095	873	0.200	53.902	40.654
c03	exterior	Ø87.694±0.395	0.698	0.185	0.087	0.103	730	0.200	40.434	43.847
c04	exterior	Ø87.470±0.506	0.999	0.211	0.089	0.121	727	0.200	26.104	43.735
c05	exterior	Ø79.055±0.472	0.887	0.191	0.084	0.119	858	0.200	11.775	39.527
c06	interior	Ø44.866±3.131	5.255	1.840	0.650	0.888	370	0.200	84.031	22.433
c06_s	interior sep.	Ø43.747±0.196	0.338	0.064	0.024	0.036	332	0.200	84.031	21.874
c07	interior	Ø44.470±2.323	4.206	1.450	0.465	0.671	321	0.200	76.021	22.235
c07_s	interior sep.	Ø44.696±0.205	0.377	0.085	0.036	0.055	326	0.200	76.021	22.348

Table 8: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel HV004.
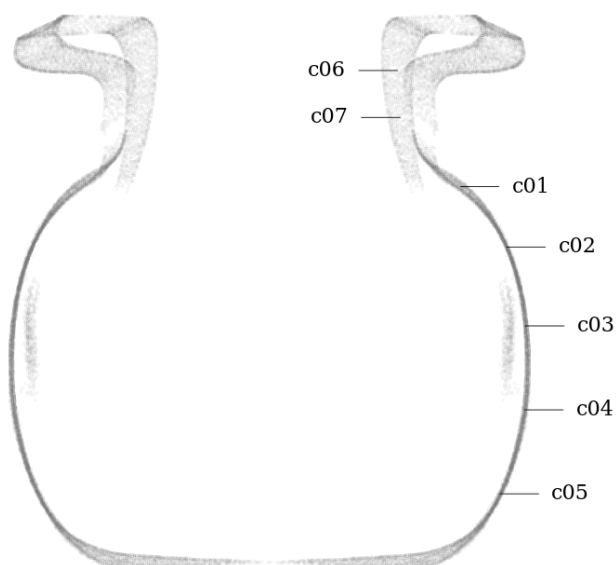
Figure 62: Circularity measurement sample locations,
full mesh aligned to exterior surface

Figure 63: Circularity measurement sample location,
separately aligned interior mesh

Samples perpendicular to the surface curvature

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø2.5770±0.0363	0.0626	0.0184	0.0064	0.0087	430	0.0079	2.5280	1.2885
c02	exterior	Ø3.2018±0.0171	0.0282	0.0051	0.0019	0.0035	764	0.0079	2.1221	1.6009
c03	exterior	Ø3.4564±0.0157	0.0283	0.0069	0.0023	0.0033	716	0.0079	1.5919	1.7282
c04	exterior	Ø3.4418±0.0200	0.0382	0.0081	0.0034	0.0046	706	0.0079	1.0277	1.7209
c05	exterior	Ø3.1123±0.0147	0.0283	0.0067	0.0026	0.0042	687	0.0079	0.4636	1.5561
c06	interior	Ø1.7343±0.1075	0.2071	0.0713	0.0247	0.0321	365	0.0079	3.3083	0.8672
c06_s	interior sep.	Ø1.7220±0.0079	0.0133	0.0025	0.0010	0.0014	332	0.0079	3.3083	0.8610
c07	interior	Ø1.7667±0.0829	0.1644	0.0573	0.0193	0.0247	332	0.0079	2.9930	0.8834
c07_s	interior sep.	Ø1.7585±0.0075	0.0148	0.0033	0.0014	0.0020	324	0.0079	2.9930	0.8792

Table 9: Detailed circularity measurements at selected samples in z-plane, vessel HV004.

Samples in the Z-plane

Tag	Area	Measured deviation[8]	Residuals				Sample size	Slice		
			Range	RMSD[9]	MAD[10]	SD		Height	Z coord.	Radius[11]
		in	in	in	in	in		in	in	in
c01	exterior	Ø2.5638±0.0597	0.1081	0.0301	0.0111	0.0147	1350	0.0079	2.5280	1.2819
c02	exterior	Ø3.2011±0.0195	0.0313	0.0055	0.0020	0.0038	873	0.0079	2.1221	1.6005
c03	exterior	Ø3.4525±0.0156	0.0275	0.0073	0.0034	0.0041	730	0.0079	1.5919	1.7263
c04	exterior	Ø3.4437±0.0199	0.0393	0.0083	0.0035	0.0048	727	0.0079	1.0277	1.7218
c05	exterior	Ø3.1124±0.0186	0.0349	0.0075	0.0033	0.0047	858	0.0079	0.4636	1.5562
c06	interior	Ø1.7664±0.1233	0.2069	0.0724	0.0256	0.0350	370	0.0079	3.3083	0.8832
c06_s	interior sep.	Ø1.7223±0.0077	0.0133	0.0025	0.0010	0.0014	332	0.0079	3.3083	0.8612
c07	interior	Ø1.7508±0.0915	0.1656	0.0571	0.0183	0.0264	321	0.0079	2.9930	0.8754
c07_s	interior sep.	Ø1.7597±0.0081	0.0149	0.0034	0.0014	0.0022	326	0.0079	2.9930	0.8798

Table 10: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel HV004.

Comparison of circularity on the full vessel surface

Metric

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.829	0.665	1.386	0.106	0.081	0.194	0.198	0.123	0.416	326	0.200
Interior	4.324	1.923	6.604	0.649	0.253	1.017	1.454	0.492	2.325	110	0.200
Interior separate	0.392	0.281	0.777	0.050	0.034	0.085	0.084	0.063	0.138	110	0.200

Table 11: Detailed circularity measurements at selected samples in z-plane, vessel HV004.

Samples in the z-plane

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	mm	mm	mm	mm	mm	mm	mm	mm	mm		mm
Exterior	0.908	0.672	3.552	0.118	0.088	0.532	0.209	0.138	0.917	324	0.200
Interior	4.553	2.849	6.748	0.722	0.437	1.189	1.541	0.938	2.374	109	0.200
Interior separate	0.395	0.286	0.778	0.054	0.035	0.094	0.085	0.062	0.145	109	0.200

Table 12: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel HV004.

Imperial

Samples perpendicular to the surface curvature

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.829	0.665	1.386	0.106	0.081	0.194	0.198	0.123	0.416	326	0.200
Interior	4.324	1.923	6.604	0.649	0.253	1.017	1.454	0.492	2.325	110	0.200
Interior separate	0.392	0.281	0.777	0.050	0.034	0.085	0.084	0.063	0.138	110	0.200

Table 13: Detailed circularity measurements at selected samples in z-plane, vessel HV004.

Samples in the z-plane

Area	Range			Standard Deviation			RMSD			Slices	Slice height
	Median	Min.	Max.	Median	Min.	Max.	Median	Min.	Max.		
	in	in	in	in	in	in	in	in	in		in
Exterior	0.908	0.672	3.552	0.118	0.088	0.532	0.209	0.138	0.917	324	0.200
Interior	4.553	2.849	6.748	0.722	0.437	1.189	1.541	0.938	2.374	109	0.200
Interior separate	0.395	0.286	0.778	0.054	0.035	0.094	0.085	0.062	0.145	109	0.200

Table 14: Detailed circularity measurements at selected samples perpendicular to vessel curvature, vessel HV004.

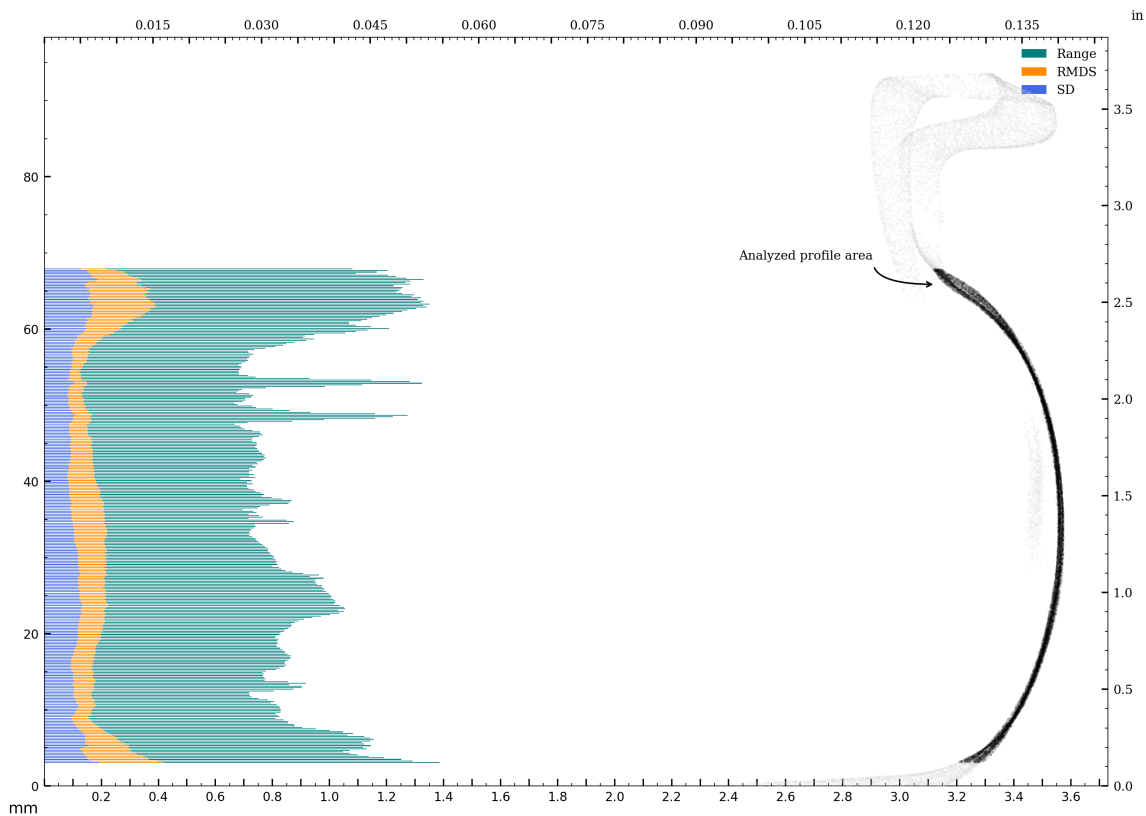## Circularity analysis of exterior surface - perpendicular to surface curvature

Figure 64: Circularity of exterior surface - perpendicular to surface curvature.

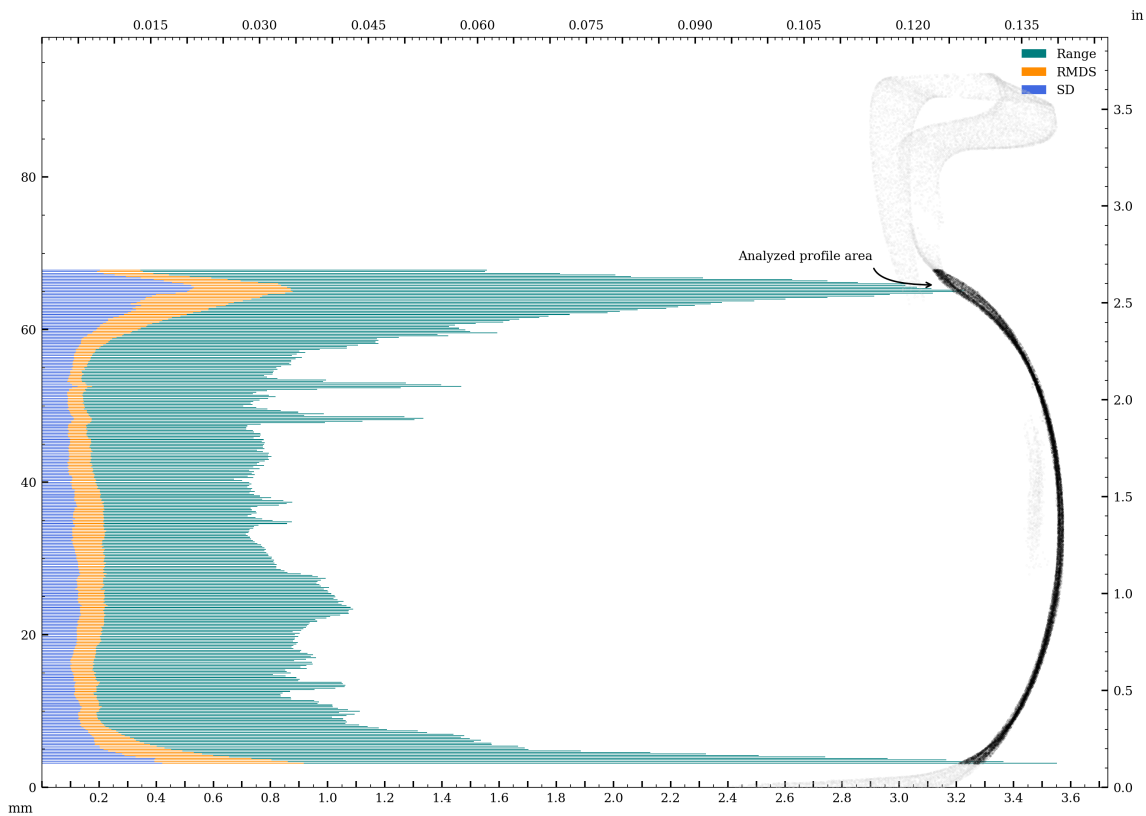## Circularity analysis of exterior surface - in z-plane

Figure 65: Circularity of exterior surface - in z-plane.

Circularity analysis of exterior surface, perpendicular to surface curvature, Standard Deviation and Root Mean Squared Deviation
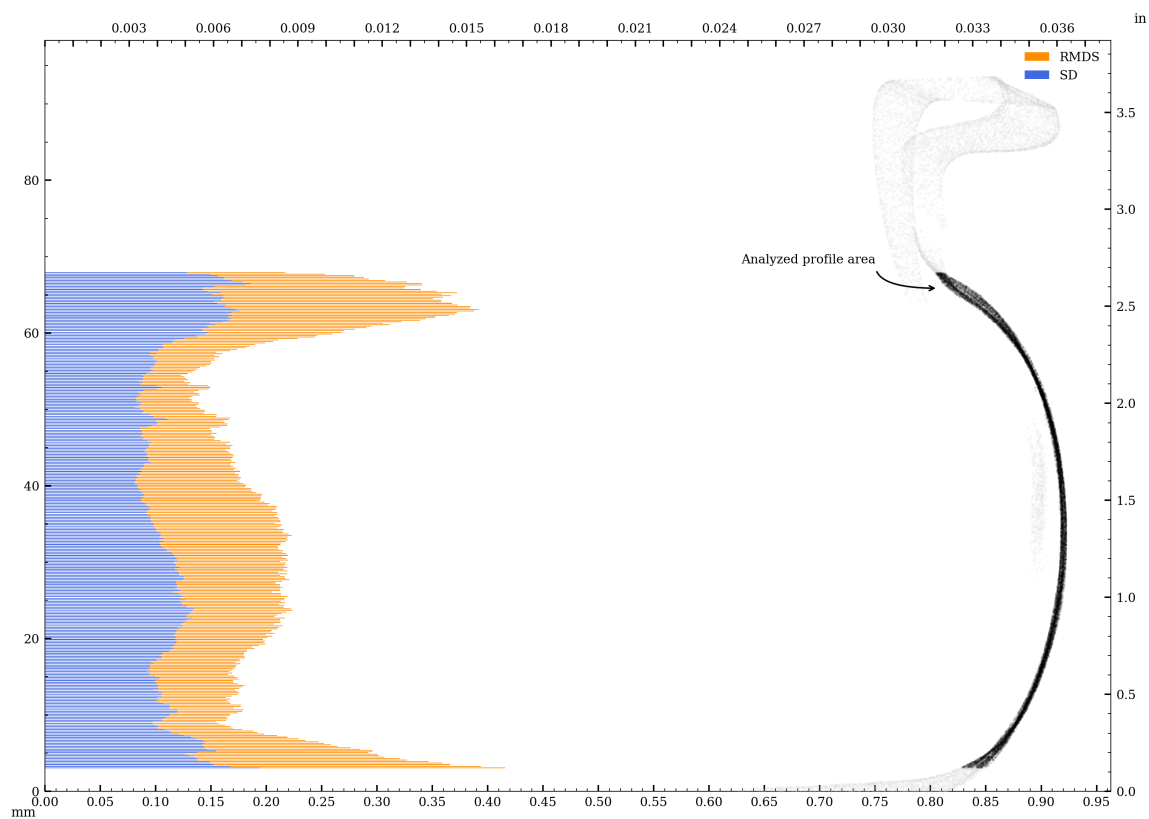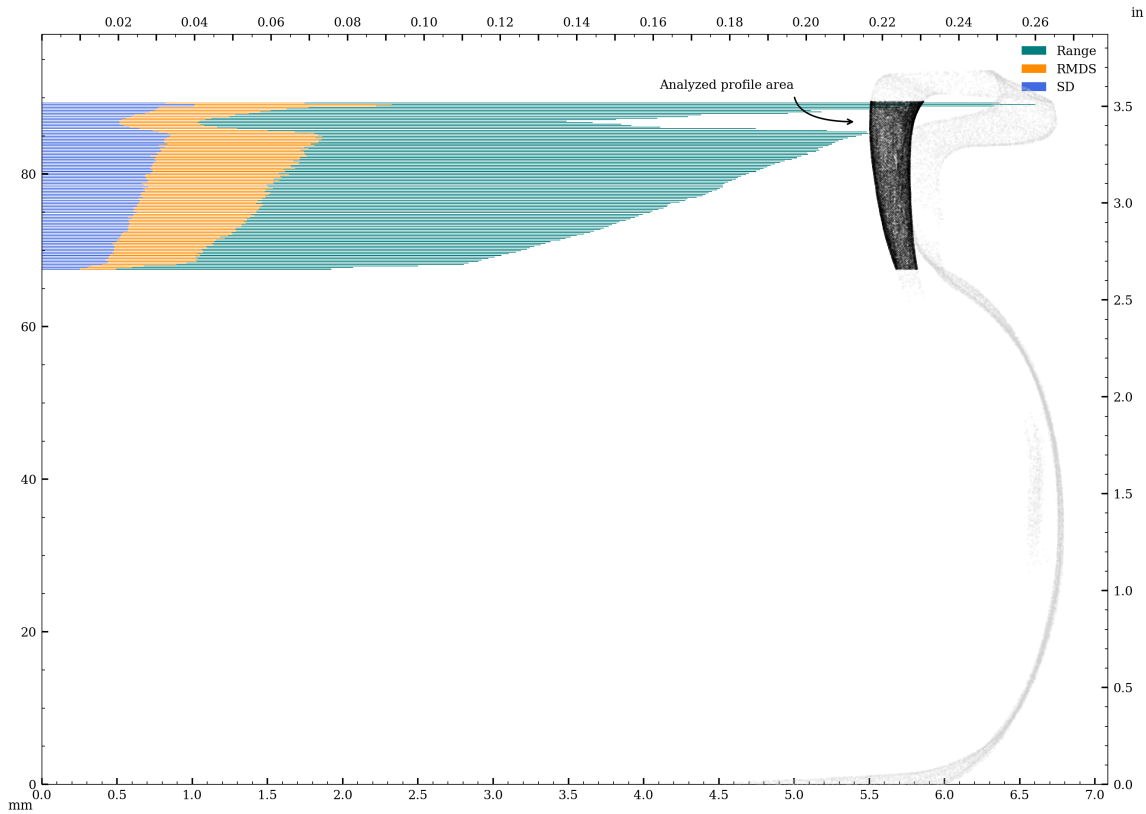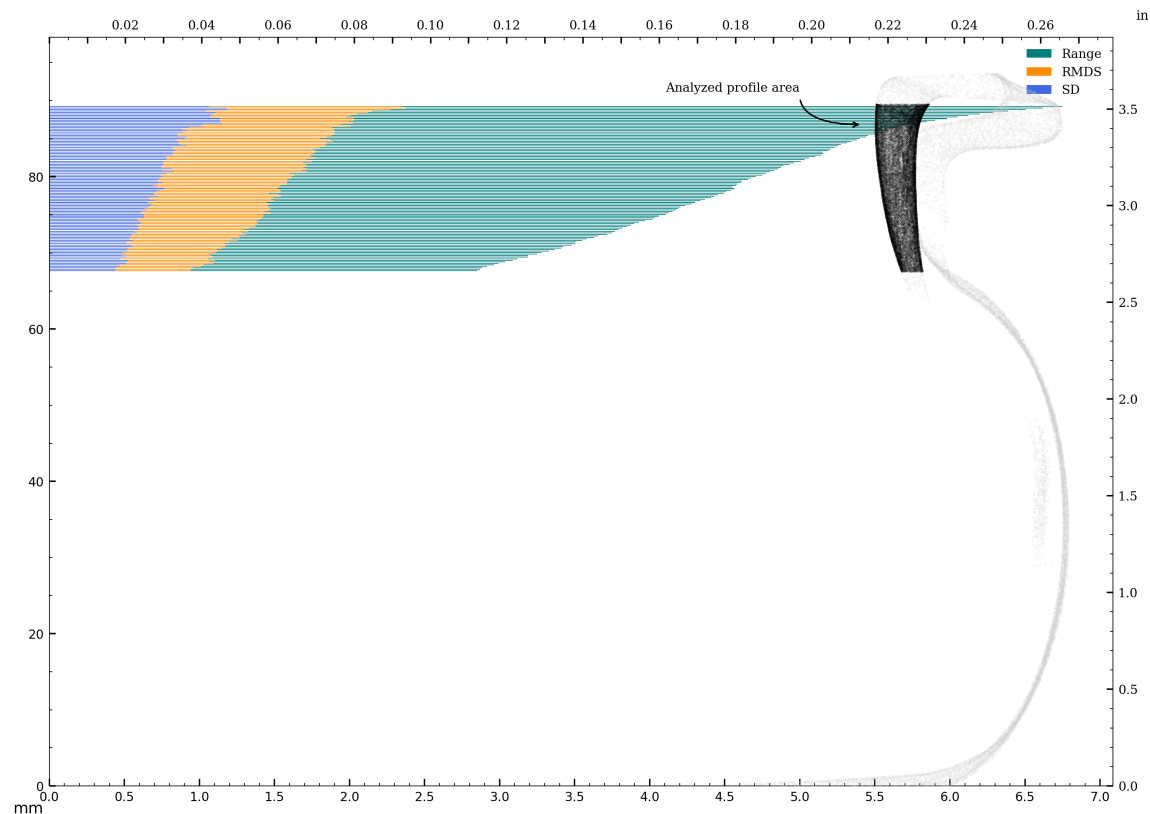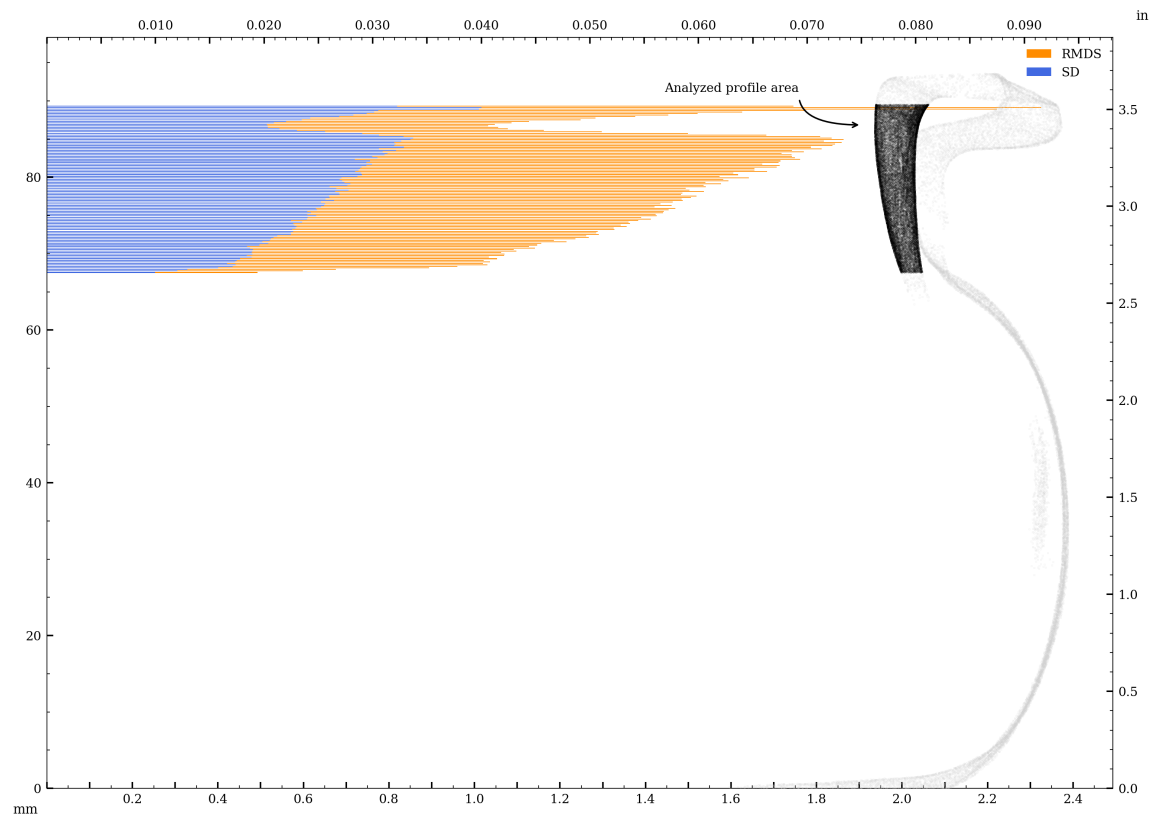

Figure 66: Vessel circularity of exterior surface, perpendicular to surface curvature, standard deviation and median absolute deviation.

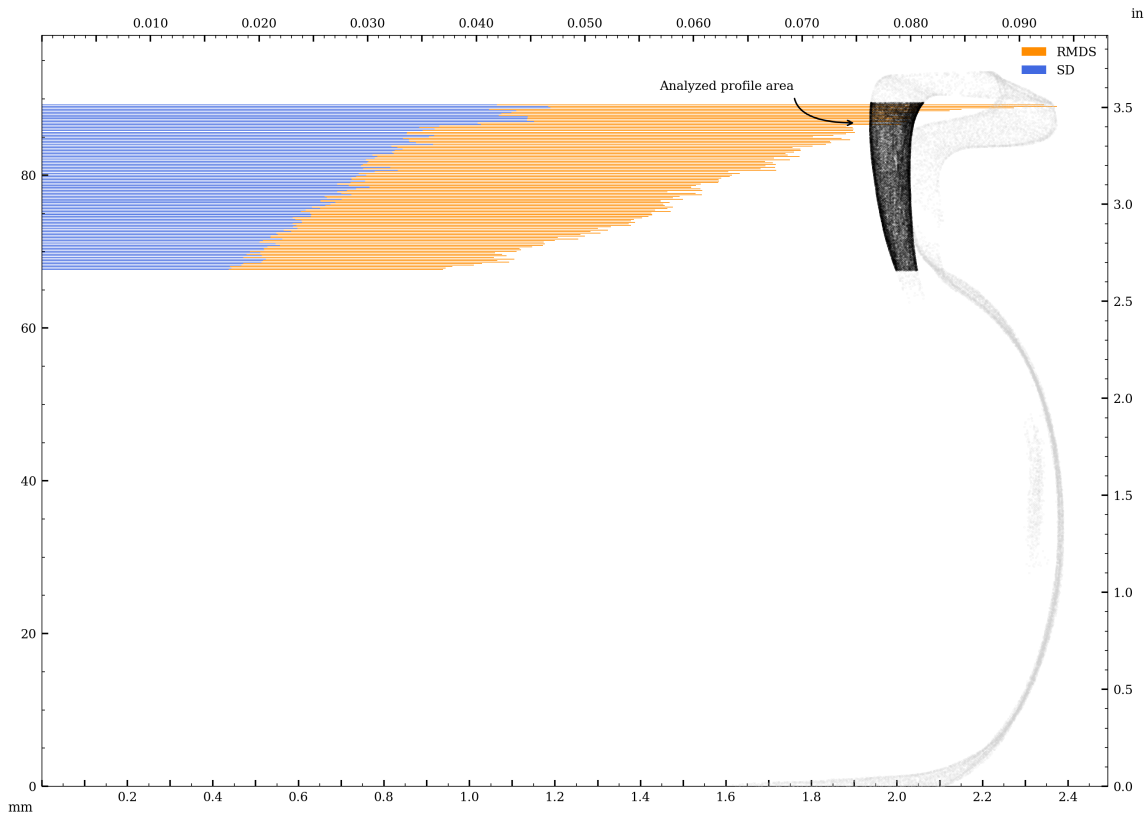## Circularity analysis of exterior surface, in z-plane, Standard Deviation and Root Mean Squared Deviation
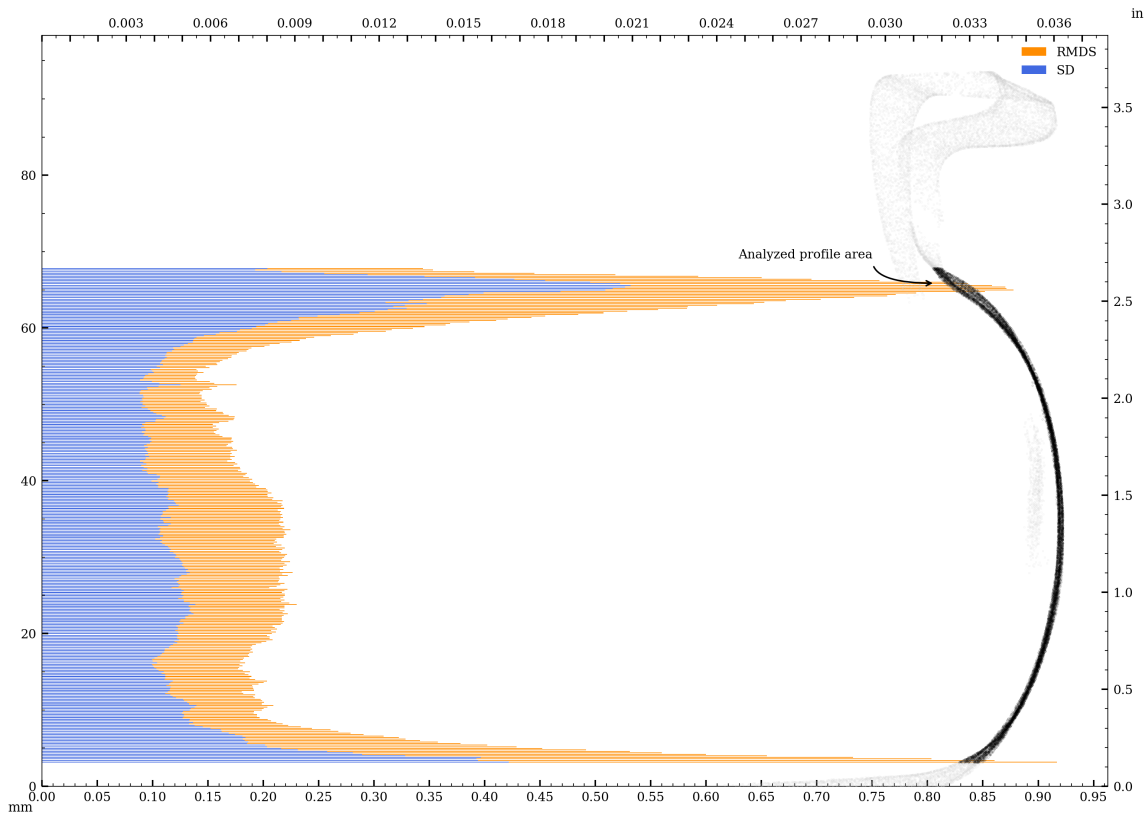
Figure 67: Vessel circularity of exterior surface, in z-plane, standard deviation and median absolute deviation.

Circularity analysis of interior surface - perpendicular to surface curvature

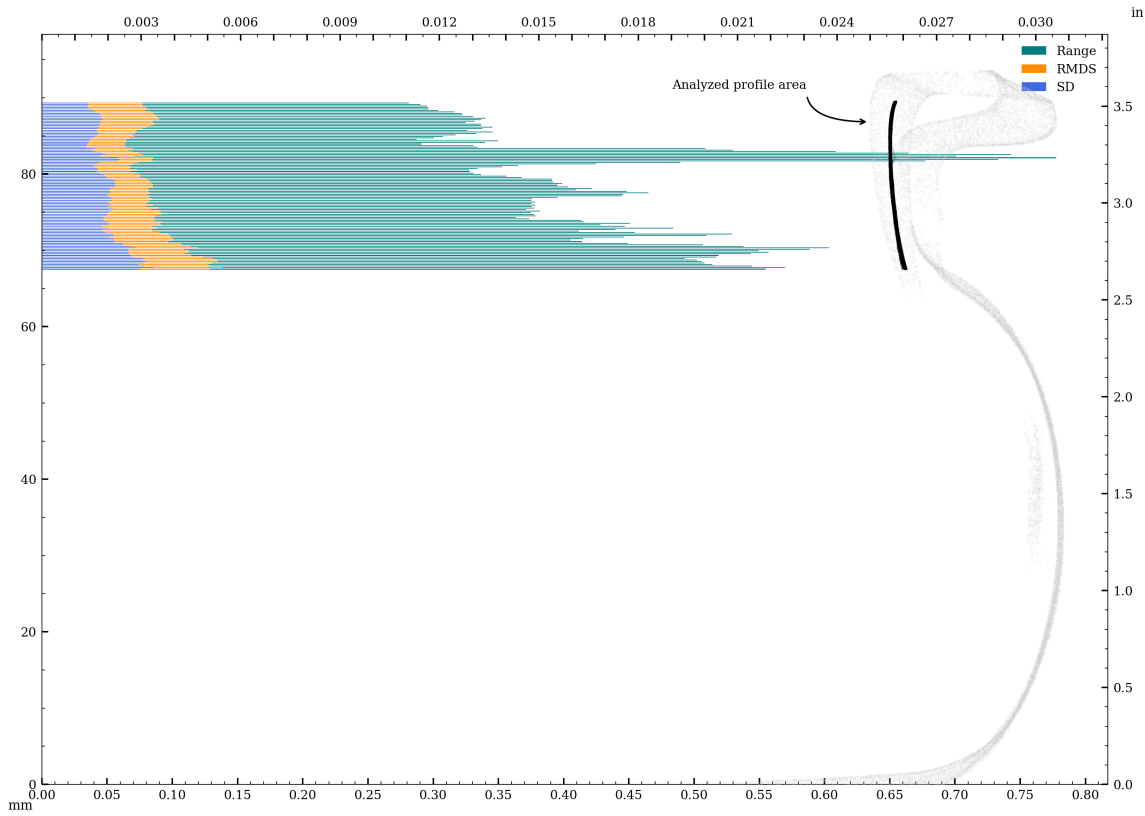Figure 68: Circularity of interior surface - perpendicular to surface curvature.

Circularity analysis of interior surface - in z-plane

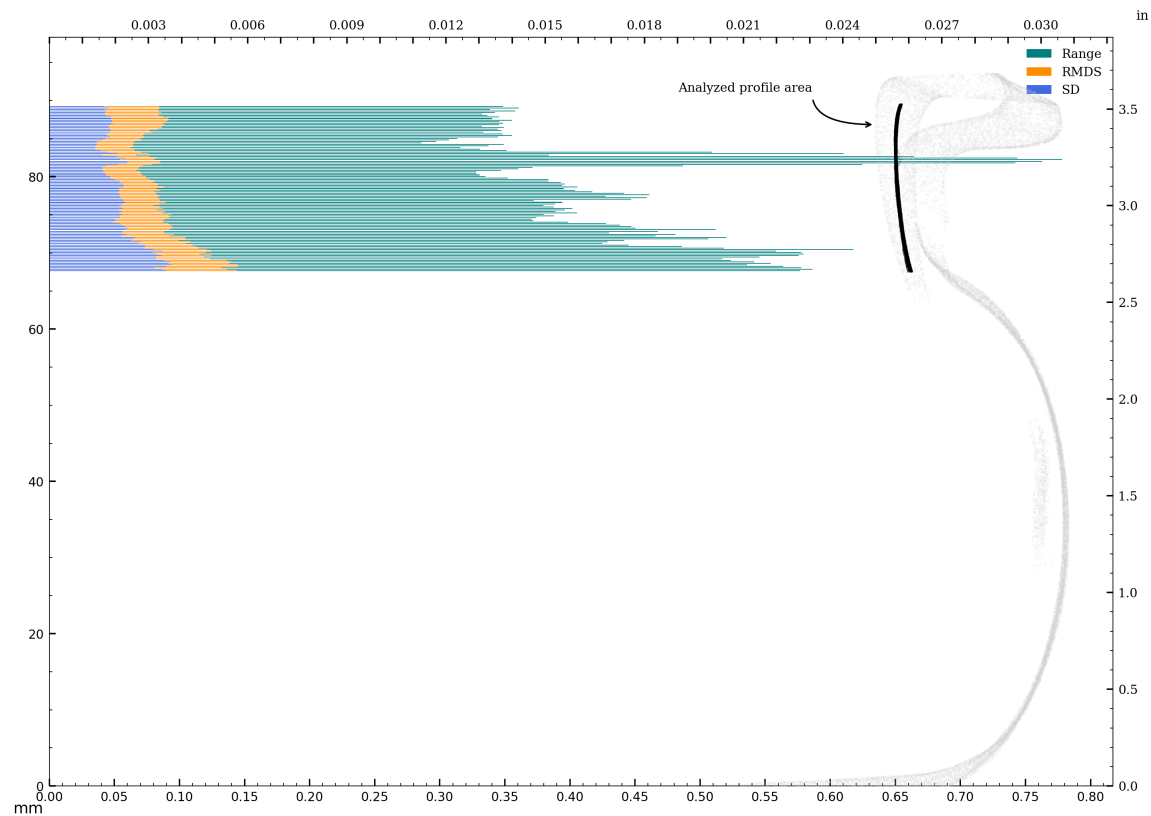Figure 69: Circularity of interior surface - in z-plane.

Circularity analysis of interior surface, perpendicular to surface curvature, Standard Deviation and Root Mean Squared Deviation
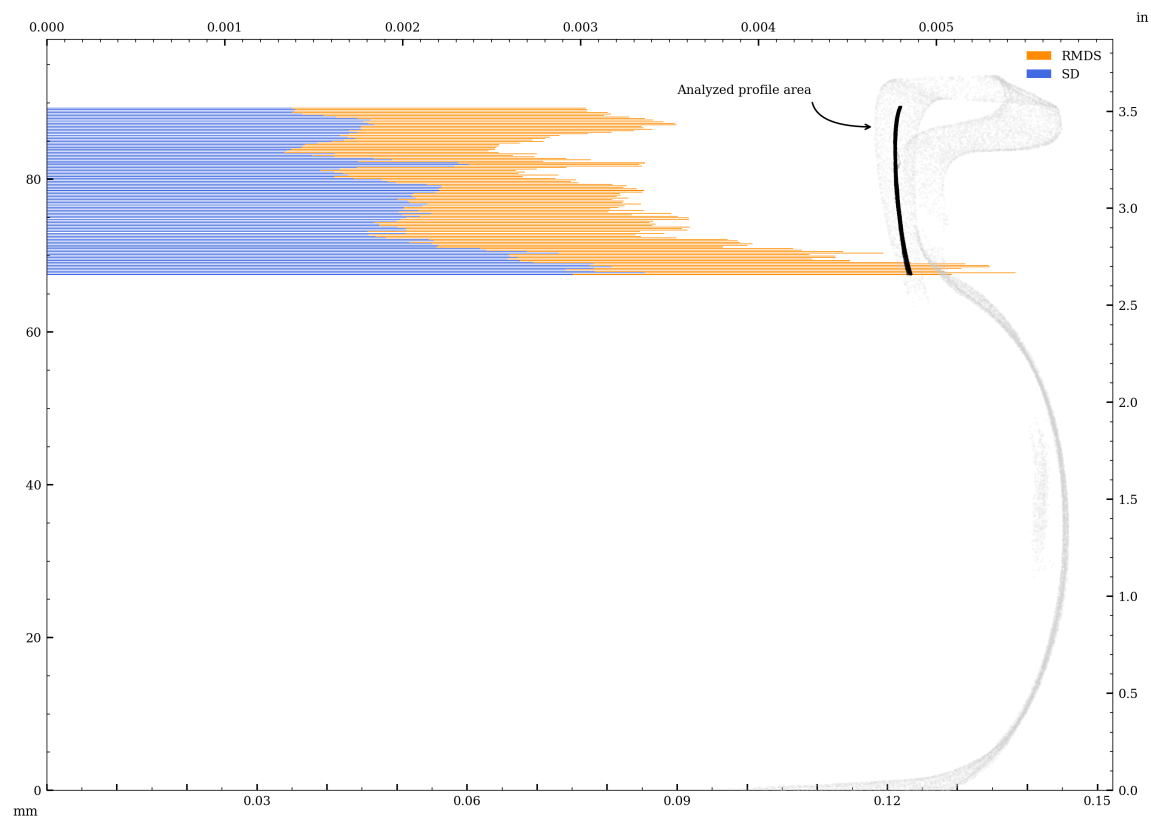

Figure 70: Vessel circularity of interior surface, perpendicular to surface curvature, standard deviation and median absolute deviation.

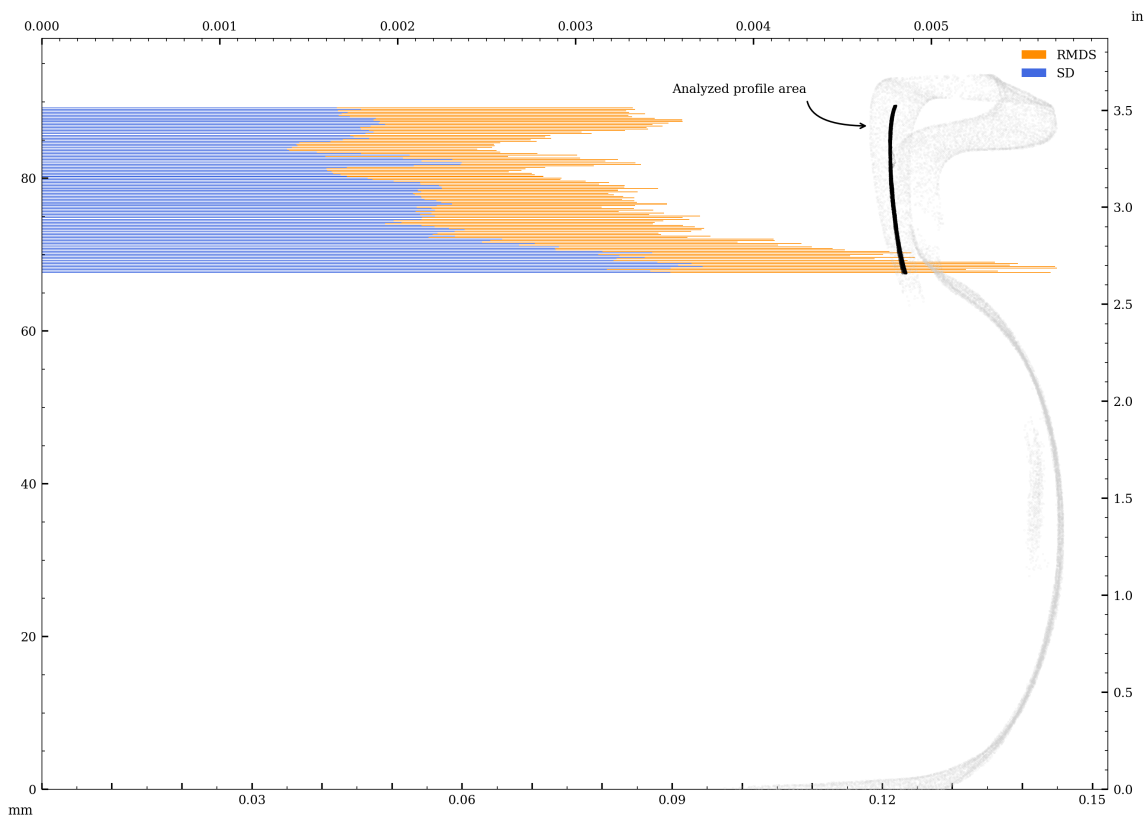## Circularity analysis of interior surface, in z-plane, Standard Deviation and Root Mean Squared Deviation

Figure 71: Vessel circularity of interior surface, in z-plane, standard deviation and median absolute deviation.

Circularity analysis of interior separately aligned surface - perpendicular to surface curvature

Figure 72: Circularity of interior_separate surface - perpendicular to surface curvature.

Circularity analysis of interior separately aligned surface - in z-plane

Figure 73: Circularity of interior_separate surface - in z-plane.

Circularity analysis of interior separately aligned surface, perpendicular to surface curvature, Standard Deviation and Root Mean Squared Deviation

Figure 74: Vessel circularity of interior_separate surface, perpendicular to surface curvature, standard deviation and median absolute deviation.

Circularity analysis of interior separately aligned surface, in z-plane, Standard Deviation and Root Mean Squared Deviation

Figure 75: Vessel circularity of interior_separate surface, in z-plane, standard deviation and median absolute deviation.

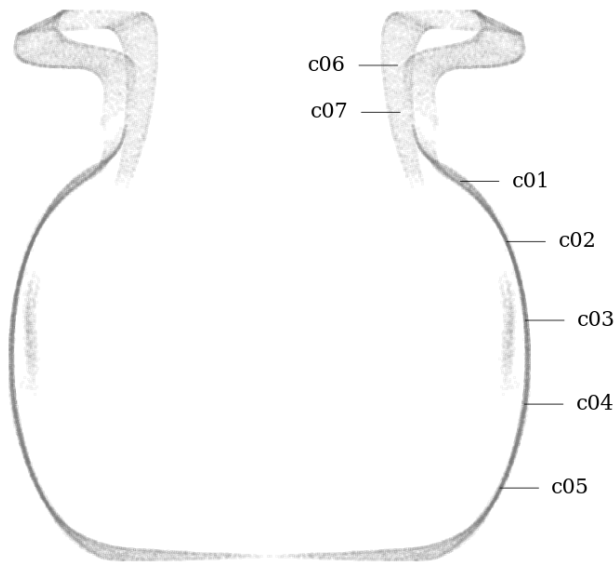# Appendix B - Comparison Of Concentricity Measurements (Z-plane vs. surface-perpendicular)

Figure 76: Circularity measurement sample locations, full mesh aligned to exterior surface

Figure 77: Circularity measurement sample location, separately aligned interior mesh

Concentricity measurements perpendicular to surface curvature

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	μm
c01	z-axis	1.012	1350	5.103	5.103	1.616	1.616	0.628	0.628	62, −1010
c02	z-axis	0.048	873	0.859	0.782	0.157	0.144	0.103	0.091	4, −48
c03	z-axis	0.078	730	0.908	0.908	0.209	0.209	0.111	0.111	7, 78
c04	z-axis	0.048	727	0.979	0.979	0.216	0.216	0.124	0.124	−39, −28
c05	z-axis	0.015	858	0.896	0.896	0.191	0.191	0.118	0.118	12, −9
c06	z-axis	2.559	370	11.775	11.775	4.213	4.213	1.786	1.786	552, 2499
c06_s	z-axis	0.015	332	0.344	0.309	0.067	0.066	0.041	0.040	−3, −15
c07	z-axis	2.051	321	9.438	9.438	3.333	3.333	1.482	1.482	522, 1984
c07_s	z-axis	0.037	326	0.405	0.405	0.096	0.096	0.059	0.059	−13, 35

Concentricity measurements in z-plane

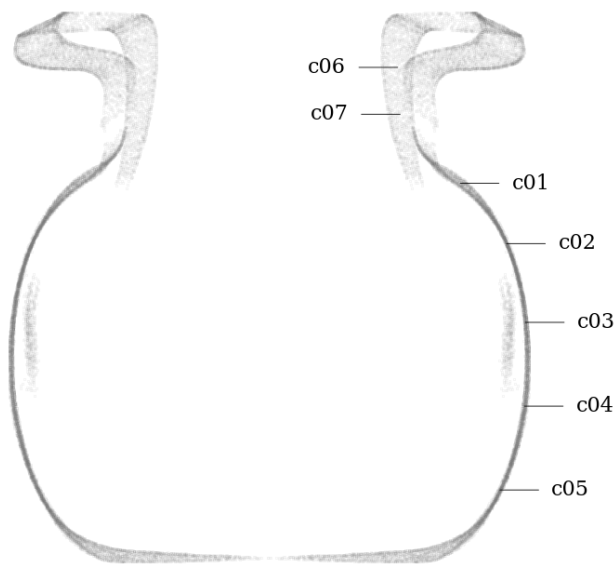| Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		mm		mm	mm	mm	mm	mm	mm	μm
c01	z-axis	1.012	1350	5.103	5.103	1.616	1.616	0.628	0.628	62, −1010
c02	z-axis	0.048	873	0.859	0.782	0.157	0.144	0.103	0.091	4, −48
c03	z-axis	0.078	730	0.908	0.908	0.209	0.209	0.111	0.111	7, 78
c04	z-axis	0.048	727	0.979	0.979	0.216	0.216	0.124	0.124	−39, −28
c05	z-axis	0.015	858	0.896	0.896	0.191	0.191	0.118	0.118	12, −9
c06	z-axis	2.559	370	11.775	11.775	4.213	4.213	1.786	1.786	552, 2499
c06_s	z-axis	0.015	332	0.344	0.309	0.067	0.066	0.041	0.040	−3, −15
c07	z-axis	2.051	321	9.438	9.438	3.333	3.333	1.482	1.482	522, 1984
c07_s	z-axis	0.037	326	0.405	0.405	0.096	0.096	0.059	0.059	−13, 35

Figure 78: Circularity measurement sample locations,
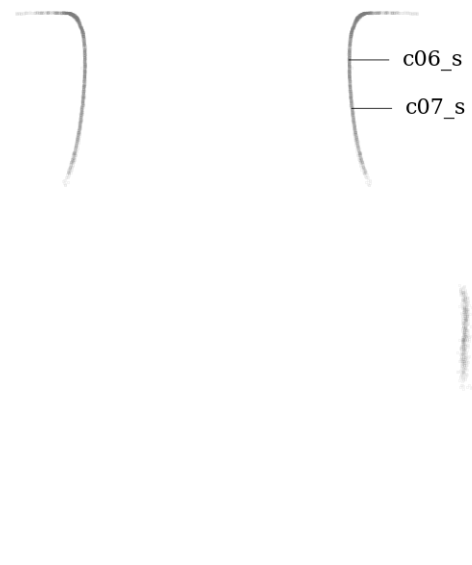full mesh aligned to exterior surface

Figure 79: Circularity measurement sample location,
separately aligned interior mesh

Concentricity measurements perpendicular to surface curvature

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0399	1350	0.2009	0.2009	0.0636	0.0636	0.0247	0.0247	2.4, −39.8
c02	z-axis	0.0019	873	0.0338	0.0308	0.0062	0.0057	0.0041	0.0036	0.2, −1.9
c03	z-axis	0.0031	730	0.0358	0.0358	0.0082	0.0082	0.0044	0.0044	0.3, 3.1
c04	z-axis	0.0019	727	0.0385	0.0385	0.0085	0.0085	0.0049	0.0049	−1.5, −1.1
c05	z-axis	0.0006	858	0.0353	0.0353	0.0075	0.0075	0.0046	0.0046	0.5, −0.4
c06	z-axis	0.1007	370	0.4636	0.4636	0.1659	0.1659	0.0703	0.0703	21.7, 98.4
c06_s	z-axis	0.0006	332	0.0135	0.0122	0.0027	0.0026	0.0016	0.0016	−0.1, −0.6
c07	z-axis	0.0808	321	0.3716	0.3716	0.1312	0.1312	0.0584	0.0584	20.6, 78.1
c07_s	z-axis	0.0015	326	0.0159	0.0159	0.0038	0.0038	0.0023	0.0023	−0.5, 1.4

Concentricity measurements in z-plane

Tag	Reference	Deviation	Sample size	Circle fit residuals analysis for sample listed in Tag column						
				Range full	Range inliers	RMSD full	RMDS inliers	SD full	SD inliers	Center (x,y)
		in		in	in	in	in	in	in	thou
c01	z-axis	0.0399	1350	0.2009	0.2009	0.0636	0.0636	0.0247	0.0247	2.4, −39.8
c02	z-axis	0.0019	873	0.0338	0.0308	0.0062	0.0057	0.0041	0.0036	0.2, −1.9
c03	z-axis	0.0031	730	0.0358	0.0358	0.0082	0.0082	0.0044	0.0044	0.3, 3.1
c04	z-axis	0.0019	727	0.0385	0.0385	0.0085	0.0085	0.0049	0.0049	−1.5, −1.1
c05	z-axis	0.0006	858	0.0353	0.0353	0.0075	0.0075	0.0046	0.0046	0.5, −0.4
c06	z-axis	0.1007	370	0.4636	0.4636	0.1659	0.1659	0.0703	0.0703	21.7, 98.4
c06_s	z-axis	0.0006	332	0.0135	0.0122	0.0027	0.0026	0.0016	0.0016	−0.1, −0.6
c07	z-axis	0.0808	321	0.3716	0.3716	0.1312	0.1312	0.0584	0.0584	20.6, 78.1
c07_s	z-axis	0.0015	326	0.0159	0.0159	0.0038	0.0038	0.0023	0.0023	−0.5, 1.4